



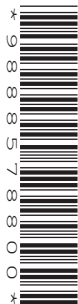
Oxford Cambridge and RSA

Tuesday 20 June 2023 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae

A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Section A
Pure Mathematics

- 1 Using logarithms, solve the equation

$$4^{2x+1} = 5^x,$$

giving your answer correct to 3 significant figures.

[3]

- 2 (a) Express $3 \sin x - 4 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the value of α correct to 4 significant figures. [3]
- (b) Hence solve the equation $3 \sin x - 4 \cos x = 2$ for $0^\circ < x < 90^\circ$, giving your answer correct to 3 significant figures. [2]

- 3 The cubic polynomial $f(x)$ is defined by $f(x) = x^3 + px + q$, where p and q are constants.

- (a) (i) Given that $f'(2) = 13$, find the value of p . [2]
- (ii) Given also that $(x - 2)$ is a factor of $f(x)$, find the value of q . [2]

The curve $y = f(x)$ is translated by the vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

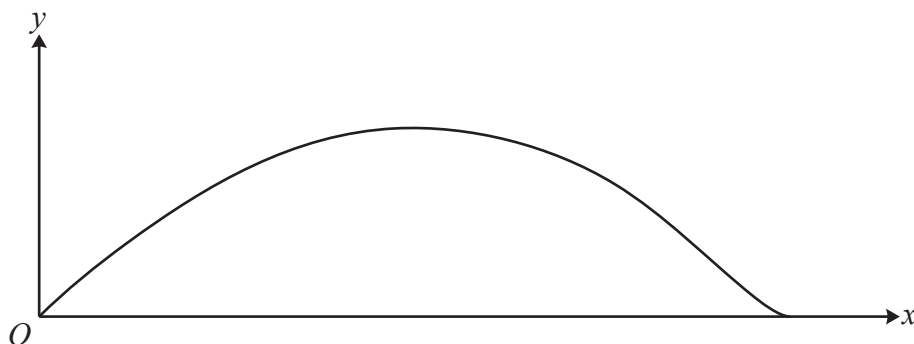
- (b) Using the values from part (a), determine the equation of the curve after it has been translated. Give your answer in the form $y = x^3 + ax^2 + bx + c$, where a , b and c are integers to be found. [4]

- 4 A circle C has equation $x^2 + y^2 - 6x + 10y + k = 0$.

- (a) Find the set of possible values of k . [2]
- (b) It is given that $k = -46$.

Determine the coordinates of the **two** points on C at which the gradient of the tangent is $\frac{1}{2}$. [5]

- 5 A mathematics department is designing a new emblem to place on the walls outside its classrooms. The design for the emblem is shown in the diagram below.



The emblem is modelled by the region between the x -axis and the curve with parametric equations

$$x = 1 + 0.2t - \cos t, \quad y = k \sin^2 t,$$

where k is a positive constant and $0 \leq t \leq \pi$.

Lengths are in metres and the area of the emblem must be 1 m^2 .

(a) Show that $k \int_0^\pi (0.2 + \sin t - 0.2 \cos^2 t - \sin t \cos^2 t) dt = 1$. [3]

(b) Determine the exact value of k . [6]

- 6 The first, third and fourth terms of an arithmetic progression are u_1 , u_3 and u_4 respectively, where

$$u_1 = 2 \sin \theta, \quad u_3 = -\sqrt{3} \cos \theta, \quad u_4 = \frac{7}{2} \sin \theta,$$

$$\text{and } \frac{1}{2}\pi < \theta < \pi.$$

(a) Determine the exact value of θ . [3]

(b) Hence determine the value of $\sum_{r=1}^{100} u_r$. [3]

- 7 A car C is moving horizontally in a straight line with velocity $v \text{ ms}^{-1}$ at time t seconds, where $v > 0$ and $t \geq 0$. The acceleration, $a \text{ ms}^{-2}$, of C is modelled by the equation

$$a = v \left(\frac{8t}{7 + 4t^2} - \frac{1}{2} \right).$$

- (a) In this question you must show detailed reasoning.

Find the times when the acceleration of C is zero.

[3]

At $t = 0$ the velocity of C is 17.5 ms^{-1} and at $t = T$ the velocity of C is 5 ms^{-1} .

