Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE

In Mathematics (9MA0) Paper 2

Pure Mathematics 2

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate’s response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - o.e. – or equivalent (and appropriate)
   - dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper
   - ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),\ \text{where}\ |pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\ |pq|=|c|\ \text{and}\ |mn|=|a|$, leading to $x=\ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to $x=\ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n\rightarrow x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n\rightarrow x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Assessment Objectives

Assessment Objective	Definition
A01	Use and apply standard techniques
A02	Reason, interpret and communicate mathematically
A03	Solve problems within mathematics and in other contexts

Elements

Element	Definition
1.1a	Select routine procedures
1.1b	Correctly carry out routine procedures
1.2	Accurately recall facts, terminology and definitions
2.1	Construct rigorous mathematical arguments (including proofs)
2.2a	Make deductions
2.2b	Make inferences
2.3	Assess the validity of mathematical arguments
2.4	Explain their reasoning
2.5	Uses mathematical language and notation correctly
3.1a	Translate problems in mathematical contexts into mathematical processes
3.1b	Translate problems in non-mathematical contexts into mathematical processes
3.2a	Interpret solutions to problems in their original context
3.2b	Evaluate (the) accuracy and limitations (of solutions to problems)
3.3	Translate situations in context into mathematical models
3.4	Use mathematical models
3.5a	Evaluate the outcomes of modelling in context
3.5b	Recognise the limitations of models
3.5c	Where appropriate, explain how to refine (models)

Question	Scheme	Marks	AOs
1	$2^x \times 4^y = \frac{1}{2\sqrt{2}} \quad \left\{= \frac{\sqrt{2}}{4}\right\}$		
Special Case	If 0 marks are scored on application of the mark scheme then allow Special Case B1 M0 A0 (total of 1 mark) for any of • $2^x \times 4^y \to 2^{x+2y}$ • $2^x \times 4^y \to 4^{\frac{1}{2}x+y}$ • $\frac{1}{2^x 2\sqrt{2}} \to 2^{-x-\frac{3}{2}}$ • $\log 2^x + \log 4^y \to x\log 2 + y\log 4$ or $x\log 2 + 2y\log 2$ • $\ln 2^x + \ln 4^y \to x\ln 2 + y\ln 4$ or $x\ln 2 + 2y\ln 2$ • $y = \log\left(\frac{1}{2^x\, 2\sqrt{2}}\right)$ o.e. {base of 4 omitted}		
Way 1	$2^x \times 2^{2y} = 2^{-\frac{3}{2}}$	B1	1.1b
	$2^{x+2y} = 2^{-\frac{3}{2}} \Rightarrow x + 2y = -\frac{3}{2} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	
Way 2	$\log(2^x \times 4^y) = \log\left(\frac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\frac{1}{2\sqrt{2}}\right)$ $\Rightarrow x\log 2 + y\log 4 = \log 1 - \log(2\sqrt{2}) \Rightarrow y = ...$	M1	2.1
	$y = \frac{-\log(2\sqrt{2}) - x\log 2}{\log 4} \quad \left\{\Rightarrow y = -\frac{1}{2}x - \frac{3}{4}\right\}$	A1	1.1b
		(3)	
Way 3	$\log(2^x \times 4^y) = \log\left(\frac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow \log 2^x + y\log 4 = \log\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow y = ...$	M1	2.1
	$y = \frac{\log\left(\frac{1}{2\sqrt{2}}\right) - \log(2^x)}{\log 4} \quad \left\{\Rightarrow y = -\frac{1}{2}x - \frac{3}{4}\right\}$	A1	1.1b
		(3)	
Way 4	$\log_2(2^x \times 4^y) = \log_2\left(\frac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log_2 2^x + \log_2 4^y = \log_2\left(\frac{1}{2\sqrt{2}}\right) \Rightarrow x + 2y = -\frac{3}{2} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	
			(3 marks)

Question	Scheme	Marks	AOs
Way 5	$4^{\frac{1}{2}x} \times 4^{y} = 4^{-\frac{3}{4}}$	B1	1.1b
	$4^{\frac{1}{2}x+y} = 4^{-\frac{3}{4}} \Rightarrow \frac{1}{2}x + y = -\frac{3}{4} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	

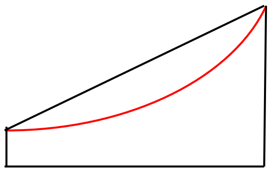
Notes for Question 1

	Way 1
B1:	Writes a correct equation in powers of 2 only
M1:	Complete process of writing a correct equation in powers of 2 only and using correct index laws to obtain y written as a function of x.
A1:	$y = -\frac{1}{2}x - \frac{3}{4}$ o.e.
	Way 2, Way 3 and Way 4
B1:	Writes a correct equation involving logarithms
M1:	Complete process of writing a correct equation involving logarithms and using correct log laws to obtain y written as a function of x.
A1:	$y = \frac{-\log(2\sqrt{2}) - x\log 2}{\log 4}$ or $y = \frac{-\ln(2\sqrt{2}) - x\ln 2}{\ln 4}$ or $y = \frac{\log\left(\frac{1}{2\sqrt{2}}\right) - \log(2^x)}{\log 4}$ or $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$ o.e.
	Way 5
B1:	Writes a correct equation in powers of 4 only
M1:	Complete process of writing a correct equation in powers of 4 only and using correct index laws to obtain y written as a function of x.
A1:	$y = -\frac{1}{2}x - \frac{3}{4}$ o.e.
Note:	Allow equivalent results for A1 where y is written as a function of x
Note:	You can ignore subsequent working following on from a correct answer.
Note:	Allow B1 for $2^x \times 4^y = \frac{1}{2\sqrt{2}} \Rightarrow 4^y = \frac{1}{2^x 2\sqrt{2}} \Rightarrow \log_4(4^y) = \log_4\left(\frac{1}{2^x 2\sqrt{2}}\right)$ followed by M1 A1 for $y = \log_4\left(\frac{1}{2^x 2\sqrt{2}}\right)$ or $y = \log_4\left(\frac{2^{-x}}{2\sqrt{2}}\right)$ or $y = \log_4\left(\frac{\sqrt{2}}{4(2^x)}\right)$ or $y = -\log_4\left(2^{x+\frac{3}{2}}\right)$ or $y = -\log_4(\sqrt{2}(2^{x+1}))$

Question	Scheme	Marks	AOs
2	Time (s): 0, 5, 10, 15, 20, 25 Speed (m s^{-1}): 2, 5, 10, 18, 28, 42		
(a)	Uses an allowable method to estimate the area under the curve. E.g.	M1	3.1a
	Way 1: an attempt at the trapezium rule (see below)		
	Way 2: $\{s=\}\left(\frac{2+42}{2}\right)(25)\ \{=550\}$		
	Way 3: $42=2+25(a) \Rightarrow a=1.6 \Rightarrow s=2(25)+(0.5)(1.6)(25)^2\{=550\}$		
	Way 4: $\{d=\}(2)(5)+5(5)+10(5)+18(5)+28(5)\ \{=63(5)=315\}$		
	Way 5: $\{d=\}\ 5(5)+10(5)+18(5)+28(5)+42(5)\ \{=103(5)=515\}$		
	Way 6: $\{d=\}\ \frac{315+515}{2}\ \{=415\}$		
	Way 7: $\{d=\}\left(\frac{2+5+10+18+28+42}{6}\right)(25)\ \{=437.5\}$		
	$\frac{1}{2}\times(5)\times[2+2(5+10+18+28)+42]$ or $\frac{1}{2}\times[\text{"315"}+\text{"515"}]$	M1	1.1b
	$=415\ \{\text{m}\}$	A1	1.1b
		(3)	
(b) Alt 1	**<u>Uses a Way 1, Way 2, Way 3, Way 5, Way 6 or Way 7 method in (a)</u>**		
	Overestimate **and** a relevant explanation e.g. • {top of} trapezia lie above the curve • Area of trapezia > area under curve • An appropriate diagram which gives reference to the extra area • Curve is convex • $\frac{d^2y}{dx^2}>0$ • Acceleration is {continually} increasing • The gradient of the curve is {continually} increasing • All the rectangles are above the curve (Way 5)	B1ft	2.4
		(1)	
(b) Alt 2	**<u>Uses a Way 4 method in (a)</u>**		
	Underestimate **and** a relevant explanation e.g. • All the rectangles are below the curve	B1ft	2.4
		(1)	
			(4 marks)

Notes for Question 2

(a)	
M1:	A low-level problem-solving mark for using an allowable method to estimate the area under the curve. E.g.
	Way 1: See scheme. Allow $\lambda(2+2(5+10+18+28)+42);\ \lambda>0$ for 1st M1
	Way 2: Uses $s=\left(\frac{u+v}{2}\right)t$ which is equivalent to finding the area of a large trapezium
	Way 3: Complete method using a uniform acceleration equation.
	Way 4: Sums rectangles lying below the curve. Condone a slip on one of the speeds.
	Way 5: Sums rectangles lying above the curve. Condone a slip on one of the speeds.
	Way 6: Average the result of Way 3 and Way 4. Equivalent to Way 1.
	Way 7: Applies (average speed)×(time)

