



Oxford Cambridge and RSA

# A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

**Friday 15 June 2018 – Afternoon**

**Time allowed: 2 hours**



**You must have:**

- Printed Answer Booklet

**You may use:**

- a scientific or graphical calculator

## INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

## INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

## Formulae

### A Level Mathematics A (H240)

#### Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

#### Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

#### Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

#### Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

#### Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

#### Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

#### Small angle approximations

$$\sin \theta \approx \theta, \cos \theta \approx 1 - \frac{1}{2}\theta^2, \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

**Trigonometric identities**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

**Numerical methods**

Trapezium rule:  $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$ , where  $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

**Probability**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**Standard deviation**

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

**The binomial distribution**

If  $X \sim B(n, p)$  then  $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ , Mean of  $X$  is  $np$ , Variance of  $X$  is  $np(1-p)$

**Hypothesis test for the mean of a normal distribution**

If  $X \sim N(\mu, \sigma^2)$  then  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$  and  $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

**Percentage points of the normal distribution**

If  $Z$  has a normal distribution with mean 0 and variance 1 then, for each value of  $p$ , the table gives the value of  $z$  such that  $P(Z \leq z) = p$ .

$p$	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$z$	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

## Section A: Pure Mathematics

Answer **all** the questions.

- 1 A circle with centre  $C$  has equation  $x^2 + y^2 + 8x - 2y - 7 = 0$ .

Find

- (i) the coordinates of  $C$ , [2]  
 (ii) the radius of the circle. [1]

- 2 Solve the equation  $|2x - 1| = |x + 3|$ . [3]

- 3 **In this question you must show detailed reasoning.**

A gardener is planning the design for a rectangular flower bed. The requirements are:

- the length of the flower bed is to be 3 m longer than the width,
- the length of the flower bed must be at least 14.5 m,
- the area of the flower bed must be less than  $180 \text{ m}^2$ .

The width of the flower bed is  $x$  m.

By writing down and solving appropriate inequalities in  $x$ , determine the set of possible values for the width of the flower bed. [6]

- 4 **In this question you must show detailed reasoning.**

The functions  $f$  and  $g$  are defined for all real values of  $x$  by

$$f(x) = x^3 \quad \text{and} \quad g(x) = x^2 + 2.$$

- (i) Write down expressions for

(a)  $fg(x)$ , [1]

(b)  $gf(x)$ . [1]

- (ii) Hence find the values of  $x$  for which  $fg(x) - gf(x) = 24$ . [6]

- 5 (i) Use the trapezium rule, with two strips of equal width, to show that

$$\int_0^4 \frac{1}{2 + \sqrt{x}} dx \approx \frac{11}{4} - \sqrt{2}. \quad [5]$$

- (ii) Use the substitution  $x = u^2$  to find the exact value of

$$\int_0^4 \frac{1}{2 + \sqrt{x}} dx. \quad [6]$$

- (iii) Using your answers to parts (i) and (ii), show that

$$\ln 2 \approx k + \frac{\sqrt{2}}{4},$$

where  $k$  is a rational number to be determined. [2]

- 6 It is given that the angle  $\theta$  satisfies the equation  $\sin\left(2\theta + \frac{1}{4}\pi\right) = 3 \cos\left(2\theta + \frac{1}{4}\pi\right)$ .

- (i) Show that  $\tan 2\theta = \frac{1}{2}$ . [3]

- (ii) Hence find, in surd form, the exact value of  $\tan \theta$ , given that  $\theta$  is an obtuse angle. [5]

- 7 The gradient of the curve  $y = f(x)$  is given by the differential equation

$$(2x - 1)^3 \frac{dy}{dx} + 4y^2 = 0$$

and the curve passes through the point  $(1, 1)$ . By solving this differential equation show that

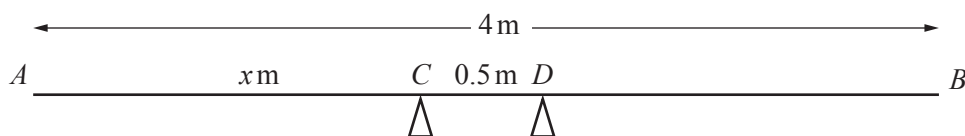
$$f(x) = \frac{ax^2 - ax + 1}{bx^2 - bx + 1},$$

where  $a$  and  $b$  are integers to be determined. [9]

## Section B: Mechanics

Answer **all** the questions.

