



Oxford Cambridge and RSA

# A Level Mathematics B (MEI)

## H640/01 Pure Mathematics and Mechanics Question Paper

**Wednesday 6 June 2018 – Morning**

**Time allowed: 2 hours**



**You must have:**

- Printed Answer Booklet

**You may use:**

- a scientific or graphical calculator

### INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

### INFORMATION

- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets [ ].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

## Formulae A Level Mathematics B (MEI) (H640)

**Arithmetic series**

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

**Geometric series**

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

**Binomial series**

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

**Differentiation**

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient Rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

**Differentiation from first principles**

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

**Integration**

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

**Small angle approximations**

$\sin \theta \approx \theta$ ,  $\cos \theta \approx 1 - \frac{1}{2}\theta^2$ ,  $\tan \theta \approx \theta$  where  $\theta$  is measured in radians

**Trigonometric identities**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

**Numerical methods**

Trapezium rule:  $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$ , where  $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

**Probability**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**Sample variance**

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation,  $s = \sqrt{\text{variance}}$

**The binomial distribution**

If  $X \sim B(n, p)$  then  $P(X = r) = {}^nC_r p^r q^{n-r}$  where  $q = 1 - p$

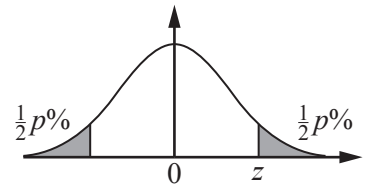
Mean of  $X$  is  $np$

**Hypothesis testing for the mean of a Normal distribution**

If  $X \sim N(\mu, \sigma^2)$  then  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$  and  $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

**Percentage points of the Normal distribution**

$p$	10	5	2	1
$z$	1.645	1.960	2.326	2.576

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Answer **all** the questions

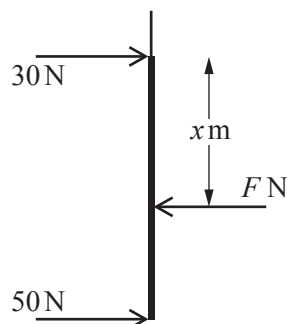
**Section A** (23 marks)

- 1 Show that  $(x-2)$  is a factor of  $3x^3 - 8x^2 + 3x + 2$ . [3]
- 2 By considering a change of sign, show that the equation  $e^x - 5x^3 = 0$  has a root between 0 and 1. [2]
- 3 **In this question you must show detailed reasoning.**  
Solve the equation  $\sec^2 \theta + 2 \tan \theta = 4$  for  $0^\circ \leq \theta < 360^\circ$ . [4]
- 4 Rory pushes a box of mass 2.8 kg across a rough horizontal floor against a resistance of 19 N. Rory applies a constant horizontal force. The box accelerates from rest to  $1.2 \text{ m s}^{-1}$  as it travels 1.8 m.  
(i) Calculate the acceleration of the box. [2]  
(ii) Find the magnitude of the force that Rory applies. [2]
- 5 The position vector  $\mathbf{r}$  metres of a particle at time  $t$  seconds is given by  
$$\mathbf{r} = (1 + 12t - 2t^2)\mathbf{i} + (t^2 - 6t)\mathbf{j}.$$
  
(i) Find an expression for the velocity of the particle at time  $t$ . [2]  
(ii) Determine whether the particle is ever stationary. [2]
- 6 Aleela and Baraka are saving to buy a car. Aleela saves £50 in the first month. She increases the amount she saves by £20 each month.  
(i) Calculate how much she saves in two years. [2]  
Baraka also saves £50 in the first month. The amount he saves each month is 12% more than the amount he saved in the previous month.  
(ii) Explain why the amounts Baraka saves each month form a geometric sequence. [1]  
(iii) Determine whether Baraka saves more in two years than Aleela. [3]

Answer **all** the questions

**Section B** (77 marks)

- 7 A rod of length 2 m hangs vertically in equilibrium. Parallel horizontal forces of 30 N and 50 N are applied to the top and bottom and the rod is held in place by a horizontal force  $F$  N applied  $x$  m below the top of the rod as shown in Fig. 7.



**Fig. 7**

- (i) Find the value of  $F$ . [1]
- (ii) Find the value of  $x$ . [2]
- 8 (i) Show that  $8 \sin^2 x \cos^2 x$  can be written as  $1 - \cos 4x$ . [3]
- (ii) Hence find  $\int \sin^2 x \cos^2 x \, dx$ . [3]

- 9 A pebble is thrown horizontally at  $14\text{ m s}^{-1}$  from a window which is  $5\text{ m}$  above horizontal ground. The pebble goes over a fence  $2\text{ m}$  high  $d\text{ m}$  away from the window as shown in Fig. 9. The origin is on the ground directly below the window with the  $x$ -axis horizontal in the direction in which the pebble is thrown and the  $y$ -axis vertically upwards.