Notes for Question 2 Continued	
(a)	***continued***
M1:	Correct trapezium rule method with $h=5$. Condone a slip on one of the speeds. The '2' and '42' should be in the correct place in the [......].
A1:	415
Note:	Units do not have to be stated
Note:	Give final A0 for giving a final answer with incorrect units. e.g. give final A0 for 415 km or 415ms^{-1}
Note:	Only the 1st M1 can only be scored for Way 2, Way 3, Way 4, Way 5 and Way 7 methods
Note:	Full marks in part (a) can only be scored by using a Way 1 or a Way 6 method.
Note:	Give M0 M0 A0 for $\{d=\}\ 2(5)+5(5)+10(5)+18(5)+28(5)+42(5)\ \ \{=105(5)=525\}$ (i.e. using too many rectangles)
Note	Condone M1 M0 A0 for $\left[\frac{(2+10)}{2}(10)+\frac{(10+18)}{2}(5)+\frac{(18+28)}{2}(5)+\frac{(28+42)}{2}(5)\right]=395\text{ m}$
Note:	Give M1 M1 A1 for $5\left[\frac{(2+5)}{2}+\frac{(5+10)}{2}+\frac{(10+18)}{2}+\frac{(18+28)}{2}+\frac{(28+42)}{2}\right]=415\text{ m}$
Note:	Give M1 M1 A1 for $\frac{5}{2}(2+42)+5(5+10+18+28)=415\text{ m}$
Note:	<u>**Bracketing mistake:**</u> **Unless the final calculated answer implies that the method has been applied correctly** give M1 M0 A0 for $\frac{5}{2}(2)+2(5+10+18+28)+42\ \ \{=169\}$ give M1 M0 A0 for $\frac{5}{2}(2+42)+2(5+10+18+28)\ \ \{=232\}$
Note:	Give M0 M0 A0 for a Simpson's Rule Method
(b)	**Alt 1**
B1ft:	**This mark depends on both an answer to part (a) being obtained and the first M in part (a)** See scheme
Note:	Allow the explanation "curve concaves upwards"
Note:	Do not allow explanations such as "curve is concave" or "curve concaves downwards"
Note:	Do not allow explanation "gradient of the curve is positive"
Note:	Do not allow explanations which refer to "friction" or "air resistance"
Note:	The diagram opposite is sufficient as an explanation. It must show the top of a trapezium lying above the curve. 
(b)	**Alt 2**
B1ft:	**This mark depends on both an answer to part (a) being obtained and the first M in part (a)** See scheme
Note:	Do not allow explanations which refer to "friction" or "air-resistance"

Question	Scheme	Marks	AOs
3 (a)	Allow explanations such as • student should have worked in radians • they did not convert degrees to radians • 40 should be in radians • θ should be in radians • angle (or θ) should be $\frac{40\pi}{180}$ or $\frac{2\pi}{9}$ • correct formula is $\pi r^2\left(\frac{\theta}{360}\right)$ {where θ is in degrees} • correct formula is $\pi r^2\left(\frac{40}{360}\right)$	B1	2.3
		(1)	
(b) Way 1	{Area of sector = } $\frac{1}{2}(5^2)\left(\frac{2\pi}{9}\right)$	M1	1.1b
	$= \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}	A1	1.1b
		(2)	
(b) Way 2	{Area of sector = } $\pi(5^2)\left(\frac{40}{360}\right)$	M1	1.1b
	$= \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}	A1	1.1b
		(2)	
			(3 marks)

Notes for Question 3

(a)	
B1:	Explains that the formula use is only valid when angle *AOB* is applied in radians. See scheme for examples of suitable explanations.
(b)	**Way 1**
M1:	Correct application of the sector formula using a correct value for θ in radians
Note:	Allow exact equivalents for θ e.g. $\theta = \frac{40\pi}{180}$ or θ in the range [0.68, 0.71]
A1*:	Accept $\frac{25}{9}\pi$ or awrt 8.73 **Note:** Ignore the units
(b)	**Way 2**
M1:	Correct application of the sector formula in degrees
A1:	Accept $\frac{25}{9}\pi$ or awrt 8.73 **Note:** Ignore the units.
Note:	Allow exact equivalents such as $\frac{50}{18}\pi$
Note:	Allow M1 A1 for $500\left(\frac{\pi}{180}\right) = \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}

Question	Scheme		Marks	AOs
4	$C_1: x = 10\cos t,\ y = 4\sqrt{2}\sin t,\ 0 \le t < 2\pi;\ C_2: x^2 + y^2 = 66$			
Way 1	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$		M1	3.1a
	$100(1-\sin^2 t) + 32\sin^2 t = 66$	$100\cos^2 t + 32(1-\cos^2 t) = 66$	M1 A1	2.1 1.1b
	$100 - 68\sin^2 t = 66 \Rightarrow \sin^2 t = \frac{1}{2}$ $\Rightarrow \sin t = ...$	$68\cos^2 t + 32 = 66 \Rightarrow \cos^2 t = \frac{1}{2}$ $\Rightarrow \cos t = ...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the *x*-coordinate and value of the corresponding *y*-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	
Way 2	$\{\cos^2 t + \sin^2 t = 1 \Rightarrow\}\ \left(\frac{x}{10}\right)^2 + \left(\frac{y}{4\sqrt{2}}\right)^2 = 1\ \{\Rightarrow 32x^2 + 100y^2 = 3200\}$		M1	3.1a
	$\frac{x^2}{100} + \frac{66 - x^2}{32} = 1$	$\frac{66 - y^2}{100} + \frac{y^2}{32} = 1$	M1 A1	2.1 1.1b
	$32x^2 + 6600 - 100x^2 = 3200$ $x^2 = 50 \Rightarrow x = ...$	$2112 - 32y^2 + 100y^2 = 3200$ $y^2 = 16 \Rightarrow y = ...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the corresponding *x*-coordinate or *y*-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	
Way 3	$\{C_2: x^2 + y^2 = 66 \Rightarrow\}\ x = \sqrt{66}\cos\alpha,\ y = \sqrt{66}\sin\alpha$ $\{C_1 = C_2 \Rightarrow\}\ 10\cos t = \sqrt{66}\cos\alpha,\ 4\sqrt{2}\sin t = \sqrt{66}\sin\alpha$ $\{\cos^2\alpha + \sin^2\alpha = 1 \Rightarrow\}\ \left(\frac{10\cos t}{\sqrt{66}}\right)^2 + \left(\frac{4\sqrt{2}\sin t}{\sqrt{66}}\right)^2 = 1$		M1	3.1a
	then continue with applying the mark scheme for Way 1			
Way 4	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$		M1	3.1a
	$100\left(\frac{1+\cos 2t}{2}\right) + 32\left(\frac{1-\cos 2t}{2}\right) = 66$		M1 A1	2.1 1.1b
	$50 + 50\cos 2t + 16 - 16\cos 2t = 66 \Rightarrow 34\cos 2t + 66 = 66$ $\Rightarrow \cos 2t = ...$		dM1	1.1b
	Substitutes their solution back into the original equation(s) to get the value of the *x*-coordinate and value of the *y*-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	
	Note: Give final A0 for writing $x = 5\sqrt{2},\ y = -4$ followed by $S = (-4, 5\sqrt{2})$			
				(6 marks)

Notes for Question 4

	Way 1
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 1: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1:	Uses the identity $\sin^2 t + \cos^2 t \equiv 1$ to achieve an equation in $\sin^2 t$ only or $\cos^2 t$ only
A1:	A correct equation in $\sin^2 t$ only or $\cos^2 t$ only
dM1: **Note:**	**dependent on both the previous M marks** Rearranges to make $\sin t = ...$ where $-1 \le \sin t \le 1$ or $\cos t = ...$ where $-1 \le \cos t \le 1$ Condone 3rd M1 for $\sin^2 t = \frac{1}{2} \Rightarrow \sin t = \frac{1}{4}$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$
	Way 2
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 2: A complete process of using $\cos^2 t + \sin^2 t \equiv 1$ to convert the parametric equation for C_1 into a Cartesian equation for C_1
M1:	Complete valid attempt to write an equation in terms of x only or y only not involving trigonometry
A1:	A correct equation in x only or y only not involving trigonometry
dM1: **Note:**	**dependent on both the previous M marks** Rearranges to make $x = ...$ or $y = ...$ their x^2 or their y^2 must be >0 for this mark
M1: **Note:**	See scheme their x^2 and their y^2 must be >0 for this mark
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$
	Way 3
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 3: A complete process of writing C_2 in parametric form, combining the parametric equations of C_1 and C_2 and applying $\cos^2\alpha + \sin^2\alpha \equiv 1$ to give an equation in one variable (i.e. t) only.
	then continue with applying the mark scheme for Way 1
	Way 4
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 4: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1: **Note:**	Uses the identities $\cos 2t \equiv 2\cos^2 t - 1$ and $\cos 2t \equiv 1 - 2\sin^2 t$ to achieve an equation in $\cos 2t$ only At least one of $\cos 2t \equiv 2\cos^2 t - 1$ or $\cos 2t \equiv 1 - 2\sin^2 t$ must be correct for this mark.
A1:	A correct equation in $\cos 2t$ only
dM1:	**dependent on both the previous M marks** Rearranges to make $\cos 2t = ...$ where $-1 \le \cos 2t \le 1$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$

Question	Scheme	Marks	AOs

4	$C_1: x = 10\cos t,\ y = 4\sqrt{2}\sin t,\ \ 0 \le t < 2\pi;\ \ C_2: x^2 + y^2 = 66$		
Way 5	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66(\sin^2 t + \cos^2 t)$	M1	2.1
		A1	1.1b
	$100\cos^2 t + 32\sin^2 t = 66\sin^2 t + 66\cos^2 t \Rightarrow 34\cos^2 t = 34\sin^2 t$ $\Rightarrow \tan t = ...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the *x*-coordinate and value of the corresponding *y*-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a
		(6)	

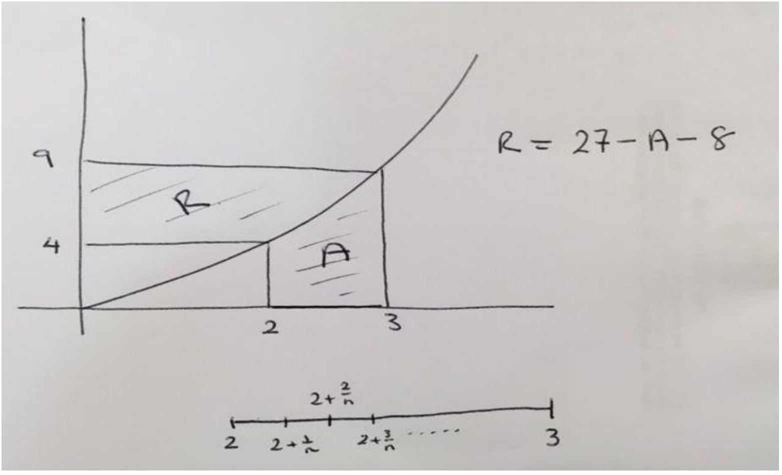
	Way 5
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 5: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. *t*) only.
M1:	Uses the identity $\sin^2 t + \cos^2 t \equiv 1$ to achieve an equation in $\sin^2 t$ only and $\cos^2 t$ only **with no constant term**
A1:	A correct equation in $\sin^2 t$ and $\cos^2 t$ containing no constant term
dM1:	**dependent on both the previous M marks** Rearranges to make $\tan t = ...$
M1:	See scheme
A1:	Selects the correct coordinates for *S* Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$

