A-level

MATHEMATICS

7357/1

Paper 1

Mark scheme

June 2021

Version: 1.0 Final Mark Scheme

216A7357/1/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1	Ticks the correct box	1.1b	B1	$\left\{x : x < -\frac{7}{2} \text{ or } x > 3\right\}$
	Question Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x}$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles correct answer	2.2a	R1	6
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4	Ticks the correct box	2.1	R1	There exists a non-zero rational and an irrational whose product is rational.
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Uses negative reciprocal to obtain an equation with the correct gradient.	1.1a	M1	$4y+3x=c$ $4\times2+3\times15=53$ $4y+3x=53$
	Obtains correct equation ACF ISW once ACF seen Eg $y=-\frac{3}{4}x+\frac{53}{4}$ $y-2=-\frac{3}{4}(x-15)$	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Begins to solve $3y-4x=21$ and their $4y+3x=53$ with elimination of one variable or better. Or obtains correct point of intersection for $3y-4x=21$ and their $4y+3x=53$	1.1a	M1	$3y-4x=21$ $4y+3x=53$ $y=11$ $x=3$ $(3-15)^2+(11-2)^2=12^2+9^2$ $=225$ $\text{Distance}=15$
	Uses distance formula to find the distance between (15, 2) and another point other than the origin. PI correct distance or square of correct distance	1.1a	M1	
	Uses distance formula for (15, 2) and their point of intersection to find distance or distance^2	3.1a	M1	
	Obtains 15 CAO	1.1b	A1	
	Subtotal		**4**	

	Question Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Obtains $a+8d=3$ OE	1.1b	B1	$a+8d=3$ $\frac{21}{2}(2a+20d)=42$ $a+10d=2$ $a=7$ $d=-0.5$
	Obtains $\frac{21}{2}(2a+20d)=42$ OE	1.1b	B1	
	Begins to solve their $a+8d=3$ $\frac{21}{2}(2a+20d)=42$ with elimination of one variable or better. For their equations condone only the following slips $a+9d=3$ $\frac{21}{2}(2a+20d)=21$ PI correct a and d	3.1a	M1	
	Obtains correct a and d	1.1b	A1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Obtains at least one correct (unsimplified) expression for S_n or T_n FT their non-zero values of a and d for S_n PI by simplified correct equation.	1.1b	B1F	$\frac{n}{2}(14-0.5(n-1))=\frac{n}{2}(-36+0.75(n-1))$ $n=0\ or\ 41$ Hence n = 41
	Equates their expressions S_n and T_n with at least one correct. FT their non-zero values of a and d for S_n And finds a non-zero value of n PI by n = 41	3.1a	M1	
	Deduces correct value of n = 41	2.2a	R1	
	Subtotal		**3**	

	Question Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Rearranges to the form $\mathrm{f}(x)=0$ and evaluates $\mathrm{f}(x)$ at least once in the interval [1.5,1.6]	1.1a	M1	$x^2 = x^3 + x - 3$ $\Rightarrow x^3 - x^2 + x - 3 = 0$ $\mathrm{f}(x) = x^3 - x^2 + x - 3$ $\mathrm{f}(1.5) = -0.375 < 0$ $\mathrm{f}(1.6) = 0.136 > 0$ Hence α lies between 1.5 and 1.6
	Completes argument with correct evaluation either side of root and reference to change of sign	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Isolates x^3 or divides by x and cancels terms Eg $x = x^2 + 1 - \frac{3}{x}$ OE Condone one slip in cancelling or one sign error	1.1a	M1	$x^2 = x^3 + x - 3$ $x^3 = x^2 - x + 3$ $x^2 = x - 1 + \frac{3}{x}$
	Completes argument to show the given result. Three terms need not be in the given order	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)	Obtains any one correct value to at least three decimal places, ignoring labels.	1.1a	M1	$x_2 = 1.5811$ $x_3 = 1.5743$ $x_4 = 1.5748$
	Obtains x_2, x_3 and x_4 correct to four decimal places or better If no labels only accept answers in clearly the correct order with no extras seen beyond x_4	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(d)	States an interval of the correct width, which includes 1.5743 and 1.5748. Condone strict inequalities. Condone correct inequality in words.	2.2a	R1	$1.574 \le \alpha \le 1.575$
	Subtotal		**1**	