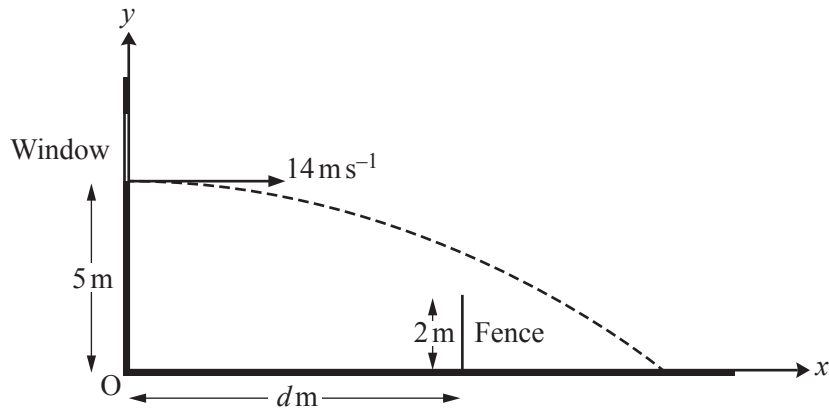


Fig. 9

- (i) Find the time the pebble takes to reach the ground. [3]
- (ii) Find the cartesian equation of the trajectory of the pebble. [4]
- (iii) Find the range of possible values for  $d$ . [3]
- 10 Fig. 10 shows the graph of  $y = (k - x)\ln x$  where  $k$  is a constant ( $k > 1$ ).

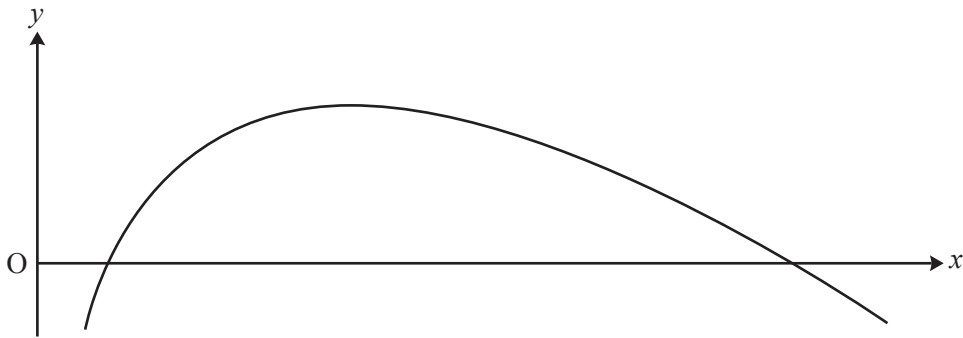
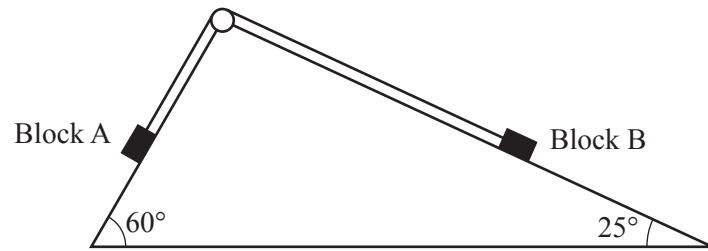


Fig. 10

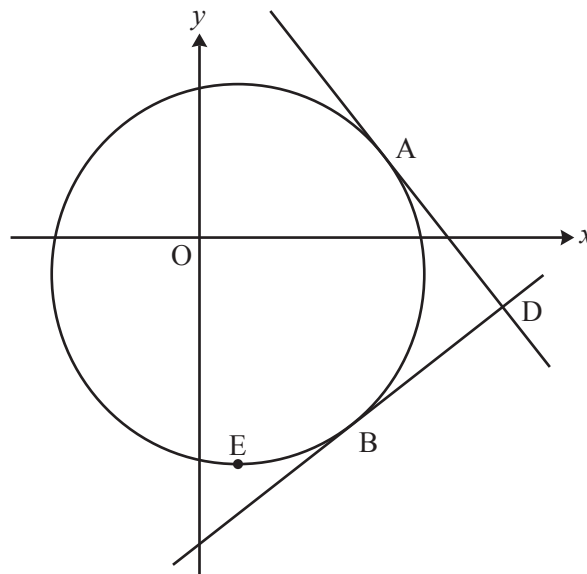
- Find, in terms of  $k$ , the area of the finite region between the curve and the  $x$ -axis. [8]

- 11** Fig. 11 shows two blocks at rest, connected by a light inextensible string which passes over a smooth pulley. Block A of mass  $4.7\text{ kg}$  rests on a smooth plane inclined at  $60^\circ$  to the horizontal. Block B of mass  $4\text{ kg}$  rests on a rough plane inclined at  $25^\circ$  to the horizontal. On either side of the pulley, the string is parallel to a line of greatest slope of the plane. Block B is on the point of sliding up the plane.