Question	Scheme	Marks	AOs
5	States $\left\{\lim\limits_{\delta x\to 0}\sum_{x=4}^{9}\sqrt{x}\,\delta x \text{ is}\right\}\ \int_4^9 \sqrt{x}\,\mathrm{d}x$	B1	1.2
	$=\left[\frac{2}{3}x^{\frac{3}{2}}\right]_4^9$	M1	1.1b
	$=\frac{2}{3}\times 9^{\frac{3}{2}}-\frac{2}{3}\times 4^{\frac{3}{2}} = \frac{54}{3}-\frac{16}{3}$		
	$=\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7	A1	1.1b
		(3)	
		(3 marks)	

Notes for Question 5

B1:	States $\int_4^9 \sqrt{x}\,\mathrm{d}x$ with or without the '$\mathrm{d}x$'
M1:	Integrates $\sqrt{x}$ to give $\lambda x^{\frac{3}{2}}$; $\lambda \neq 0$
A1:	See scheme
Note:	You can imply B1 for $\left[\lambda x^{\frac{3}{2}}\right]_4^9$ or for $\lambda\times 9^{\frac{3}{2}}-\lambda\times 4^{\frac{3}{2}}$
Note:	Give B0 for $\int_1^9 \sqrt{x}\,\mathrm{d}x-\int_1^3 \sqrt{x}\,\mathrm{d}x$ or for $\int_3^9 \sqrt{x}\,\mathrm{d}x$ without reference to a correct $\int_4^9 \sqrt{x}\,\mathrm{d}x$
Note:	Give B1 M1 A1 for no working leading to a correct $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\int_4^9 \sqrt{x}\,\mathrm{d}x=\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\left[\frac{2}{3}x^{\frac{3}{2}}+c\right]_4^9=\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for no working followed by an answer $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give M0 A0 for use of a trapezium rule method to give an answer of awrt 12.7, but allow B1 if $\int_4^9 \sqrt{x}\,\mathrm{d}x$ is seen in a trapezium rule method
Note:	Otherwise, give B0 M0 A0 for using the trapezium rule to give an answer of awrt 12.7

Notes for Question 5 Continued	
Alt	The following method is correct:



$$\text{Area }(A) = \lim_{n\to\infty}\sum_{i=1}^{n}(x_i - x_{i-1})\mathrm{f}(x_i) = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left(2+\frac{i}{n}\right)^2$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}4 + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{4i}{n}\right) + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{i^2}{n^2}\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}4 + \frac{4}{n^2}\sum_{i=1}^{n}i + \frac{1}{n^3}\sum_{i=1}^{n}i^2\right]$$

$$= \lim_{n\to\infty}\left[\frac{4n}{n} + \frac{4}{n^2}\left(\frac{1}{2}n(n+1)\right) + \frac{1}{n^3}\left(\frac{1}{6}n(n+1)(2n+1)\right)\right]$$

$$= \lim_{n\to\infty}\left[\frac{4}{n} + \frac{4n^2+4n}{2n^2} + \frac{2n^3+3n^2+n}{6n^3}\right]$$

$$= \lim_{n\to\infty}\left[4 + 2 + \frac{2}{n} + \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right]$$

$$= 4 + 2 + \frac{1}{3} = \frac{19}{3}$$

So, $\lim_{\delta x\to 0}\sum_{x=4}^{9}\sqrt{x}\,\delta x = \text{Area}(R) = (3\times 9) - (2\times 4) - \frac{19}{3}$

$$= \frac{38}{3} \quad \text{or} \quad 12\frac{2}{3} \quad \text{or} \quad \text{awrt } 12.7$$

Question	Scheme	Marks	AOs
6 (a)	$\text{gg}(0) = \text{g}((0-2)^2+1) = \text{g}(5) = 4(5)-7 = 13$	M1	2.1
		A1	1.1b
		(2)	
(b)	Solves either $(x-2)^2+1=28 \Rightarrow x = ...$ or $4x-7=28 \Rightarrow x = ...$	M1	1.1b
	At least one critical value $x = 2-3\sqrt{3}$ or $x = \frac{35}{4}$ is correct	A1	1.1b
	Solves both $(x-2)^2+1=28 \Rightarrow x = ...$ and $4x-7=28 \Rightarrow x = ...$	M1	1.1b
	Correct final answer of '$x < 2-3\sqrt{3}$, $x > \frac{35}{4}$'	A1	2.1
	Note: Writing awrt -3.20 or a truncated -3.19 or a truncated -3.2 in place of $2-3\sqrt{3}$ is accepted for any of the A marks	**(4)**	
(c)	<u>h is a one-one</u> {function (or mapping) so has an inverse} <u>g is a many-one</u> {function (or mapping) so does not have an inverse}	B1	2.4
		(1)	
(d) Way 1	$\left\{\text{h}^{-1}(x) = -\frac{1}{2} \Rightarrow\right\}$ $x = \text{h}\left(-\frac{1}{2}\right)$	M1 B1 on epen	1.1b
	$x = \left(-\frac{1}{2}-2\right)^2+1$ **Note:** Condone $x = \left(\frac{1}{2}-2\right)^2+1$	M1	1.1b
	$\Rightarrow x = 7.25$ only **cso**	A1	2.2a
		(3)	
(d) Way 2	$\{\text{their h}^{-1}(x)\} = \pm 2 \pm \sqrt{x \pm 1}$	M1	1.1b
	Attempts to solve $\pm 2 \pm \sqrt{x \pm 1} = -\frac{1}{2} \Rightarrow \pm\sqrt{x \pm 1} = ...$	M1	1.1b
	$\Rightarrow x = 7.25$ only **cso**	A1	2.2a
		(3)	
		(10 marks)	

Notes for Question 6

(a)	
M1:	Uses a complete method to find $\text{gg}(0)$. E.g. • Substituting $x = 0$ into $(0-2)^2+1$ and the result of this into the relevant part of $\text{g}(x)$ • Attempts to substitute $x = 0$ into $4((x-2)^2+1)-7$ or $4(x-2)^2-3$
A1:	$\text{gg}(0) = 13$
(b)	
M1:	See scheme
A1:	See scheme
M1:	See scheme
A1:	Brings all the strands of the problem together to give a correct solution.
Note:	You can ignore inequality symbols for any of the M marks
Note:	If a 3TQ is formed (e.g. $x^2-4x-23=0$) then a correct method for solving a 3TQ is required for the relevant method mark to be given.
Note:	Writing $(x-2)^2+1=28 \Rightarrow (x-2)+1=\sqrt{28} \Rightarrow x=-1+\sqrt{28}$ (i.e. taking the square-root of each term to solve $(x-2)^2+1=28$ is not considered to be an acceptable method)
Note:	Allow set notation. E.g. $\{x \in \mathbb{R} : x < 2-3\sqrt{3} \cup x > 8.75\}$ is fine for the final A mark

Notes for Question 6 Continued	
(b)	***continued***
Note:	Give final A0 for $\{ x \in \mathbb{R} : x < 2 - 3\sqrt{3} \cap x > 8.75\}$
Note:	Give final A0 for $2 - 3\sqrt{3} > x > 8.75$
Note:	Allow final A1 for their writing a final answer of "$x < 2 - 3\sqrt{3}$ and $x > \frac{35}{4}$"
Note:	Allow final A1 for a final answer of $x < 2 - 3\sqrt{3},\ x > \frac{35}{4}$
Note:	Writing $2 - \sqrt{27}$ in place of $2 - 3\sqrt{3}$ is accepted for any of the A marks
Note:	Allow final A1 for a final answer of $x < -3.20,\ x > 8.75$
Note:	Using 29 instead of 28 is M0 A0 M0 A0
(c)	
B1:	A correct explanation that conveys the underlined points
Note:	A minimal acceptable reason is "h is a one-one and g is a many-one"
Note:	Give B1 for "h^{-1} is one-one and g^{-1} is one-many"
Note:	Give B1 for "h is a one-one and g is not"
Note:	Allow B1 for "g is a many-one and h is not"
(d)	**Way 1**
M1:	Writes $x = h\left(-\frac{1}{2}\right)$
M1:	See scheme
A1:	Uses $x = h\left(-\frac{1}{2}\right)$ to deduce that $x = 7.25$ only, **cso**
(d)	**Way 2**
M1:	See scheme
M1:	See scheme
A1:	Use a correct $h^{-1}(x) = 2 - \sqrt{x-1}$ to deduce that $x = 7.25$ only, **cso**
Note:	Give final A0 cso for $2 + \sqrt{x-1} = -\frac{1}{2} \Rightarrow \sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Give final A0 cso for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow \sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Give final A1 cso for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow -\sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Allow final A1 for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow \pm\sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$