	Question Total		**7**	

Q	Marking instructions	AO	Mark	Typical solution
8(a)	Recalls and uses $\sin 2\theta = 2\sin\theta\cos\theta$	1.2	B1	$9\sin^2\theta + \sin 2\theta = 8$ $9\sin^2\theta + 2\sin\theta\cos\theta = 8$ $9 + 2\cot\theta = 8\,\text{cosec}^2\,\theta$ $9 + 2\cot\theta = 8\left(\cot^2\theta + 1\right)$ $8\cot^2\theta - 2\cot\theta - 1 = 0$
	Uses $\cot^2\theta + 1 = \text{cosec}^2\,\theta$ Or $\tan^2\theta + 1 = \sec^2\theta$ Condone a sign error	1.1a	M1	
	Divides through by $\cos^2\theta$ or $\sin^2\theta$	1.1a	M1	
	Completes rearrangement to achieve given result. **AG**	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Mark	Typical solution
8(b)	Solves to give values of cot θ or tan θ PI by sight of 2 and – 4 or $-\frac{1}{4}$ and $\frac{1}{2}$ or by two correct answers	1.1a	M1	$\cot\theta = -\frac{1}{4}$ or $\cot\theta = \frac{1}{2}$ tan θ = – 4 or tan θ = 2 θ = 1.82 θ = 1.82 + π = 4.96 θ = 1.11 θ = 1.11 + π = 4.25
	Obtains two correct values of θ. Condone AWRT correct answers.	1.1b	A1	θ = 1.11, 1.82, 4.25, 4.96
	Obtains all four solutions with no additional solutions or errors. Ignore additional solutions outside the interval. AWRT 1.11, 1.82, 4.25, 4.96 CAO	1.1b	A1	
	Subtotal		**3**	

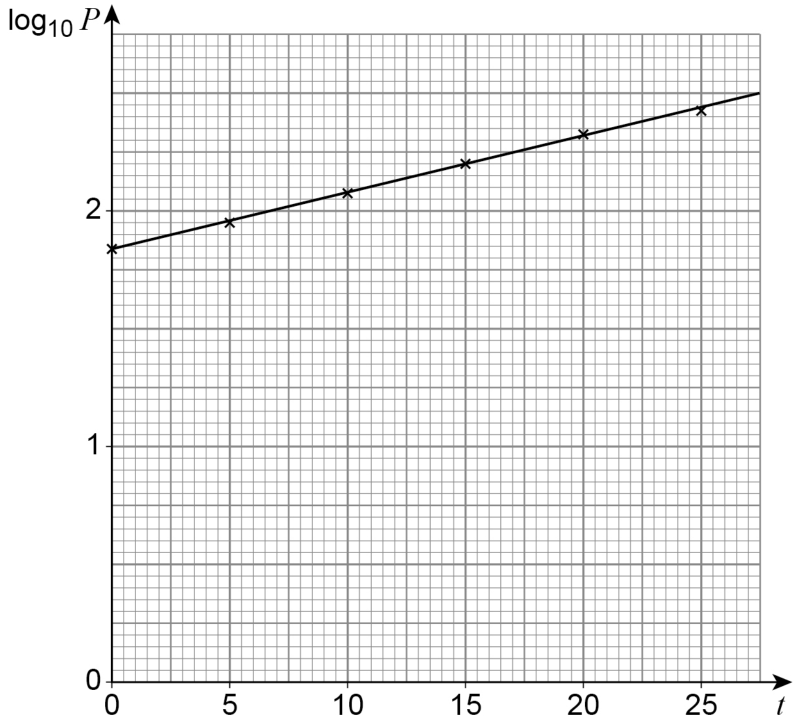
Q	Marking instructions	AO	Mark	Typical solution
8(c)	Sets $2x-\frac{\pi}{4}$ equal to at least one of their solutions. PI by a correct answer	3.1a	M1	$2x-\frac{\pi}{4}=1.107..,1.815..$ x = 0.9, 1.3
	Obtains correct AWRT values. Correct values should be rounded from 0.94627… and 1.300058… ISW once correct answers seen. CSO Condone extra values outside of the interval.	1.1b	A1	
	Subtotal		**2**	

