OCR
Oxford Cambridge and RSA

GCE

Mathematics A

Unit **H240/01**: Pure Mathematics

Advanced GCE

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for A Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			$m = \frac{17-5}{4-1}$ (= 4)	**M1**	**1.1a**	Attempt to find gradient of *AB*	Fraction must be correct way around, with coordinates used in a consistent order in the numerator and denominator
			$m_{\text{perp}} = -\frac{1}{4}$	**M1**	**1.1a**	Attempt gradient of perpendicular line	Use $m_1m_2 = -1$ with their numerical gradient Could be implied by the gradient that they use in the equation of the line
			$y-8=-\frac{1}{4}(x-2)$	**M1**	**1.1a**	Attempt equation of line through (2, 8), using their attempt at a perpendicular gradient	Either substitute into $y - y_1 = m(x - x_1)$ or use $y = mx + c$, as far as attempting c If not correct, then their gradient must be either the negative or the reciprocal of their original gradient eg if $m_1 = 4$, then $m_2 = -4$ or $\frac{1}{4}$ would be allowed
			$x + 4y = 34$	**A1**	**1.1**	Obtain $x + 4y = 34$	oe in required form ie with x and y terms on one side of the equation and a constant term on the other a, b and c do not have to be integers eg accept $\frac{1}{4}x + y = \frac{17}{2}$ (but not $\frac{34}{4}$)
				[4]			

Question			Answer	Marks	AO	Guidance	
2	(i)		$0.5 \times 0.5\{(e^{0} + e^{2^2}) + 2(e^{0.5^2} + e^{1^2} + e^{1.5^2})\}$	B1	1.1	Obtain all five ordinates and no others	Allow decimal equivs (1, 1.284, 2.718, 9.488, 54.598) – 3sf or better B0 if other ordinates seen unless clearly not intended to be used
				M1	1.1a	Use correct structure for trapezium rule with $h = 0.5$	Big brackets need to be seen or implied (40.9 is the result of no brackets) y-values must be correctly placed Must be using attempts at all 5 y-values Must be attempting area between $x = 0$ and $x = 2$
			= 20.6	A1	1.1	Obtain 20.6, or better	Allow more accurate answers in the range [20.64, 20.65)
				[3]			
	(ii)		Use more trapezia, of a narrower width, over the same interval	B1	2.4	Convincing reason	Condone just 'more trapezia' or 'narrower trapezia' Could refer to 'strips' not 'trapezia'
				[1]			

Question			Answer	Marks	AO	Guidance	
3			**DR**				
			$(x^2 - 5)(x^2 + 1) = 0$	**M1**	**3.1a**	Attempt to solve disguised quadratic, which has first been rearranged to a useable form **DR** so method must be seen	Substitution or direct factorisation Could use quadratic formula M0 for $(x - 5)(x + 1)$, or equiv with formula, unless clear substitution of $x = x^2$
			$x^2 = 5$	**A1**	**1.1**	Obtain at least $x^2 = 5$	Could be implied by their explicit substitution eg $u = 5$, where $u = x^2$ May still have $x^2 = -1$ as well, but A0 if any other value for x^2
			$x^2 \geq 0$, so $x^2 + 1 = 0$ has no real solutions	**B1**	**2.3**	Explicitly reject $x^2 + 1 = 0$, with reasoning	eg negative numbers cannot be square rooted or $x^2 \neq -1$ as x is real $x^2 \neq -1$ is insufficient without further reasoning Must be sensible reason, not just 'math error' or 'not possible' Could say that there are only imaginary (or not real) roots (condone 'complex' roots) Could say x^2 cannot be negative, but B0 for x^2 must be positive (or equiv as an inequality)
			$x = \pm\sqrt{5}$	**A1**	**1.1**	Obtain $x = \pm\sqrt{5}$	A0 if any extra roots. Both roots required, and must be exact
				[4]			