Question	Scheme	Marks	AOs
7	£y is the total cost of making x bars of soap Bars of soap are sold for £2 each		
(a)	$y = kx + c$ {where k and c are constants}	B1	3.3
	Note: Work for (a) cannot be recovered in (b) or (c)	**(1)**	
(b) **Way 1**	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	Applies (800, their 1100) and (300, their 680) to give two equations $1100 = 800k + c$ and $680 = 300k + c \Rightarrow k, c = ...$	dM1	1.1b
	Solves correctly to find $k = 0.84$, $c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	**(3)**	
(b) **Way 2**	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	Complete method for finding both $k = ...$ and $c = ...$ e.g. $k = \frac{1100 - 680}{800 - 300}$ $\{= 0.84\}$ $(800, 1100) \Rightarrow 1100 = 800(0.84) + c \Rightarrow c = ...$	dM1	1.1b
	Solves to find $k = 0.84$, $c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	**(3)**	
(b) **Way 3**	Either • $x = 800 \Rightarrow y = 2(800) - 500$ $\{= 1100 \Rightarrow (x, y) = (800, 1100)\}$ • $x = 300 \Rightarrow y = 2(300) + 80$ $\{= 680 \Rightarrow (x, y) = (300, 680)\}$	M1	3.1b
	$\{y = 0.84x + 428 \Rightarrow\}$ $x = 800 \Rightarrow y = (0.84)(800) + 428 = 1100$ $x = 300 \Rightarrow y = (0.84)(300) + 428 = 680$	dM1	1.1b
	Hence $y = 0.84x + 428$ *	A1*	2.1
		(3)	
(c)	Allow any of {0.84, in £s} represents • the ***cost*** of {making} each extra bar {of soap} • the direct ***cost*** of {making} a bar {of soap} • the marginal ***cost*** of {making} a bar {of soap} • the ***cost*** of {making} a bar {of soap} **(**Condone this answer) **Note: Do not allow** • {0.84, in £s} is the profit per bar {of soap} • {0.84, in £s} is the (selling) price per bar {of soap}	B1	3.4
		(1)	
(d) **Way 1**	{Let n be the least number of bars required to make a profit}		
	$2n = 0.84n + 428 \Rightarrow n = ...$ (Condone $2x = 0.84x + 428 \Rightarrow x = ...$)	M1	3.4
	Answer of 369 {bars}	A1	3.2a
		(2)	
(d) **Way 2**	• Trial 1: $n = 368 \Rightarrow y = (0.84)(368) + 428 \Rightarrow y = 737.12$ {revenue $= 2(368) = 736$ or loss = 1.12} • Trial 2: $n = 369 \Rightarrow y = (0.84)(369) + 428 \Rightarrow y = 737.96$ {revenue $= 2(369) = 738$ or profit = 0.04} leading to an answer of 369 {bars}	M1 A1	3.4 3.2a
		(2)	
			(7 marks)

Notes for Question 7	
(a)	
B1:	Obtains a correct form of the equation. E.g. $y = kx + c$; $k \neq 0, c \neq 0$. **Note:** Must be seen in (a)
Note:	Ignore how the constants are labelled – as long as they appear to be constants. e.g. *k*, *c*, *m* etc.
(b)	**Way 1**
M1:	Translates the problem into the model by finding either • $y = 2(800) - 500$ for $x = 800$ • $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** See scheme
A1:	See scheme – no errors in their working
Note	Allow 1st M1 for any of • $1600 - y = 500$ • $600 - y = -80$
(b)	**Way 2**
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** See scheme
A1:	See scheme – no error in their working
(b)	**Way 3**
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** Uses the model to test both points $(800$, their $1100)$ and $(300$, their $680)$
A1:	Confirms $y = 0.84x + 428$ is true for both $(800, 1100)$ and $(300, 680)$ and gives a conclusion
Note:	Conclusion could be "$y = 0.84x + 428$" or "QED" or "proved"
Note:	Give 1st M0 for $500 = 800k + c,\ 80 = 300k + c \Rightarrow k = \dfrac{500-80}{800-300} = 0.84$
(c)	
B1:	see scheme
Note:	Also condone B1 for "rate of change of cost", "cost of {making} a bar", "constant of proportionality for cost per bar of soap" or "rate of increase in cost",
Note:	Do not allow reasons such as "price increase or decrease", "rate of change of the bar of soap" or "decrease in cost"
Note:	Give B0 for incorrect use of units. E.g. Give B0 for "the cost of making each extra bar of soap is £84" Condone the use of £0.84p

Notes for Question 7 Continued	
(d)	**Way 1**
M1:	Using the model and constructing an argument leading to a critical value for the number of bars of soap sold. See scheme.
A1:	369 only. Do not accept decimal answers.
(d)	**Way 2**
M1:	Uses either 368 or 369 to find the cost $y = ...$
A1:	Attempts both trial 1 and trial 2 to find both the cost $y = ...$ and arrives at an answer of 369 only. Do not accept decimal answers.
Note:	You can ignore inequality symbols for the method mark in part (d)
Note:	Give M1 A1 for no working leading to 369 {bars}
Note:	Give final A0 for $x > 369$ or $x > 368$ or $x \geq 369$ without $x = 369$ or 369 stated as their final answer
Note:	Condone final A1 for **in words** “at least 369 bars must be made/sold”
Note:	**Special Case:** Assuming a profit of £1 is required and achieving $x = 370$ scores special case M1A0

Question	Scheme	Marks	AOs
8 (i) Way 1	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = 20\left(\frac{1}{2}\right)^4 + 20\left(\frac{1}{2}\right)^5 + 20\left(\frac{1}{2}\right)^6 + \ldots$		
	$= \frac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= (1.25)(2)\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 2	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=1}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=1}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \frac{10}{1-\frac{1}{2}} - (10+5+2.5)$ or $= \frac{10}{1-\frac{1}{2}} - \frac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 20 - 17.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 3	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=0}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=0}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \frac{20}{1-\frac{1}{2}} - (20+10+5+2.5)$ or $= \frac{20}{1-\frac{1}{2}} - \frac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 40 - 37.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(ii) Way 1	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\}$		
	$= \log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \ldots\ldots + \log_5\left(\frac{50}{49}\right) = \log_5\left(\frac{3}{2}\times\frac{4}{3}\times\ldots\times\frac{50}{49}\right)$	M1	1.1b
		M1	3.1a
	$= \log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$ *	A1*	2.1
		(3)	
(ii) Way 2	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\} \sum_{n=1}^{48}(\log_5(n+2) - \log_5(n+1))$	M1	1.1b
	$= (\log_5 3 + \log_5 4 + \ldots\ldots + \log_5 50) - (\log_5 2 + \log_5 3 + \ldots\ldots + \log_5 49)$	M1	3.1a
	$= \log_5 50 - \log_5 2$ or $\log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$*	A1*	2.1
		(3)	
			(6 marks)

Notes for Question 8	
(i)	**Way 1**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a complete strategy of applying $\frac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$
A1:	2.5 o.e.
(i)	**Way 2**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a completely correct strategy of applying $\frac{10}{1-\frac{1}{2}} - (10 + 5 + 2.5)$ or $\frac{10}{1-\frac{1}{2}} - \frac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$
A1:	2.5 o.e.
(i)	**Way 3**
M1:	Applies $\frac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a completely correct strategy of applying $\frac{20}{1-\frac{1}{2}} - (20+10+5+2.5)$ or $\frac{20}{1-\frac{1}{2}} - \frac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$
A1:	2.5 o.e.
Note:	Give M1 M1 A1 for a correct answer of 2.5 from no working in (i)
(ii)	**Way 1**
M1:	Some evidence of applying the addition law of logarithms as part of a valid proof
M1:	Begins to solve the problem by just writing (or by combining) at least three terms including • **either** the first two terms and the last term • **or** the first term and the last two terms
Note:	The 2nd mark can be gained by writing any of • listing $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{4}{3}\right), \log_5\left(\frac{50}{49}\right)$ or $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{49}{48}\right), \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + ... + \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2}\right) + ... + \log_5\left(\frac{49}{48}\right) + \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2} \times \frac{4}{3} \times ... \times \frac{50}{49}\right)$ ***{this will also gain the 1st M1 mark}*** • $\log_5\left(\frac{3}{2} \times ... \times \frac{49}{48} \times \frac{50}{49}\right)$ ***{this will also gain the 1st M1 mark}***
A1*:	Correct proof leading to a correct answer of 2
Note:	Do not allow the 2nd M1 if $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{4}{3}\right)$ are listed and $\log_5\left(\frac{50}{49}\right)$ is used for the first time in their applying the formula $S_{48} = \frac{48}{2}\left(\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{50}{49}\right)\right)$
Note:	<u>**Listing all 48 terms**</u> Give M0 M1 A0 for $\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \log_5\left(\frac{5}{4}\right) + ... + \log_5\left(\frac{50}{49}\right) = 2$ {lists all terms} Give M0 M0 A0 for $0.2519... + 0.1787... + 0.1386... + ... + 0.0125... = 2$ {all terms in decimals}

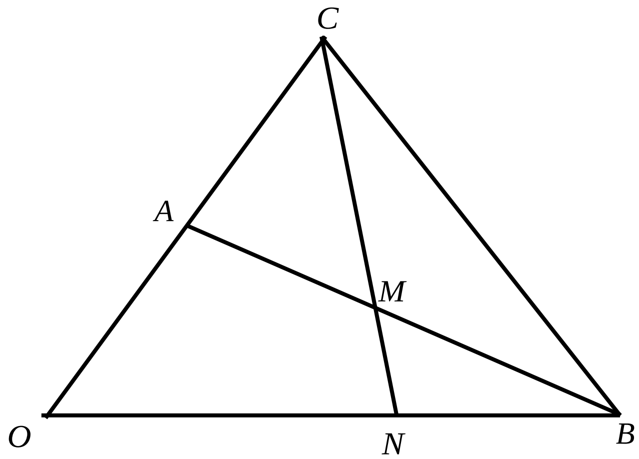
Notes for Question 8	
(ii)	**Way 2**
M1:	Uses the subtraction law of logarithms to give $\log_5\left(\frac{n+2}{n+1}\right) \to \log_5(n+2) - \log_5(n+1)$
M1:	Begins to solve the problem by writing at least three terms for each of $\log_5(n+2)$ and $\log_5(n+1)$ including • **either** the first two terms and the last term for both $\log_5(n+2)$ and $\log_5(n+1)$ • **or** the first term and the last two terms for both $\log_5(n+2)$ and $\log_5(n+1)$
Note:	This mark can be gained by writing any of • $(\log_5 3 + \log_5 4 + ... + \log_5 50) - (\log_5 2 + \log_5 3 + ... + \log_5 49)$ • $(\log_5 3 + ... + \log_5 49 + \log_5 50) - (\log_5 2 + ... + \log_5 48 + \log_5 49)$ • $(\log_5 3 + \log_5 4 + ... + \log_5 50) - (\log_5 2 + \log_5 3 + ... + \log_5 49)$ • $(\log_5 3 - \log_5 2) + (\log_5 4 - \log_5 3) + ... + (\log_5 50 - \log_5 49)$ • $\log_5 3 - \log_5 2,\ ......, \log_5 49 - \log_5 48,\ \log_5 50 - \log_5 49$
A1*:	Correct proof leading to a correct answer of 2
Note:	The base of 5 can be omitted for the M marks in part (ii), but the base of 5 must be included in the final line (as shown on the mark scheme) of their solution.
Note:	If a student uses a mixture of a Way 1 or Way 2 method, then award the best Way 1 mark only or the best Way 2 mark only.
Note:	Give M1 M0 A0 (1st M for implied use of subtraction law of logarithms) for $\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 91.8237... - 89.8237... = 2$
Note:	Give M1 M1 A1 for $\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = \sum_{n=1}^{48} (\log_5(n+2) - \log_5(n+1))$ $= \log_5(3 \times 4 \times .... \times 50) - \log_5(2 \times 3 \times .... \times 49)$ $= \log_5\left(\frac{50!}{2}\right) - \log_5(49!)$ or $= \log_5(25 \times 49!) - \log_5(49!)$ $= \log_5 25 = 2$