	Question Total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Takes $\log_{10}$ of both sides to obtain $\log_{10} P = \log_{10}\left(A \times 10^{kt}\right)$ Or States that $A = 10^c$	1.1a	M1	$\log_{10} P = \log_{10}\left(A \times 10^{kt}\right)$ $\log_{10} P = \log_{10} A + \log_{10} 10^{kt}$ $\log_{10} P = \log_{10} A + kt$
	Obtains $\log_{10} P = \log_{10} A + \log_{10} 10^{kt}$ Or $P = 10^{kt+c}$	1.1b	A1	
	Completes rigorous argument to show $\log_{10} P = \log_{10} A + kt$ Or $\log_{10} P = kt + c$	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(i)	Completes table.	1.1b	B1	see table below
	Subtotal		**1**	

t	0	5	10	15	20	25
$\log_{10} P$	1.88	1.97	2.08	2.19	2.31	2.41

Q	Marking instructions	AO	Marks	Typical solution
9(b)(ii)	Plots at least four points correctly. Allow +/- one small square	1.1a	M1	
	Draws a ruled line of best fit from t=0 to t=25 or better CSO	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)(i)	Calculates the gradient of the graph either using the line of best fit or two points from the table of values	1.1a	M1	$k=\dfrac{2.41-1.88}{25}$ $=0.0212$ ≈ 0.02
	Obtains a value of k which rounds to 0.02	1.1b	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(c)(ii)	Infers the value of A Uses 75 from data or uses $10^{their\ intercept}$	2.2b	B1F	A=75
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(d)	Substitutes $t=50$ into their model of the form $P=A\times10^{0.02t}$ PI by 750	3.4	M1	$P=75\times10^{0.02\times50}$ 750 million tonnes
	Obtains the value for the number of tonnes of annual global production of plastics. Follow through their 70 < A <90	3.2a	A1F	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
9(e)	Forms an equation or inequality using their model of the form $P=A\times10^{0.02t}$ and $P=8000$	3.4	M1	$8000=75\times10^{0.02\times t}$ $t=101.401$ 2082
	Obtains t=101.4 AWFW [97.44,102.90]	1.1b	A1F	
	Interprets their answer as a year by calculating their (integer part of t)+1980+1, provided their t>50	3.2a	A1F	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
9(f)	Gives a **reason in context** why the model for the **production** of plastics will be inappropriate. Eg It is not appropriate to extrapolate the future global production of plastics from the date provided. The global production of plastics may decrease in the future.	3.5b	E1	The world will produce less plastics to be more environmentally friendly.
	Subtotal		**1**	

	Question Total		**15**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)	Recalls $\tan x = \frac{\sin x}{\cos x}$	1.2	B1	$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$ $= \frac{\cos x \cos x - (-\sin x)\sin x}{\cos^2 x}$ $= \frac{\sin^2 x + \cos^2 x}{\cos^2 x}$ $= \frac{1}{\cos^2 x}$ $= \sec^2 x$
	Uses the correct quotient rule. Condone sign error in differentiation of sin or cosine.	1.1a	M1	
	Completes rigorous argument to show the required result. Use of $\sin^2 x + \cos^2 x = 1$ or $\tan^2 x + 1 = \sec^2 x$ must be explicit. Must include $\frac{d}{dx}(\tan x) = \ldots$ or $\frac{dy}{dx} = \ldots$	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)	Writes down an integral of the form $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x\,dx$, $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (1 - \tan^2 x)\,dx$ Condone missing or incorrect limits and missing dx	3.1a	M1	Area under curve $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x\,dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x - 1\,dx$ $= [\tan x - x]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$ $= \left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) - \left(\tan\left(-\frac{\pi}{4}\right) - -\frac{\pi}{4}\right)$ $= 1 - \frac{\pi}{4} + 1 - \frac{\pi}{4}$ $= 2 - \frac{\pi}{2}$ Area of shaded region $\frac{\pi}{2} - \left(2 - \frac{\pi}{2}\right)$ $= \pi - 2$
	Uses $\tan^2 x + 1 = \sec^2 x$ to write integrand in a form which can be integrated, condone sign error.	3.1a	M1	
	Integrates their expression of the form $A\sec^2 x + B$	1.1b	A1F	
	Forms an expression for or evaluates the area of the relevant rectangle. $2\frac{\pi}{4}\tan^2\frac{\pi}{4}$ or $\frac{\pi}{4}\tan^2\frac{\pi}{4}$ Could be implicit within their integral	1.1b	B1	
	Completes rigorous argument to show the required result. Substitution of consistent limits should be explicit and no slips in algebra. Use of dx is required. AG	2.1	R1	
	Subtotal		**5**	