Question			Answer	Marks	AO	Guidance	
4			If n is even then n can be written as $2m$. $n^3+3n-1 = 8m^3 + 6m - 1$	**E1**	**2.1**	Consider when n is even	Substitute $2m$ or equiv Must include reasoning, including that $2m$ represents an even number
			$= 2(4m^3 + 3m) - 1$ For all m, $2(4m^3 + 3m)$ is even, hence $2(4m^3 + 3m) - 1$ is odd	**E1**	**2.4**	Conclude from useable form	Must be of a form where odd can be easily deduced **SR E1** for If n is even, n^3 is even, $3n$ is even, hence n^3+3n is even + even = even and therefore n^3+3n-1 is even – odd = odd Each step must be justified
			If n is odd then n can be written as $2m + 1$ $n^3+3n-1 = 8m^3 + 12m^2 + 6m + 1 + 6m + 3 - 1$ $= 8m^3 + 12m^2 + 12m + 3$	**E1**	**2.1**	Consider when n is odd	Substitute $2m + 1$ or equiv Must include reasoning, including that $2m + 1$ represents an odd number
			$= 2(4m^3 + 6m^2 + 6m) + 3$ For all m, $2(4m^3 + 6m^2 + 6m)$ is even, hence $2(4m^3 + 6m^2 + 6m) + 3$ is odd	**E1**	**2.4**	Conclude from useable form	Must be of a form where odd can be easily deduced **SR E1** for If n is odd, n^3 is odd, $3n$ is odd, hence n^3+3n is odd + odd = even and therefore n^3+3n-1 is even – odd = odd Each step must be justified
				[4]			

Question			Answer	Marks	AO	Guidance	
5	(i)		centre is (–3, 1)	**B1**	1.1	Correct centre of circle	Allow $x = -3$, $y = 1$
			$(x+3)^2 - 9 + (y-1)^2 - 1 - 10 = 0$ $(x+3)^2 + (y-1)^2 = 20$	**M1**	1.1a	Attempt to complete the square twice	Allow for $(x \pm 3)^2 \pm 9 + (y \pm 1)^2 \pm 1$ seen $(x \pm 3)^2 + (y \pm 1)^2 - 10 = 0$ is M0 as no evidence of subtracting the constant terms to complete the squares Or attempt to use $r^2 = g^2 + f^2 - c$
			radius = $2\sqrt{5}$ or $\sqrt{20}$	**A1**	1.1	Correct radius	From correct working only, including correct factorisation Allow $r = 4.47$, or better
				[3]			
	(ii)		$x^2 + (2x-3)^2 + 6x - 2(2x-3) - 10 = 0$ OR $(x+3)^2 + (2x-4)^2 = 20$	**M1**	3.1a	Substitute the linear equation into the quadratic equation	Either substitute for y, or an attempt at x Either use the given expanded equation or their attempt at a factorised equation
			$x^2 - 2x + 1 = 0$	**A1**	1.1	Correct three term quadratic	Must be three terms, but not necessarily on same side of equation
			$x = 1$	**A1**	1.1	BC, or from any valid method	A0 if additional incorrect x value
			(1, –1)	**A1**	2.1	A0 if additional points also given	Allow $x = 1$, $y = -1$
				[4]			
	(iii)		The line is a tangent to the circle at (1, –1)	**B1ft**	2.2a	Correct deduction Strict follow-through on their number of roots from **(ii)**	Allow just mention of 'tangent' Allow other correct statements such as the line and the circle only touch once
				[1]			

Question			Answer	Marks	AO	Guidance	
6	**(i)**		$f(x) = (x - 3)(2x^2 - x - 1)$	**M1**	**2.2a**	Attempt complete division by $(x - 3)$	Must be complete attempt Division – must be subtracting on each line (allow one error) Coefficient matching – valid attempt at all 3 coefficients Inspection – must give three correct terms on expansion Synthetic division – allow one error
				A1	**1.1**	Obtain correct quotient	Could be seen in division Cannot be implied by $A = 2$ etc
			$f(x) = (x - 3)(2x + 1)(x - 1)$	**A1**	**1.1**	Obtain fully factorised $f(x)$	Must be as product of all 3 linear factors Correct answer gets full marks, but an incorrect factorisation such as $(x - 3)(x + \frac{1}{2})(x - 1)$ is M0 unless method is seen
				[3]			
	(ii)		Sketch of positive cubic	**B1**	**1.2**	Three roots and two stationary points	Ignore any intercepts for this mark
			$(-0.5, 0)$, $(1, 0)$, $(3, 0)$, $(0, 3)$	**B1**	**2.2a**	All intercepts correctly indicated	ft their three factors Could be given as coordinates, or just values marked on relevant axes BOD if coordinates transposed as long as marked on correct axis
				[2]			