Question	Scheme	Marks	AOs
9 (a) Way 1	$\{d = kV^n \Rightarrow\}$ $\log_{10} d = \log_{10} k + n\log_{10} V$ or $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ seen or used as part of their argument	M1	2.1
	Alludes to $d = kV^n$ and gives a full explanation by comparing their result with a linear model e.g. $Y = MX + C$	A1	2.4
	$\{k =\}$ $10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
9 (a) Way 2	$\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ or $\log_{10} d = \log_{10} k + n\log_{10} V$ seen or used as part of their argument	M1	2.1
	$\{d = kV^n \Rightarrow\}$ $\log_{10} d = \log_{10}(kV^n)$ $\Rightarrow \log_{10} d = \log_{10} k + \log_{10} V^n \Rightarrow \log_{10} d = \log_{10} k + n\log_{10} V$	A1	2.4
	$\{k =\}$ $10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(a) Way 3	Starts from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$	M1	2.1
	$\log_{10} d = m\log_{10} V + c \Rightarrow d = 10^{m\log_{10} V + c} \Rightarrow d = 10^c V^m \Rightarrow d = kV^n$ **or** $\log_{10} d = m\log_{10} V - 1.77 \Rightarrow d = 10^{m\log_{10} V - 1.77}$ $\Rightarrow d = 10^{-1.77} V^m \Rightarrow d = kV^n$	A1	2.4
	$\{k =\}$ $10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(b)	$\{d = 20, V = 30 \Rightarrow\}$ $20 = k(30)^n$ or $\log_{10} 20 = \log_{10} k + n\log_{10} 30$	M1	3.4
	$20 = k(30)^n \Rightarrow \log 20 = \log k + n\log 30 \Rightarrow n = \dfrac{\log 20 - \log k}{\log 30} \Rightarrow n = ...$ $\log_{10} 20 = \log_{10} k + n\log_{10} 30 \Rightarrow n = \dfrac{\log_{10} 20 - \log_{10} k}{\log_{10} 30} \Rightarrow n = ...$	M1	1.1b
	$\{n = \text{awrt } 2.08 \Rightarrow\}$ $d = (0.017)V^{2.08}$ **or** $\log_{10} d = -1.77 + 2.08\log_{10} V$	A1	1.1b
	Note: You can recover the A1 mark for a correct model equation given in part (c)	**(3)**	
(c)	$d = (0.017)(60)^{2.08}$	M1	3.4
	• $13.333... + 84.918... = 98.251... \Rightarrow$ Sean stops in time	M1	3.1b
	• $100 - 13.333... = 86.666...$ & $d = 84.918 \Rightarrow$ Sean stops in time	A1ft	3.2a
		(3)	
			(9 marks)

ADVICE: Ignore labelling (a), (b), (c) when marking this question

Note: Give B0 in (a) for $10^{-1.77} = 0.01698...$ without reference to 0.017 in the same part

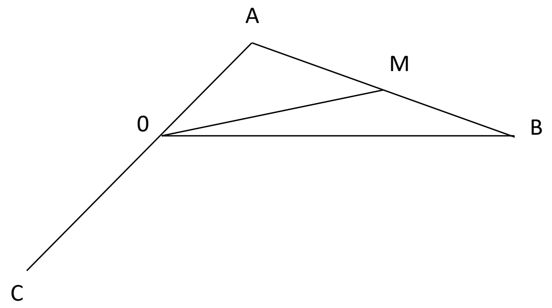
Notes for Question 9	
Note:	In their solution to (a) and/or (b) condone writing $\log$ in place of $\log_{10}$
(a)	**Way 1**
M1:	See scheme
A1:	See scheme
B1*:	See scheme
(a)	**Way 2**
M1:	See scheme
A1:	Starts from $d = kV^n$ (which they do not have to state) and progresses to $\log_{10} d = \log_{10} k + n\log_{10} V$ with an intermediate step in their working.
B1*:	See scheme
(a)	**Way 3**
M1:	Starts their argument from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$
A1:	Mathematical explanation is seen by showing any of either • $\log_{10} d = m\log_{10} V + c \rightarrow d = 10^c V^m$ or $d = kV^n$ • $\log_{10} d = m\log_{10} V - 1.77 \rightarrow d = 10^{-1.77} V^m$ or $d = kV^n$ with no errors seen in their working
B1*:	See scheme
Note:	Allow B1 for $\log_{10} 0.017 = -1.77$ or $\log 0.017 = -1.77$
(b)	
M1:	Applies $V = 30$ and $d = 20$ to their model **(correct way round)**
M1:	Applies $(V, d) = (30, 20)$ or $(20, 30)$ and applies logarithms correctly leading to $n = ...$
A1:	$d = (0.017)V^{2.08}$ or $\log_{10} d = -1.77 + 2.08\log_{10} V$ or $\log_{10} d = \log_{10}(0.017) + 2.08\log_{10} V$
Note:	Allow $k =$ awrt 0.017 and/or $n =$ awrt 2.08 in their final model equation
Note:	M0 M1 A0 is a possible score for (b)
(c)	
M1:	Applies $V = 60$ to their exponential model or their logarithmic model
M1:	Uses their model in a correct problem-solving process of either • adding a "thinking distance" to their value of their *d* to find an overall stopping distance • applying 100 – "thinking distance" and finds their value of *d*
Note:	$\frac{1}{75}$ or 48 are examples of acceptable thinking distances
A1ft:	**Either** adds 13.3… to their *d* to find a total stopping distance and gives a correct ft conclusion **or** finds their *d* and a comparative 86.666…(m) or awrt 87 (m) and gives a correct ft conclusion
Note:	The thinking distance must be dimensionally correct for the M1 mark. i.e. $0.8\times$ their velocity
Note:	A thinking distance of awrt 13 and a value of *d* in the range [81.5, 88.5] are required for A1ft
Note:	Allow "Sean stops in time" or "Yes he stops in time" or "he misses the puddle" as relevant conclusions.
Note:	A mark of M0 M1 A0 is possible in (c)

Question	Scheme	Marks	AOs
10			
	$\overrightarrow{OA} = \mathbf{a},\ \overrightarrow{OB} = \mathbf{b}$		
(a)	$\left\{\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \overrightarrow{CA} + \frac{1}{2}\overrightarrow{AB} \Rightarrow\right\}\ \overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$	M1	3.1a
	$\left\{\overrightarrow{CM} = \overrightarrow{CB} + \overrightarrow{BM} = \overrightarrow{CB} + \frac{1}{2}\overrightarrow{BA} \Rightarrow\right\}\ \overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$		
	$\Rightarrow \overrightarrow{CM} = -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ (***needs to be simplified and seen in (a) only***)	A1	1.1b
		(2)	
(b)	$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$	M1	1.1b
	$\overrightarrow{ON} = 2\mathbf{a} + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$ *	A1*	2.1
		(2)	
(c) **Way 1**	$\left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = ...$	M1	2.2a
	$\lambda = \frac{4}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}\ \left\{\Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b}\right\} \Rightarrow ON : NB = 2:1$ *	A1*	2.1
		(2)	
(c) **Way 2**	$\overrightarrow{ON} = \mu\mathbf{b} \Rightarrow \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b} = \mu\mathbf{b}$		
	$\mathbf{a}:\ \left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = ...\ \left\{\mathbf{b}:\ \frac{1}{2}\lambda = \mu\ \ \&\ \lambda = \frac{4}{3} \Rightarrow \mu = \frac{2}{3}\right\}$	M1	2.2a
	$\lambda = \frac{4}{3}$ or $\mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}\ \left\{\Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b}\right\} \Rightarrow ON : NB = 2:1$ *	A1*	2.1
		(2)	
			(6 marks)

Question	Scheme	Marks	AOs
10 (c) Way 3	$\overrightarrow{OB} = \overrightarrow{ON} + \overrightarrow{NB} \Rightarrow \mathbf{b} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b} + K\mathbf{b}$		
	$\mathbf{a}:\ \left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = ...\ \left\{\mathbf{b}:\ 1 = \frac{1}{2}\lambda + K\ \ \&\ \ \lambda = \frac{4}{3} \Rightarrow K = \frac{1}{3}\right\}$	M1	2.2a
	$\lambda = \frac{4}{3}$ or $K = \frac{1}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}$ or $\overrightarrow{NB} = \frac{1}{3}\mathbf{b} \Rightarrow ON : NB = 2:1$ *	A1	2.1
		(2)	
10 (c) Way 4	$\overrightarrow{ON} = \mu\mathbf{b}\ \&\ \overrightarrow{CN} = k\overrightarrow{CM} \Rightarrow \overrightarrow{CO} + \overrightarrow{ON} = k\overrightarrow{CM}$		
	$-2\mathbf{a} + \mu\mathbf{b} = k\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$		
	$\mathbf{a}: -2 = -\frac{3}{2}k \Rightarrow k = \frac{4}{3},\quad \mathbf{b}:\ \mu = \frac{1}{2}k \Rightarrow \mu = \frac{1}{2}\left(\frac{4}{3}\right) = ...$	M1	2.2a
	$\mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}\ \left\{\Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b}\right\} \Rightarrow ON : NB = 2:1$ *	A1	2.1
		(2)	