	Question Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
11	Separates variables. To obtain an equation of the form $\int \frac{A}{y^2}\mathrm{d}y = \int Bx^2\mathrm{d}x$ Or $\int \frac{A}{y^2}\frac{\mathrm{d}y}{\mathrm{d}x}\mathrm{d}x = \int Bx^2\mathrm{d}x$ Must have integral signs, and consistent dy and dx Condone x instead of x^2	3.1a	M1	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6}(xy)^2$ $\frac{1}{y^2}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6}x^2$ $\int \frac{1}{y^2}\mathrm{d}y = \int \frac{1}{6}x^2\mathrm{d}x$ $-y^{-1} = \frac{x^3}{18} + c$ $(1,6) \Rightarrow -\frac{1}{6} = \frac{1}{18} + c$ $\Rightarrow c = -\frac{2}{9}$
	Integrates one of their integrals of the form above correctly.	1.1a	M1	
	Obtains correct integrated equation. Condone missing +c	1.1b	A1	y cannot equal zero in
	Substitutes $(1,6)$ to determine their constant of integration.	1.1a	M1	$-y^{-1} = \frac{x^3}{18} - \frac{2}{9}$
	Explains why y cannot equal zero follow through their equation in which y cannot be zero.	2.4	E1F	as y^{-1} is undefined at $y = 0$ therefore C does not intersect the x-axis
	Substitutes $x = 0$ into their integrated equation to obtain the y-intercept.	3.1a	M1	
	Obtains y = 4.5 Correctly deduces and states that that C intersects the axes at (exactly/only) one point. Also, states the coordinate $(0,4.5)$ Also, must have stated that C does not intersect the x-axis CSO	1.1b 2.2a	A1 R1	$x = 0 \Rightarrow -y^{-1} = -\frac{2}{9} \Rightarrow y = \frac{9}{2}$ Hence the curve crosses the y-axis at $(0,4.5)$
	Total		**8**	

Q	Marking instructions	AO	Mark	Typical solution
12(a)	Substitutes y = 0 to form an equation for x PI x = 4	3.1a	M1	$(x+y)^2 = 4y+2x+8$ $y=0 \Rightarrow x^2 = 8+2x$ $\Rightarrow x = 4 \text{ or } -2$ $x = 4$ at P $x^2+2xy+y^2 = 4y+2x+8$ $2x+2y+2x\frac{dy}{dx}+2y\frac{dy}{dx} = 4\frac{dy}{dx}+2$ $x=4, y=0$ $\Rightarrow 8+8\frac{dy}{dx} = 4\frac{dy}{dx}+2$ $4\frac{dy}{dx} = -6$ $\Rightarrow \frac{dy}{dx} = -\frac{3}{2}$
	Obtains x = 4 ignore any other value.	1.1b	A1	
	Expands and uses product rule to obtain the derivative of their Axy term. or Uses chain rule to obtain $2(x+y)\left(1+\frac{dy}{dx}\right)$ Condone missing brackets.	3.1a	M1	
	Uses implicit differentiation correctly to obtain the derivative of $4y$ or y^2.	1.1b	B1	
	Obtains correct equation from correct differentiation.	1.1b	A1	
	Substitutes x = 4 and y = 0 into $\frac{dy}{dx} = \frac{2-2x-2y}{2x+2y-4}$ OE and obtains $-\frac{3}{2}$ If substituting into an earlier equation must reach $\frac{dy}{dx} = -\frac{3}{2}$ (AG)	2.1	R1	
	Subtotal		**6**	