- (b) By setting up and solving a differential equation, show that T satisfies the equation

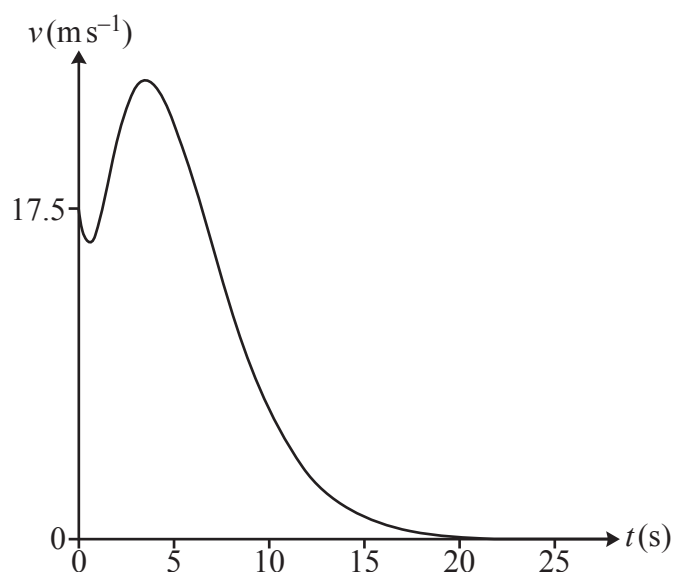
$$T = 2 \ln \left(\frac{7 + 4T^2}{2} \right).$$

[6]

- (c) Use an iterative formula, based on the equation in part (b), to find the value of T , giving your answer correct to 4 significant figures. Use an initial value of 11.25 and show the result of each step of the iteration process.

[2]

- (d) The diagram below shows the velocity-time graph for the motion of C .



Find the time taken for C to decelerate from travelling at its maximum speed until it is travelling at 5 ms^{-1} .

[1]

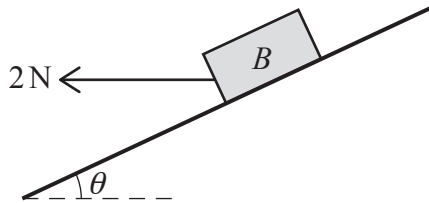
Section B
Mechanics

- 8 A particle P moves with constant acceleration $(3\mathbf{i} - 2\mathbf{j})\text{ms}^{-2}$. At time $t = 4$ seconds, P has velocity $6\mathbf{i}\text{ms}^{-1}$.

Determine the speed of P at time $t = 0$ seconds.

[4]

9



A block B of weight 10 N lies at rest in equilibrium on a rough plane inclined at θ to the horizontal. A horizontal force of magnitude 2 N , acting above a line of greatest slope, is applied to B (see diagram).

- (a) Complete the diagram in the Printed Answer Booklet to show all the forces acting on B . [1]

It is given that B remains at rest and the coefficient of friction between B and the plane is 0.8 .

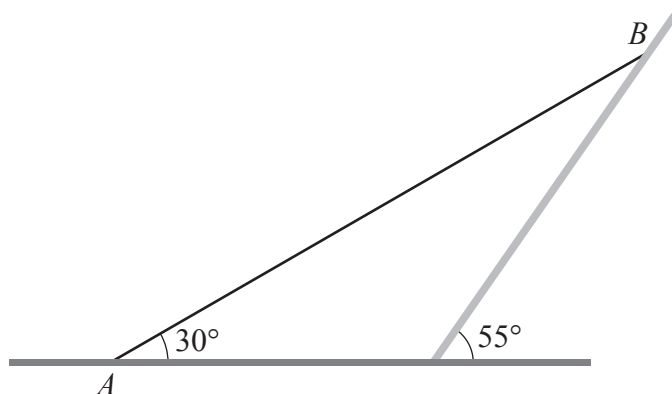
- (b) Determine the greatest possible value of $\tan \theta$. [5]

- 10 A particle P of mass $m\text{ kg}$ is moving on a smooth horizontal surface under the action of two constant horizontal forces $(-4\mathbf{i} + 2\mathbf{j})\text{N}$ and $(a\mathbf{i} + b\mathbf{j})\text{N}$. The resultant of these two forces is $\mathbf{R}\text{N}$. It is given that $\mathbf{R}$ acts in a direction which is parallel to the vector $-\mathbf{i} + 3\mathbf{j}$.

- (a) Show that $3a + b = 10$. [3]

It is given that $a = 6$ and that P moves with an acceleration of magnitude $5\sqrt{10}\text{ms}^{-2}$.