Notes for Question 10

(a)	
M1:	Valid attempt to find $\overrightarrow{CM}$ using a combination of known vectors **a** and **b**
A1:	A simplified correct answer for $\overrightarrow{CM}$
Note:	Give M1 for $\overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$ or for $\left\{\overrightarrow{CM} = \overrightarrow{OM} - \overrightarrow{OC} \Rightarrow\right\}$ $\overrightarrow{CM} = \frac{1}{2}(\mathbf{a} + \mathbf{b}) - 2\mathbf{a}$ **only o.e.**
(b)	
M1:	Uses $\overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$
A1*:	Correct proof
Note:	**<u>Special Case</u>** Give SC M1 A0 for the solution $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \lambda\overrightarrow{CM}$ $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)\ \left\{= \left(\frac{1}{2} - \frac{3}{2}\lambda\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\lambda\right)\mathbf{b}\right\}$
Note:	**<u>Alternative 1:</u>** Give M1 A1 for the following alternative solution: $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \mu\overrightarrow{CM}$ $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \mu\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) = \left(\frac{1}{2} - \frac{3}{2}\mu\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\mu\right)\mathbf{b}$ $\mu = \lambda - 1 \Rightarrow \overrightarrow{ON} = \left(\frac{1}{2} - \frac{3}{2}(\lambda - 1)\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}(\lambda - 1)\right)\mathbf{b} \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$
(c)	**Way 1, Way 2 and Way 3**
M1:	Deduces that $\left(2 - \frac{3}{2}\lambda\right) = 0$ and attempts to find the value of λ
A1*:	Correct proof
(c)	**Way 4**
M1:	Complete attempt to find the value of μ
A1*:	Correct proof

<table>
<tr><th colspan="2">Notes for Question 10 Continued</th></tr>
<tr><td>Note:</td><td>Part (b) and part (c) can be marked together.</td></tr>
<tr><td rowspan="3">(a) Special Case</td><td><u>Special Case where the point C is believed to be below the origin O</u>

</td></tr>
<tr><td>Give Special Case M1 A0 in part (a) for $\left\{\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} \Rightarrow\right\}$ $\overrightarrow{CM} = 3\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$</td></tr>
<tr><td>$\left\{\text{which leads to } \overrightarrow{CM} = \frac{5}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right\}$</td></tr>
</table>

Question	Scheme	Marks	AOs
11 (a) Way 1	$\{y = x^x \Rightarrow\}\ \ln y = x\ln x$	B1	1.1a
	$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln x$	M1	1.1b
		A1	2.1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\ \frac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = \mathrm{e}^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	**(5)**	
(a) Way 2	$\{y = x^x \Rightarrow\}\ y = \mathrm{e}^{x\ln x}$	B1	1.1a
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{x}{x} + \ln x\right)\mathrm{e}^{x\ln x}$	M1	1.1b
		A1	2.1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\ \frac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = \mathrm{e}^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	**(5)**	
(b) Way 1	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$1.8... < 2$ and $2.1... > 2$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
(c)	Attempts $x_{n+1} = 2x_n^{1-x_n}$ at least once with $x_1 = 1.5$ Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63	M1	1.1b
	$\{x_4 = 1.67313... \Rightarrow\}\ x_4 = 1.673$ (3 dp) **cao**	A1	1.1b
		(2)	
(d)	Give 1st B1 for any of • oscillates • periodic • non-convergent • divergent • fluctuates • goes up and down • 1, 2, 1, 2, 1, 2 • alternates (condone) || Give B1 B1 for any of • periodic {sequence} with period 2 • oscillates between 1 and 2	B1	2.5
	Condone B1 B1 for any of • fluctuates between 1 and 2 • keep getting 1, 2 • alternates between 1 and 2 • goes up and down between 1 and 2 • 1, 2, 1, 2, 1, 2, …	B1	2.5
		(2)	
		(11 marks)	

Note	**A common solution** A maximum of 3 marks (i.e. B1 1st M1 and 2nd M1) can be given for the solution $\log y = x\log x \Rightarrow \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \log x$ $\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\ 1 + \log x = 0 \Rightarrow x = 10^{-1}$
	• 1st B1 for $\log y = x\log x$ • 1st M1 for $\log y \to \lambda\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}$; $\lambda \neq 0$ or $x\log x \to 1 + \log x$ or $\frac{x}{x} + \log x$ • 2nd M1 can be given for $1 + \log x = 0 \Rightarrow \log x = k \Rightarrow x = ...;\quad k \neq 0$

Question	Scheme	Marks	AOs
11 (b) Way 2	For $x^x - 2$, attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$-0.16... < 0$ and $0.12... > 0$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
11 (b) Way 3	For $\ln y = x\ln x$, attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$0.608... < 0.69...$ and $0.752... > 0.69...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
11 (b) Way 4	For $\log y = x\log x$, attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp	M1	1.1b
	$0.264... < 0.301...$ and $0.326... > 0.301...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	

Notes for Question 11

(a)	**Way 1**
B1:	$\ln y = x\ln x$. Condone $\log_x y = x\log_x x$ or $\log_x y = x$
M1:	For either $\ln y \to \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}$ or $x\ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln x$ or $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{x} + \ln x$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = y(1+\ln x)$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = x^x(1+\ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = \mathrm{e}^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give no marks for no working leading to 0.368
Note:	Give M0 A0 M0 A0 for $\ln y = x\ln x \to x = 0.368$ with no intermediate working
(a)	**Way 2**
B1:	$y = \mathrm{e}^{x\ln x}$
M1:	For either $y = \mathrm{e}^{x\ln x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(\ln x)\mathrm{e}^{x\ln x}$ or $x\ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{x}{x} + \ln x\right)\mathrm{e}^{x\ln x}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = (1+\ln x)\mathrm{e}^{x\ln x}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = x^x(1+\ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = \mathrm{e}^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give B1 M1 A0 M1 A1 for the following solution: $\{y = x^x \Rightarrow\}$ $\ln y = x\ln x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln x \Rightarrow 1 + \ln x = 0 \Rightarrow x = \mathrm{e}^{-1}$ or awrt 0.368

Notes for Question 11 Continued	
(b)	**Way 1**
M1:	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} =$ awrt $1.8...$ and $1.6^{1.6} =$ awrt $2.1...$, reason (e.g. $1.8... < 2$ and $2.1... > 2$ or states C cuts through $y = 2$), C continuous and conclusion
(b)	**Way 2**
M1:	Attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ correct to awrt 1 dp, reason (e.g. $-0.16... < 0$ and $0.12... > 0$, sign change or states C cuts through $y = 0$), C continuous and conclusion
(b)	**Way 3**
M1:	Attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ correct to awrt 1 dp, reason (e.g. $0.608... < 0.69...$ and $0.752... > 0.69...$ or states they are either side of $\ln 2$), C continuous and conclusion.
(b)	**Way 4**
M1:	Attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp
A1:	Both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ correct to awrt 2 dp, reason (e.g. $0.264... < 0.301...$ and $0.326... > 0.301...$ or states they are either side of $\log 2$), C continuous and conclusion.
(c)	
M1:	An attempt to use the given or their formula once. Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63
A1:	States $x_4 = 1.673$ **cao** (to 3 dp)
Note:	Give M1 A1 for stating $x_4 = 1.673$
Note:	M1 can be implied by stating their final answer $x_4 =$ awrt 1.673
Note:	$x_2 = 1.63299...$, $x_3 = 1.46626...$, $x_4 = 1.67313...$
(d)	
B1:	see scheme
B1:	see scheme
Note:	Only marks of B1B0 or B1B1 are possible in (d)
Note:	Give B0 B0 for "Converges in a cob-web pattern" or "Converges up and down to α"

Question	Scheme	Marks	AOs
12	$\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta$		
(a) Way 1	$\{\text{LHS}=\}\ \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$=\dfrac{\cos(3\theta-\theta)}{\sin\theta\cos\theta}\ \left\{=\dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$=\dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}\ =\ 2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 2	$\{\text{LHS}=\}\ \dfrac{\cos 2\theta\cos\theta-\sin 2\theta\sin\theta}{\sin\theta}+\dfrac{\sin 2\theta\cos\theta+\cos 2\theta\sin\theta}{\cos\theta}$		
	$=\dfrac{\cos 2\theta\cos^2\theta-\sin 2\theta\sin\theta\cos\theta+\sin 2\theta\cos\theta\sin\theta+\cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$=\dfrac{\cos 2\theta(\cos^2\theta+\sin^2\theta)}{\sin\theta\cos\theta}\ \left\{=\dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$=\dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}\ =\ 2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 3	$\{\text{RHS}=\}\ \dfrac{2\cos 2\theta}{\sin 2\theta}=\dfrac{2\cos(3\theta-\theta)}{\sin 2\theta}=\dfrac{2(\cos 3\theta\cos\theta+\sin 3\theta\sin\theta)}{\sin 2\theta}$	M1	3.1a
		A1	2.1
	$=\dfrac{2(\cos 3\theta\cos\theta+\sin 3\theta\sin\theta)}{2\sin\theta\cos\theta}$	dM1	1.1b
	$=\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}$ *	A1 *	2.1
		(4)	
(b) Way 1	$\left\{\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=4\Rightarrow\right\}\ 2\cot 2\theta=4\Rightarrow 2\left(\dfrac{1}{\tan 2\theta}\right)=4$	M1	1.1b
	Rearranges to give $\tan 2\theta=k;\ k\neq 0$ and applies $\arctan k$	dM1	1.1b
	$\left\{90°<\theta<180°,\ \tan 2\theta=\dfrac{1}{2}\Rightarrow\right\}$		
	Only one solution of $\theta=103.3°$ (1 dp) or awrt $103.3°$	A1	2.2a
		(3)	
(b) Way 2	$\left\{\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=4\Rightarrow\right\}\ 2\cot 2\theta=4\Rightarrow\dfrac{2}{\tan 2\theta}=4$	M1	1.1b
	$\dfrac{2}{\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)}=4\Rightarrow 2(1-\tan^2\theta)=8\tan\theta$ $\Rightarrow\ \tan^2\theta+4\tan\theta-1=0\Rightarrow\tan\theta=\dfrac{-4\pm\sqrt{(4)^2-4(1)(-1)}}{2(1)}$ $\{\Rightarrow\tan\theta=-2\pm\sqrt{5}\}\Rightarrow\tan\theta=k;\ k\neq 0\Rightarrow$ applies $\arctan k$	dM1	1.1b
	$\{90°<\theta<180°,\ \tan\theta=-2-\sqrt{5}\Rightarrow\}$		
	Only one solution of $\theta=103.3°$ (1 dp) or awrt $103.3°$	A1	2.2a
		(3)	
			(7 marks)