Q	Marking instructions	AO	Mark	Typical solution
12(b)	Uses $\frac{2}{3}$ and y = 0 and their x = 4 from part (a) to form equation of line.	1.1a	M1	$y = \frac{2}{3}(x-4)$ $2x-3y = 8$
	Obtains their equation in correct form.	1.1b	A1F	
	Subtotal		**2**	

	Question Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(a)	Substitutes $x=-\frac{1}{5}$ into $125x^3+150x^2+55x+6$ and obtains zero. Must see $-\frac{1}{5}$ bracketed correctly in the cubed and squared term with a multiplication sign if missing brackets in the 55x term or a further step to indicate correct evaluation eg -1 + 6 -11 +6 = 0	1.1a	M1	$125\left(-\frac{1}{5}\right)^3+150\left(-\frac{1}{5}\right)^2+55\left(-\frac{1}{5}\right)+6=0$ Since $P\left(-\frac{1}{5}\right)=0$ $(5x+1)$ must be a factor of $P(x)$
	Completes factor theorem argument to show that $(5x+1)$ is a factor of $125x^3+150x^2+55x+6$ Statement can come first but must be in the right direction AND be accompanied by the evaluation ie $P\left(-\frac{1}{5}\right)=0 \Rightarrow (5x+1)$ is a factor of $P(x)$ Accept $P\left(-\frac{1}{5}\right)=$ in front of evaluation. Not $(5x+1)$ is a factor $\Rightarrow P\left(-\frac{1}{5}\right)=0$	2.1	R1	
	Subtotal		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(b)	Obtains quadratic factor of the form $25x^2+bx+6$, or states other roots. PI by correct answer	1.1a	M1	$(5x+1)(5x+2)(5x+3)$
	Obtains second linear factor. Condone $(x+0.4)$ or $(x+0.6)$ OE PI by correct answer.	1.1a	M1	
	Obtains $(5x+1)(5x+2)(5x+3)$ OE	1.1b	A1	
	Subtotal		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(c)	Deduces $250n^3+300n^2+110n+12$ $=2(5n+1)(5n+2)(5n+3)$ FT their three factors from part (b) Condone use of a different letter to n	2.2a	M1	$250n^3+300n^2+110n+12$ $=2(5n+1)(5n+2)(5n+3)$ $(5n+1),(5n+2)$ and $(5n+3)$ are three consecutive whole numbers. The three algebraic factors must contain a multiple of 3 and must also contain a multiple of 2 and the extra 2 gives $2\times2\times3=12$ therefore $250n^3+300n^2+110n+12$ is a multiple of 12.
	Explains that the factors contain three consecutive (positive whole) numbers. Must have their three factors in a form which give consecutive positive whole numbers.	2.4	R1	
	Completes reasoned argument to show $250n^3+300n^2+110n+12$ is a multiple of 12. Reasons that the three algebraic factors must contain a multiple of 3 and must also contain a multiple of 2 and the extra 2 gives $2\times2\times3=12$ therefore $250n^3+300n^2+110n+12$ is a multiple of 12. Condone conclusion about $2(5n+1)(5n+2)(5n+3)$	2.4	R1	
	Subtotal		**3**	

	Question Total		**8**	

Q	Marking instructions	AO	Mark	Typical solution
14(a)	Uses y = 0 to obtain a non-zero value of t	3.1a	M1	$y=0\Rightarrow 4t^2-t^3=0$ $t=0\ or\ 4$ $\frac{dy}{dt}=8t-3t^2$ $\frac{dx}{dt}=2t+1$ $t=4\Rightarrow\frac{dy}{dx}=-\frac{16}{9}$
	Obtains $\frac{dy}{dt}=8t-3t^2$ or $\frac{dy}{dt}=-16$	1.1b	B1	
	Obtains $\frac{dx}{dt}=2t+1$ or $\frac{dx}{dt}=9$	1.1b	B1	
	Uses their $\frac{dy}{dt}\div$ their $\frac{dx}{dt}=\frac{dy}{dx}$ and their non-zero value of t to find a numerical expression or value for $\frac{dy}{dx}$	3.1a	M1	
	Obtains $-\frac{16}{9}$ OE	1.1b	A1	
	Subtotal		**5**	