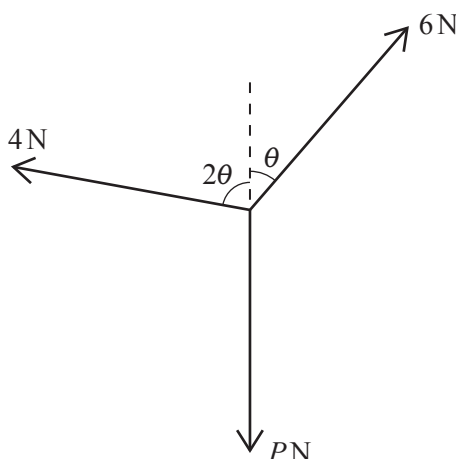
- 8 In this question  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$  denote unit vectors which are horizontal and vertically upwards respectively.
- A particle of mass 5 kg, initially at rest at the point with position vector  $\begin{pmatrix} 2 \\ 45 \end{pmatrix}$  m, is acted on by gravity and also by two forces  $\begin{pmatrix} 15 \\ -8 \end{pmatrix}$  N and  $\begin{pmatrix} -7 \\ -2 \end{pmatrix}$  N.
- (i) Find the acceleration vector of the particle. [3]
- (ii) Find the position vector of the particle after 10 seconds. [3]
- 9 A uniform plank  $AB$  has weight 100 N and length 4 m. The plank rests horizontally in equilibrium on two smooth supports  $C$  and  $D$ , where  $AC = x$  m and  $CD = 0.5$  m (see diagram).



The magnitude of the reaction of the support on the plank at  $C$  is 75 N. Modelling the plank as a rigid rod, find

- (i) the magnitude of the reaction of the support on the plank at  $D$ , [1]
- (ii) the value of  $x$ . [3]
- A stone block, which is modelled as a particle, is now placed at the end of the plank at  $B$  and the plank is on the point of tilting about  $D$ .
- (iii) Find the weight of the stone block. [3]
- (iv) Explain the limitation of modelling
- (a) the stone block as a particle, [1]
- (b) the plank as a rigid rod. [1]

- 10 Three forces, of magnitudes 4 N, 6 N and  $P$  N, act at a point in the directions shown in the diagram.



The forces are in equilibrium.

- (i) Show that  $\theta = 41.4^\circ$ , correct to 3 significant figures. [4]

- (ii) Hence find the value of  $P$ . [2]

The force of magnitude 4 N is now removed and the force of magnitude 6 N is replaced by a force of magnitude 3 N acting in the same direction.

- (iii) Find

- (a) the magnitude of the resultant of the two remaining forces, [3]

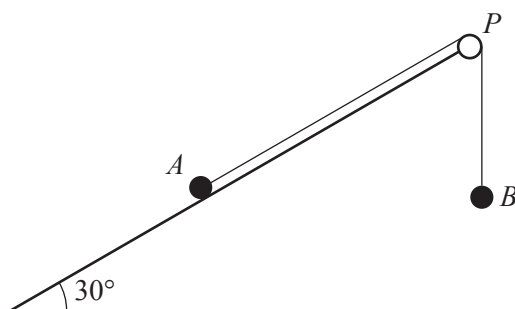
- (b) the direction of the resultant of the two remaining forces. [2]

- 11 The velocity  $v \text{ m s}^{-1}$  of a car at time  $t \text{ s}$ , during the first 20 s of its journey, is given by  $v = kt + 0.03t^2$ , where  $k$  is a constant. When  $t = 20$  the acceleration of the car is  $1.3 \text{ m s}^{-2}$ . For  $t > 20$  the car continues its journey with constant acceleration  $1.3 \text{ m s}^{-2}$  until its speed reaches  $25 \text{ m s}^{-1}$ .

- (i) Find the value of  $k$ . [3]

- (ii) Find the total distance the car has travelled when its speed reaches  $25 \text{ m s}^{-1}$ . [7]

- 12 One end of a light inextensible string is attached to a particle  $A$  of mass  $m$  kg. The other end of the string is attached to a second particle  $B$  of mass  $\lambda m$  kg, where  $\lambda$  is a constant. Particle  $A$  is in contact with a rough plane inclined at  $30^\circ$  to the horizontal. The string is taut and passes over a small smooth pulley  $P$  at the top of the plane. The part of the string from  $A$  to  $P$  is parallel to a line of greatest slope of the plane. The particle  $B$  hangs freely below  $P$  (see diagram).



The coefficient of friction between  $A$  and the plane is  $\mu$ .

- (i) It is given that  $A$  is on the point of moving down the plane.
- (a) Find the exact value of  $\mu$  when  $\lambda = \frac{1}{4}$ . [7]
- (b) Show that the value of  $\lambda$  must be less than  $\frac{1}{2}$ . [2]
- (ii) Given instead that  $\lambda = 2$  and that the acceleration of  $A$  is  $\frac{1}{4}g \text{ m s}^{-2}$ , find the exact value of  $\mu$ . [5]

**END OF QUESTION PAPER**

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