Notes for Question 12	
(a)	**Way 1 and Way 2**
M1:	Correct valid method forming a common denominator of $\sin\theta\cos\theta$ i.e. correct process of $\dfrac{(...)\cos\theta+(...)\sin\theta}{\cos\theta\sin\theta}$
A1:	Proceeds to show that the numerator of their resulting fraction simplifies to $\cos(3\theta-\theta)$ or $\cos 2\theta$
dM1:	**dependent on the previous M mark** Applies a correct $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to the common denominator $\sin\theta\cos\theta$
A1*	Correct proof
Note:	Writing $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta}{\sin\theta\cos\theta} + \dfrac{\sin 3\theta\sin\theta}{\sin\theta\cos\theta}$ is considered a correct valid method of forming a common denominator of $\sin\theta\cos\theta$ for the 1st M1 mark
Note:	Give 1st M0 e.g. for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$
Note:	Give 1st M0 e.g. for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta}=\dfrac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$
Note:	Allow 2nd M1 for stating a correct $\sin 2\theta = 2\sin\theta\cos\theta$ and for attempting to apply it to the common denominator $\sin\theta\cos\theta$
(a)	**Way 3**
M1:	Starts from RHS and proceeds to expand $\cos 2\theta$ in the form $\cos 3\theta\cos\theta \pm \sin 3\theta\sin\theta$
A1:	Shows, as part of their proof, that $\cos 2\theta = \cos 3\theta\cos\theta + \sin 3\theta\sin\theta$
dM1:	**dependent on the previous M mark** Applies $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to their denominator
A1*:	Correct proof
Note:	Allow 1st M1 1st A1 (together) for any of LHS $\rightarrow \dfrac{\cos 2\theta}{\sin\theta\cos\theta}$ or LHS $\rightarrow \dfrac{\cos 2\theta(\cos^2\theta+\sin^2\theta)}{\sin\theta\cos\theta}$ or LHS $\rightarrow \cos 2\theta(\cot\theta+\tan\theta)$ or LHS $\rightarrow \cos 2\theta\left(\dfrac{1+\tan^2\theta}{\tan\theta}\right)$ (i.e. where $\cos 2\theta$ has been factorised out)
Note:	Allow 1st M1 1st A1 for progressing as far as LHS $= ... = \cot x - \tan x$
Note:	The following is a correct alternative solution $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\frac{1}{2}(\cos 4\theta+\cos 2\theta)-\frac{1}{2}(\cos 4\theta-\cos 2\theta)}{\sin\theta\cos\theta}$ $= \dfrac{\cos 2\theta}{\sin\theta\cos\theta} = \dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *
Note:	E.g. going from $\dfrac{\cos 2\theta\cos^2\theta - \sin 2\theta\sin\theta\cos\theta + \sin 2\theta\cos\theta\sin\theta + \cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$ to $\dfrac{\cos 2\theta}{\sin\theta\cos\theta}$ with no intermediate working is 1st A0

Notes for Question 12 Continued	
(b)	**Way 1**
M1:	Evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
dM1:	**dependent on the previous M mark** Rearranges to give $\tan 2\theta = k,\ k \neq 0,$ and applies $\arctan k$
A1:	Uses $90° < \theta < 180°$ to deduce the only solution $\theta =$ awrt 103.3°
Note:	Give M0M0A0 for writing, for example, $\tan 2\theta = 2$ with no evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
Note:	1st M1 can be implied by seeing $\tan 2\theta = \frac{1}{2}$
Note:	Condone 2nd M1 for applying $\frac{1}{2}\arctan\left(\frac{1}{2}\right)$ $\{= 13.28...\}$
(b)	**Way 2**
M1:	Evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
dM1:	**dependent on the previous M mark** Applies $\tan 2\theta \equiv \frac{2\tan\theta}{1-\tan^2\theta}$, forms and uses a correct method for solving a 3TQ to give $\tan\theta = k,\ k \neq 0,$ and applies $\arctan k$
A1:	Uses $90° < \theta < 180°$ to deduce the only solution $\theta =$ awrt 103.3°
Note:	Give M1 dM1 A1 for no working leading to $\theta =$ awrt 103.3° and no other solutions
Note:	Give M1 dM1 A0 for no working leading to $\theta =$ awrt 103.3° and other solutions which can be either outside or inside the range $90° < \theta < 180°$

Question	Scheme	Marks	AOs
13 (a)	States or uses $6=\pi r^2h+\frac{2}{3}\pi r^3$	B1	1.1a
	$\Rightarrow\ h=\frac{6}{\pi r^2}-\frac{2}{3}r,\ \pi h=\frac{6}{r^2}-\frac{2}{3}\pi r,\ \pi rh=\frac{6}{r}-\frac{2}{3}\pi r^2,\ rh=\frac{6}{\pi r}-\frac{2}{3}r^2$		
	$A=\pi r^2+2\pi rh+2\pi r^2\ \{\Rightarrow A=3\pi r^2+2\pi rh\}$		
	$A=2\pi r^2+2\pi r\left(\frac{6}{\pi r^2}-\frac{2}{3}r\right)+\pi r^2$	M1	3.1a
		A1	1.1b
	$A=3\pi r^2+\frac{12}{r}-\frac{4}{3}\pi r^2\ \Rightarrow\ A=\frac{12}{r}+\frac{5}{3}\pi r^2$ *	A1*	2.1
		(4)	
(b)	$\left\{A=12r^{-1}+\frac{5}{3}\pi r^2\Rightarrow\right\}\ \frac{dA}{dr}=-12r^{-2}+\frac{10}{3}\pi r$	M1	3.4
		A1	1.1b
	$\left\{\frac{dA}{dr}=0\Rightarrow\right\}\ -\frac{12}{r^2}+\frac{10}{3}\pi r=0\Rightarrow -36+10\pi r^3=0\Rightarrow r^{\pm 3}=\ldots\left\{=\frac{18}{5\pi}\right\}$	M1	2.1
	$r=1.046447736...\Rightarrow r=$ 1.05 (m) (3 sf) or awrt 1.05 (m)	A1	1.1b
	Note: Give final A1 for correct exact values for r	**(4)**	
(c)	$A_{min}=\frac{12}{(1.046...)}+\frac{5}{3}\pi(1.046...)^2$	M1	3.4
	$\{A_{min}=17.20...\Rightarrow\}\ A=17\ (m^2)$ or $A=$ awrt 17 (m^2)	A1ft	1.1b
		(2)	
			(10 marks)

Notes for Question 13

(a)	
B1:	See scheme
M1:	Complete process of substituting their $h=\ldots$ or $\pi h=\ldots$ or $\pi rh=\ldots$ or $rh=\ldots$, where '...' $=f(r)$ into an expression for the surface area which is of the form $A=\lambda\pi r^2+\mu\pi rh;\ \lambda,\mu\neq 0$
A1:	Obtains correct simplified or un-simplified $\{A=\}\ 2\pi r^2+2\pi r\left(\frac{6}{\pi r^2}-\frac{2}{3}r\right)+\pi r^2$
A1*:	Proceeds, using rigorous and careful reasoning, to $A=\frac{12}{r}+\frac{5}{3}\pi r^2$
Note:	Condone the lack of $A=\ldots$ or $S=\ldots$ for any one of the A marks or for both of the A marks
(b)	
M1:	Uses the model (or their model) and differentiates $\frac{\lambda}{r}+\mu r^2$ to give $\alpha r^{-2}+\beta r$; $\lambda,\mu,\alpha,\beta\neq 0$
A1:	$\left\{\frac{dA}{dr}=\right\}\ -12r^{-2}+\frac{10}{3}\pi r$ o.e.
M1:	Sets their $\frac{dA}{dr}=0$ and rearranges to give $r^{\pm 3}=k,\ k\neq 0$ (**Note:** k can be positive or negative)
Note:	This mark can be implied. Give M1 (and A1) for $-36+10\pi r^3=0\rightarrow r=\left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r=\left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r=\left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$
A1:	$r=$ awrt 1.05 (ignoring units) or $r=$ awrt 105 cm
Note:	Give M0 A0 M0 A0 where $r=$ 1.05 (m) (3 sf) or awrt 1.05 (m) is found from no working.
Note:	Give final A1 for correct exact values for r. E.g. $r=\left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r=\left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r=\left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$

Notes for Question 13 Continued

Note:	Give final M0 A0 for $-\frac{12}{r^2}+\frac{10}{3}\pi r > 0 \Rightarrow r > 1.0464$
Note:	Give final M1 A1 for $-\frac{12}{r^2}+\frac{10}{3}\pi r > 0 \Rightarrow r > 1.0464... \Rightarrow r = 1.0464...$
(c)	
M1:	Substitutes their $r = 1.046...$, found from solving $\frac{\mathrm{d}A}{\mathrm{d}r}=0$ in part (b), into the model with equation $A=\frac{12}{r}+\frac{5}{3}\pi r^2$
Note:	Give M0 for substituting their *r* which has been found from solving $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}=0$ or from using $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}$ into the model with equation $A=\frac{12}{r}+\frac{5}{3}\pi r^2$
A1ft:	$\{A=\}\,17$ or $\{A=\}$ awrt 17 (ignoring units)
Note:	You can only follow through on values of *r* for $0.6 \le$ their $r \le 1.3$ (and where their *r* has been found from solving $\frac{\mathrm{d}A}{\mathrm{d}r}=0$ in part (b))

r	***A***	***A* (nearest integer)**
0.6	21.88495…	awrt 22
0.7	19.70849…	awrt 20
0.8	18.35103…	awrt 18
0.9	17.57448…	awrt 18
1.0	17.23598…	awrt 17
1.1	17.24463…	awrt 17
1.2	17.53982…	awrt 18
1.3	18.07958…	awrt 18
1.05	17.20124…	awrt 17
1.04644…	17.20105…	awrt 17