**Fig. 11**

- (i) Show that the tension in the string is  $39.9\text{ N}$  correct to 3 significant figures. [2]
- (ii) Find the coefficient of friction between the rough plane and Block B. [5]
- 12** Fig. 12 shows the circle  $(x-1)^2 + (y+1)^2 = 25$ , the line  $4y = 3x - 32$  and the tangent to the circle at the point A  $(5, 2)$ . D is the point of intersection of the line  $4y = 3x - 32$  and the tangent at A.



**Fig. 12**

- (i) Write down the coordinates of C, the centre of the circle. [1]
- (ii) (A) Show that the line  $4y = 3x - 32$  is a tangent to the circle. [4]
- (B) Find the coordinates of B, the point where the line  $4y = 3x - 32$  touches the circle. [1]
- (iii) Prove that ADBC is a square. [3]
- (iv) The point E is the lowest point on the circle. Find the area of the sector ECB. [5]

- 13 The function  $f(x)$  is defined by  $f(x) = \sqrt[3]{27-8x^3}$ . Jenny uses her scientific calculator to create a table of values for  $f(x)$  and  $f'(x)$ .

$x$	$f(x)$	$f'(x)$
0	3	0
0.25	2.9954	-0.056
0.5	2.9625	-0.228
0.75	2.8694	-0.547
1	2.6684	-1.124
1.25	2.2490	-1.977
1.5	0	ERROR

- (i) Use calculus to find an expression for  $f'(x)$  and hence explain why the calculator gives an error for  $f'(1.5)$ . [3]
- (ii) Find the first three terms of the binomial expansion of  $f(x)$ . [3]
- (iii) Jenny integrates the first three terms of the binomial expansion of  $f(x)$  to estimate the value of  $\int_0^1 \sqrt[3]{27-8x^3} dx$ . Explain why Jenny's method is valid in this case. (You do not need to evaluate Jenny's approximation.) [2]
- (iv) Use the trapezium rule with 4 strips to obtain an estimate for  $\int_0^1 \sqrt[3]{27-8x^3} dx$ . [3]

The calculator gives 2.921 174 38 for  $\int_0^1 \sqrt[3]{27-8x^3} dx$ . The graph of  $y = f(x)$  is shown in Fig. 13.

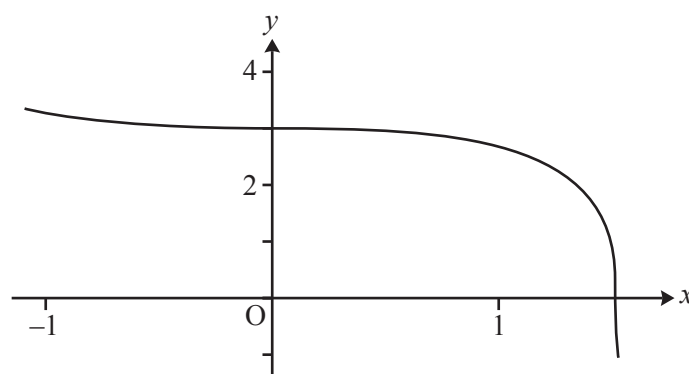


Fig. 13

- (v) Explain why the trapezium rule gives an underestimate. [1]



- 14** The velocity of a car,  $v \text{ m s}^{-1}$  at time  $t$  seconds, is being modelled. Initially the car has velocity  $5 \text{ m s}^{-1}$  and it accelerates to  $11.4 \text{ m s}^{-1}$  in 4 seconds.

In model A, the acceleration is assumed to be uniform.

- (i) Find an expression for the velocity of the car at time  $t$  using this model. [3]

- (ii) Explain why this model is not appropriate in the long term. [1]

Model A is refined so that the velocity remains constant once the car reaches  $17.8 \text{ m s}^{-1}$ .

- (iii) Sketch a velocity-time graph for the motion of the car, making clear the time at which the acceleration changes. [3]

- (iv) Calculate the displacement of the car in the first 20 seconds according to this refined model. [3]

In model B, the velocity of the car is given by

$$v = \begin{cases} 5 + 0.6t^2 - 0.05t^3 & \text{for } 0 \leq t \leq 8, \\ 17.8 & \text{for } 8 < t \leq 20. \end{cases}$$

- (v) Show that this model gives an appropriate value for  $v$  when  $t = 4$ . [1]

- (vi) Explain why the value of the acceleration immediately before the velocity becomes constant is likely to mean that model B is a better model than model A. [3]

- (vii) Show that model B gives the same value as model A for the displacement at time 20 s. [3]

**END OF QUESTION PAPER**





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