Question			Answer	Marks	AO	Guidance	
	(iii)		$\{x : x < -0.5\} \cup \{x : 1 < x < 3\}$	M1	2.2a	Identify one set of values	ft their cubic roots in **(ii),** even if not 3 real, distinct, roots Allow notation using just inequalities Allow interval notation eg $(-\infty, -0.5)$ and/or $(1, 3)$ If both sets of values given then ignore linking sign for this mark
				A1ft	2.5	Fully correct solution in set notation	ft their cubic roots in **(ii)**, as long as 3 real, distinct, roots Each set should have the correct structure ie $\{x: \quad\}$ with the sets linked by $\cup$ Allow equivs eg $\{x : x < -0.5\} \cup \{x : x > 1\} \cap \{x : x < 3\}$ eg $(-\infty, -0.5) \cup (1, 3)$ Do not accept $(x < -0.5) \cup (1 < x < 3)$
				[2]			
	(iv)		$y = 2(2x)^3 - 7(2x)^2 + 2(2x) + 3$ $= 16x^3 - 28x^2 + 4x + 3$ OR $y = (2x - 3)(4x + 1)(2x - 1)$	M1	1.2	Attempt f(2x) or f(0.5x)	Condone lack of brackets as long as implied by later work M0 if each term just multiplied or divided by 2
				A1	1.1	Obtain correct equation	Must have $y = \ldots$ Condone f(2x) = …, or f(x) = … Accept unsimplified equiv ISW an incorrect attempt to expand
				[2]			

Question			Answer	Marks	AO	Guidance	
7	(i)		$r = 0.98$	B1	3.3	Identify that r is 0.98	Allow any exact equiv, including $\frac{147}{150}$ or better
			$u_{12} = 150\times0.98^{11}$ or $u_{13} = 150\times0.98^{12}$	M1	3.4	Attempt u_{12} or u_{13} using ar^{n-1} with $a = 150$ and their r	Must attempt to evaluate at least one of the values Must be using correct formula Allow M1 even if subsequently incorrectly evaluated eg as $(ar)^{n-1}$
			$u_{12} = 120.1$ or $u_{13} = 117.7$	A1	1.1	Obtain one correct value	3sf or better
			$u_{12} > 120$ and $u_{13} < 120$, hence the thirteenth half marathon **A.G.**	A1	2.3	Conclude from both values with reference to half marathon number	Both values must be correct, and to a suitable degree of accuracy that allows comparison to 120 to be made Condone 'marathon' or 'run' instead of 'half marathon'
				[4]			**OR** B1 $r = 0.98$ or equiv M1 Equate ar^{n-1} with $a = 150$ and their r to 120 and attempt to find value for n (equation or inequality) A1 Obtain $n = 12.05$, or better Ignore inequality signs for this A1 A1 Conclusion with reference to half marathon number A0 if incorrect inequality signs
	(ii)		$\dfrac{150(1-0.98^n)}{1-0.98} = 2974$	M1*	3.1a	Equate S_N formula to 2974	Must be correct formula with $a = 150$ and their r Must be equated to 2974 Allow use of 2973.5 and/or 2974.5 as attempt to deal with 'nearest minute'

Question			Answer	Marks	AO	Guidance	
			$0.98^n = 0.6035$	**M1d***	**1.1**	Rearrange to useable form, and find value for n	Must be using correct formula for S_n Must be using correct processes to solve equation BC so may be no evidence of log use
			$n = 25$ hence Chris has run 25 half marathons	**A1**	**3.2a**	Conclude with reference to number of half marathons run	Must be put in context and not just $n = 25$ Condone 'marathons' or 'runs' instead of 'half marathons'
				[3]			
	(iii)		Does not take into account possible variations in conditions	**E1**	**3.5b**	Identify that times for individual half marathons may vary from the model	Any sensible reason eg weather, terrain, injury etc
			Assumes that times will continue to improve	**E1**	**3.5b**	Identify that improvements may not continue	Any sensible reason as to why the model has long-term limitations E0 for 'model will eventually predict 0 minutes'
				[2]			
8	**(i)**		$(1-\frac{1}{4}x)^{-\frac{1}{2}} = 1+(-\frac{1}{2})(-\frac{1}{4}x)+(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(-\frac{1}{4}x)^2$	**B1**	**1.1**	Obtain correct first two terms	Allow unsimplified coeffs
				M1	**1.1a**	Attempt third term in expansion of $(1-\frac{1}{4}x)^{-\frac{1}{2}}$	Product of attempt at binomial coefficient and $(-\frac{1}{4}x)^2$ Allow BOD on missing brackets Allow BOD on missing negative sign in third term
			$=1+\frac{1}{8}x+\frac{3}{128}x^2$	**A1**	**1.1**	Correct third term	Allow unsimplified coeffs