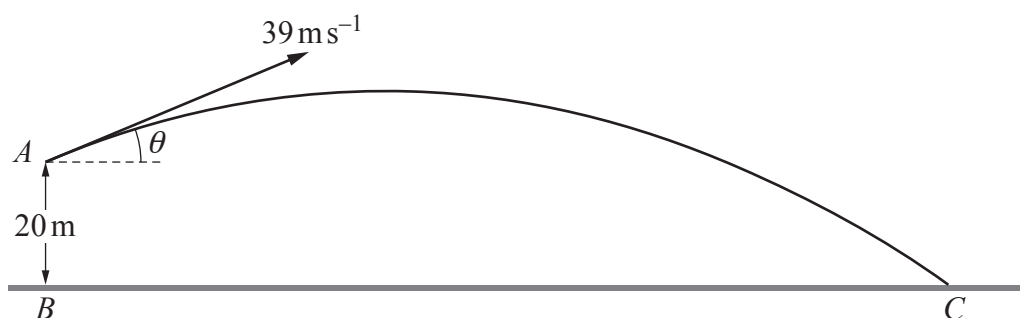
- (b) Determine the value of m . [4]



A uniform rod AB , of weight 20 N and length 2.8 m , rests in equilibrium with the end A in contact with rough horizontal ground and the end B resting against a smooth wall inclined at 55° to the horizontal. The rod, which rests in a vertical plane that is perpendicular to the wall, is inclined at 30° to the horizontal (see diagram).

- (a) Show that the magnitude of the force acting on the rod at B is 9.56 N , correct to **3** significant figures. [3]
- (b) Determine the magnitude of the contact force between the rod and the ground. Give your answer correct to **3** significant figures. [5]

12 In this question you should take the acceleration due to gravity to be 10 m s^{-2} .



A small ball P is projected from a point A with speed 39 m s^{-1} at an angle of elevation θ , where $\sin \theta = \frac{5}{13}$ and $\cos \theta = \frac{12}{13}$. Point A is 20 m vertically above a point B on horizontal ground. The ball first lands at a point C on the horizontal ground (see diagram).

The ball P is modelled as a particle moving freely under gravity.

- (a) Find the maximum height of P above the ground during its motion. [3]

The time taken for P to travel from A to C is T seconds.

- (b) Determine the value of T . [3]
 (c) State **one** limitation of the model, other than air resistance or the wind, that could affect the answer to part (b). [1]

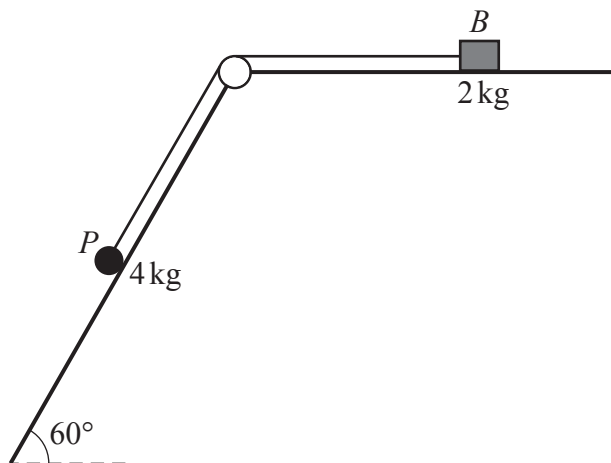
At the instant that P is projected, a second small ball Q is released from rest at B and moves towards C along the horizontal ground.

At time t seconds, where $t \geq 0$, the velocity $v \text{ m s}^{-1}$ of Q is given by

$$v = kt^3 + 6t^2 + \frac{3}{2}t,$$

where k is a positive constant.

- (d) Given that P and Q collide at C , determine the acceleration of Q immediately before this collision. [6]



The diagram shows a small block B , of mass 2 kg , and a particle P , of mass 4 kg , which are attached to the ends of a light inextensible string. The string is taut and passes over a small smooth pulley fixed at the intersection of a horizontal surface and an inclined plane. The particle can move on the inclined plane, which is rough, and which makes an angle of 60° with the horizontal. The block can move on the horizontal surface, which is also rough.

The system is released from rest, and in the subsequent motion P moves down the plane and B does not reach the pulley.

It is given that the coefficient of friction between P and the inclined plane is twice the coefficient of friction between B and the horizontal surface.

(a) Determine, in terms of g , the tension in the string. [7]

When P is moving at 2 ms^{-1} the string breaks. In the 0.5 seconds after the string breaks P moves 1.9 m down the plane.

(b) Determine the deceleration of B after the string breaks. Give your answer correct to 3 significant figures. [5]

END OF QUESTION PAPER

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