Q	Marking instructions	AO	Mark	Typical solution
14(b)(i)	Deduces b = 20 (FT t^2+t for their value of t)	2.2a	B1F	b = 20
	Subtotal		**1**	

Q	Marking instructions	AO	Mark	Typical solution
14(b)(ii)	Substitutes their $dx=(2t+1)dt$	3.1a	M1	$\frac{dx}{dt}=2t+1\Rightarrow dx=(2t+1)dt$ $A=\int_0^{20}y\,dx$ $=\int_0^4(4t^2-t^3)(2t+1)\,dt$ $=\int_0^4 8t^3+4t^2-2t^4-t^3dt$ $=\int_0^4 4t^2+7t^3-2t^4dt$
	Completes correct substitution for y and dx Condone incorrect or omission of limits.	1.1b	A1F	
	Completes rigorous argument, to show given result. t = 4 when x = 20 must be justified either here or in part (b)(i)	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Mark	Typical solution
14(b)(iii)	Evaluates A = 1856/15 or AWRT 124	1.1b	B1	$A = \frac{1856}{15}$
	Subtotal		**1**	

	Question Total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
15(a)	Uses small angle approximation for sine at least once.	1.1b	B1	$\sin x - \sin x \cos 2x \approx x - x\left(1 - \frac{(2x)^2}{2}\right)$ $\approx x - x + x\frac{4x^2}{2}$ $\approx 2x^3$
	Replaces $\cos 2x$ with $1 - \frac{(2x)^2}{2}$ Or Used double angle identity and small angle approximations Condone a sign error or missing brackets.	3.1a	M1	
	Completes rigorous argument to show the given result. Condone "=" instead of "≈"	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
15(b)	Forms an integral of the form $\int_0^{0.25} y\,\mathrm{d}x$ or better where y is their $\sqrt{8 \times 2x^3}$. Condone missing limits and dx.	3.1a	M1	$Area \approx \int_0^{0.25} \sqrt{8 \times 2x^3}\,\mathrm{d}x$ $= 4\int_0^{0.25} x^{3/2}\,\mathrm{d}x$ $= 4\left[\frac{2x^{5/2}}{5}\right]_0^{0.25}$ $= \frac{8}{5} \times 0.25^{5/2}$ $= \frac{8}{5} \times \left(\frac{1}{2}\right)^5$ $= 2^{-2} \times 5^{-1}$
	Simplifies integrand to $Bx^{3/2}$	1.1a	M1	
	Integrates their integrand of the form $Bx^{3/2}$ correctly	1.1b	A1F	
	Substitutes correct limits and completes argument to obtain correct approximation in correct form.	2.1	R1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
15(c)(i)	Explains that the limits or 6.4 and 6.3 are not small.	2.4	E1	The approximation is only valid for small values of x and 6.3 and 6.4 are not small.
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
15(c)(ii)	Explains how the limits can be changed. Examples of reasoning could include: $\sin x - \sin x \cos 2x$ is periodic OE (has a period of 2π PI) evaluating the integral over a different interval will result in the same value. Reduce/shift the limits by 2π The graph repeats Uses a substitution to bring the limits within an acceptable interval.	2.4	E1	$\sin x - \sin x \cos 2x$ repeats so evaluate the integral over a different interval. Use small values $a = 6.3 - 2\pi$ and $b = 6.4 - 2\pi$ to obtain a valid approximation.
	Deduces $a = 6.3 - 2\pi$ = AWRT 0.017 and $b = 6.4 - 2\pi$ = AWRT 0.117	2.2a	R1	
	Subtotal		**2**	

	Question Total		**10**	