Note:	Give M1 A1 for $A=17\ (\mathrm{m}^2)$ or $A=$ awrt $17\ (\mathrm{m}^2)$ from no working

Question	Scheme	Marks	AOs
14 (a)	$\{u = 4 - \sqrt{h} \Rightarrow\}\ \frac{\mathrm{d}u}{\mathrm{d}h} = -\frac{1}{2}h^{-\frac{1}{2}}$ or $\frac{\mathrm{d}h}{\mathrm{d}u} = -2(4-u)$ or $\frac{\mathrm{d}h}{\mathrm{d}u} = -2\sqrt{h}$	B1	1.1b
	$\left\{\int \frac{\mathrm{d}h}{4-\sqrt{h}} = \right\}\ \int \frac{-2(4-u)}{u}\mathrm{d}u$	M1	2.1
	$= \int \left(-\frac{8}{u} + 2\right)\mathrm{d}u$	M1	1.1b
	$= -8\ln u + 2u\ \{+c\}$	M1	1.1b
		A1	1.1b
	$= -8\ln\lvert 4-\sqrt{h}\rvert + 2(4-\sqrt{h}) + c\ =\ -8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h}\ + k$ *	A1*	2.1
		(6)	
(b)	$\left\{\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}(4-\sqrt{h})}{20} = 0 \Rightarrow\right\}\ 4-\sqrt{h} = 0$	M1	3.4
	Deduces any of $0 < h < 16,\ 0 \le h < 16,\ 0 < h \le 16,\ 0 \le h \le 16,$ $h < 16,\ h \le 16$ or all values up to 16	A1	2.2a
		(2)	
(c) Way 1	$\int \frac{1}{(4-\sqrt{h})}\mathrm{d}h\ =\ \int \frac{1}{20}t^{0.25}\mathrm{d}t$	B1	1.1b
	$-8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h}\ = \frac{1}{25}t^{1.25}\ \{+c\}$	M1	1.1b
		A1	1.1b
	$\{t=0,\ h=1 \Rightarrow\}\ -8\ln(4-1) - 2\sqrt{(1)}\ = \frac{1}{25}(0)^{1.25} + c$	M1	3.4
	$\Rightarrow c = -8\ln(3) - 2 \Rightarrow\ -8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h}\ = \frac{1}{25}t^{1.25} - 8\ln(3) - 2$ $\{h = 12 \Rightarrow\}\ -8\ln\lvert 4-\sqrt{12}\rvert - 2\sqrt{12}\ = \frac{1}{25}t^{1.25} - 8\ln(3) - 2$	dM1	3.1a
	$t^{1.25} = 221.2795202... \Rightarrow\ t = \sqrt[1.25]{221.2795...}$ or $t = (221.2795...)^{0.8}$	M1	1.1b
	$t = 75.154... \Rightarrow t =\ 75.2$ (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	**(7)**	
(c) Way 2	$\int_1^{12} \frac{20}{(4-\sqrt{h})}\mathrm{d}h\ =\ \int_0^T t^{0.25}\mathrm{d}t$	B1	1.1b
	$\left[20(-8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h})\right]_1^{12} = \left[\frac{4}{5}t^{1.25}\right]_0^T$	M1	1.1b
		A1	1.1b
	$20(-8\ln(4-\sqrt{12}) - 2\sqrt{12}) - 20(-8\ln(4-1) - 2\sqrt{1}) = \frac{4}{5}T^{1.25} - 0$	M1	3.4
		dM1	3.1a
	$T^{1.25} = 221.2795202... \Rightarrow\ T = \sqrt[1.25]{221.2795...}$ or $T = (221.2795...)^{0.8}$	M1	1.1b
	$T = 75.154... \Rightarrow T =\ 75.2$ (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	**(7)**	
			(15 marks)

Notes for Question 14	
(a)	
B1:	See scheme. Allow $\mathrm{d}u = -\frac{1}{2}h^{-\frac{1}{2}}\mathrm{d}h$, $\mathrm{d}h = -2(4-u)\mathrm{d}u$, $\mathrm{d}h = -2\sqrt{h}\,\mathrm{d}u$ o.e.
M1: **Note:**	Complete method for applying $u = 4-\sqrt{h}$ to $\int \frac{\mathrm{d}h}{4-\sqrt{h}}$ to give an expression of the form $\int \frac{k(4-u)}{u}\mathrm{d}u$; $k \neq 0$ Condone the omission of an integral sign and/or $\mathrm{d}u$
M1:	Proceeds to obtain an integral of the form $\int\left(\frac{A}{u}+B\right)\{\mathrm{d}u\}$; $A, B \neq 0$
M1:	$\int\left(\frac{A}{u}+B\right)\{\mathrm{d}u\} \to D\ln u + Eu$; $A, B, D, E \neq 0$; with or without a constant of integration
A1:	$\int\left(-\frac{8}{u}+2\right)\{\mathrm{d}u\} \to -8\ln u + 2u$; with or without a constant of integration
A1*:	**dependent on all previous marks** Substitutes $u = 4-\sqrt{h}$ into their integrated result and completes the proof by obtaining the printed result $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$. Condone the use of brackets instead of the modulus sign.
Note:	They must combine $2(4)$ and their $+c$ correctly to give $+k$
Note:	Going from $-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) + c$ to $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$, with no intermediate working or with no incorrect working is required for the final A1* mark.
Note:	Allow A1* for correctly reaching $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + c + 8$ and stating $k = c+8$
Note:	Allow A1* for correctly reaching $-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) + k = -8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$
	Alternative (integration by parts) method for the 2nd M, 3rd M and 1st A mark $\left\{\int \frac{-2(4-u)}{u}\mathrm{d}u = \int \frac{2u-8}{u}\mathrm{d}u\right\} = (2u-8)\ln u - \int 2\ln u\,\mathrm{d}u = (2u-8)\ln u - 2(u\ln u - u)\ \{+c\}$
2nd M1:	Proceeds to obtain an integral of the form $(Au+B)\ln u - \int A\ln u\{\mathrm{d}u\}$; $A, B \neq 0$
3rd M1:	Integrates to give $D\ln u + Eu$; $D, E \neq 0$; which can be simplified or un-simplified with or without a constant of integration.
Note:	Give 3rd M1 for $(2u-8)\ln u - 2(u\ln u - u)$ because it is an un-simplified form of $D\ln u + Eu$
1st A1:	Integrates to give $(2u-8)\ln u - 2(u\ln u - u)$ or $-8\ln u + 2u$ o.e. with or without a constant of integration.
(b)	
M1: **Note:**	Uses the context of the model and has an understanding that the tree keeps growing until $\frac{\mathrm{d}h}{\mathrm{d}t} = 0 \Rightarrow 4-\sqrt{h} = 0$. Alternatively, they can write $\frac{\mathrm{d}h}{\mathrm{d}t} > 0 \Rightarrow 4-\sqrt{h} > 0$ Accept $h = 16$ or 16 used in their inequality statement for this mark.
A1:	See scheme
Note:	A correct answer can be given M1 A1 from any working.

Notes for Question 14	
(c)	**Way 1**
B1:	Separates the variables correctly. dh and dt should not be in the wrong positions, although this mark can be implied by later working. Condone absence of integral signs.
M1:	Integrates $t^{0.25}$ to give $\lambda t^{1.25}$; $\lambda \neq 0$
A1:	Correct integration. E.g. $-8\ln\left|4-\sqrt{h}\right| - 2\sqrt{h} = \frac{1}{25}t^{1.25}$ or $20(-8\ln\left|4-\sqrt{h}\right| - 2\sqrt{h}) = \frac{4}{5}t^{1.25}$ $-8\ln\left|4-\sqrt{h}\right| + 2(4-\sqrt{h}) = \frac{1}{25}t^{1.25}$ or $20(-8\ln\left|4-\sqrt{h}\right| + 2(4-\sqrt{h})) = \frac{4}{5}t^{1.25}$ with or without a constant of integration, e.g. k, c or A
Note:	There is no requirement for modulus signs.
M1:	Some evidence of ***applying*** both $t=0$ and $h=1$ to their model (which can be a changed equation) which contains a constant of integration, e.g. k, c or A
dM1:	**dependent on the previous M mark** Complete process of finding their constant of integration, followed by applying $h=12$ and their constant of integration to their changed equation
M1:	Rearranges their equation to make $t^{\text{their } 1.25} = ...$ followed by a correct method to give $t = ...$; $t > 0$
Note:	$t^{\text{their } 1.25} = ...$ can be negative, but their '$t = ...$' must be positive
Note:	"their 1.25" cannot be 0 or 1 for this mark
Note:	Do not give this mark if $t^{\text{their } 1.25} = ...$ (usually $t^{0.25} = ...$) is a result of substituting $t=12$ (or $t=11$) into the given $\frac{dh}{dt} = \frac{t^{0.25}(4-\sqrt{h})}{20}$. **Note:** They will usually write $\frac{dh}{dt}$ as either 12 or 11.
A1:	awrt 75.2
(c)	**Way 2**
B1: **Note:**	Separates the variables correctly. dh and dt should not be in the wrong positions, although this mark can be implied by later working. Integral signs and limits are not required for this mark.
M1:	Same as Way 1 (ignore limits)
A1:	Same as Way 1 (ignore limits)
M1:	Applies limits of 1 and 12 to their model (i.e. to their changed expression in h) and subtracts
dM1	**dependent on the previous M mark** Complete process of applying limits of 1 and 12 and 0 and T (or 't') appropriately to their changed equation
M1:	Same as Way 1
A1:	Same as Way 1

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