Question			Answer	Marks	AO	Guidance	
			$(4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}(1-\frac{1}{4}x)^{-\frac{1}{2}} = \frac{1}{2}(1-\frac{1}{4}x)^{-\frac{1}{2}}$ $(4-x)^{-\frac{1}{2}} = \frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$	**B1ft**	**1.1**	Correct expansion of $(4-x)^{-\frac{1}{2}}$	ft as $\frac{1}{2}$ (their three term expansion) No ISW if expression subsequently spoiled by attempt to simplify eg $\times 256$
				[4]			
	(ii)		$\frac{1}{2}a = 16$ hence $a = 32$	**B1ft**	**3.1a**	Correct value of a	ft their first term from **(i)**
			$2+\frac{1}{2}b = -1$ OR $\frac{1}{16}a+\frac{1}{2}b = -1$	**M1**	**2.2a**	Attempt equation involving b and a, or their numerical a	Must be using two relevant products (and no others) equated to –1 Allow for attempting equation – do not need to actually attempt solution for M1 Allow BOD if muddle between terms and coefficients
			$b = –6$	**A1**	**1.1**	Solve to obtain $b = –6$	A0 for – $6x$ unless subsequently corrected
				[3]			
9	**(i)**		$f(x) = c + 16 – (x – 4)^2$	**M1***	**3.1a**	Attempt to identify maximum point	Full attempt to complete the square Could differentiate, equate to 0 and solve to get $8 – 2x = 0$, so $x = 4$
			$c + 16 = 19$	**M1d***	**1.1a**	Link maximum point to 19	Link the constant term of their completed square to 19 – must involve c Allow equation or inequality (including incorrect inequality) If using differentiation then link f(their $x = 4$) to 19

Question			Answer	Marks	AO	Guidance	
			$c = 3$	**A1**	**1.1**	Solve to obtain $c = 3$	A0 if given as inequality unless subsequently corrected Must come from fully correct working, so $f(x) = c + 16 - (x + 4)^2$, leading to $c + 16 = 19$ hence $c = 3$ is M1 M1 A0
				[3]			**OR** M1* Attempt to use $b^2 - 4ac = 0$ on their attempt at $f(x) - 19 = 0$ M1d* Attempt to solve their $64 - 4(-1)(c - 19) = 0$ A1 Obtain $c = 3$
	(ii)		$f(2) = c + 12$	**B1**	**1.1**	Correct f(2)	Stated or implied by being used in later method
			$f(c + 12) = c + 8(c + 12) - (c + 12)^2$	**M1***	**1.2**	Attempt correct composition of ff	Must be attempt at composition of functions so M0 for $\{f(2)\}^2$
			$-48 - 15c - c^2 = 8$ $c^2 + 15c + 56 = 0$	**M1d***	**1.1a**	Equate to 8 and rearrange to useable form	Expand and rearrange to a three term quadratic Could be implied by the two correct roots
			$c = -7, c = -8$	**A1**	**2.1**	Both correct values for c	**BC**

Question			Answer	Marks	AO		Guidance
				[4]			**OR** for the first two marks **M1*** Attempt ff(x) ie attempt at ff(x) = $c + 8(c + 8x - x^2) - (c + 8x - x^2)^2$ **M1d*** Attempt ff(2) using their ff(x), which may no longer be correct
10	**(i)**		$\frac{dx}{dt} = 1 - 2t^{-2}$	**B1**	**1.1**	Correct $\frac{dx}{dt}$	Any equivalent form
			$\frac{dy}{dt} = 1 + 2t^{-2}$	**B1**	**1.1**	Correct $\frac{dy}{dt}$	Any equivalent form
			$\frac{dy}{dx} = \frac{1 + 2t^{-2}}{1 - 2t^{-2}} = \frac{\frac{t^2+2}{t^2}}{\frac{t^2-2}{t^2}}$	**M1**	**1.1a**	Attempt correct method to combine their derivatives	Division must be correct way around
			$\frac{dy}{dx} = \frac{t^2 + 2}{t^2 - 2}$	**A1**	**1.1**	Obtain correct derivative	Allow any simplified equivalent such as $1 + \frac{4}{t^2 - 2}$
				[4]			

Question			Answer	Marks	AO	Guidance	
	(ii)		$\frac{dy}{dx} = 0 \Rightarrow t^2 + 2 = 0$	E1ft	2.2a	Justify $t^2 + 2 = 0$ for stat point	Must state that gradient (or $\frac{dy}{dx}$) = 0 (cannot be implied by method) then equate their numerator to 0 Allow use of a gradient that is no longer a fraction
			$t^2 \geq 0$, hence $t^2 + 2 = 0$ has no solutions, hence curve has no stationary points	E1	2.4	Justify no stationary points	Explain why there are no solutions eg referring to $t^2 + 2 \geq 2$ eg t^2 is always positive (as $t \neq 0$ given) eg $t^2 + 2 = 0$ has no real roots and conclude with 'no stationary points' Must now be from a fully correct derivative only
				[2]			
	(iii)		$x + y = 2t$ hence $t = \frac{1}{2}(x+y)$	B1	1.1	Correct expression for t	Any correct equation involving t along with x and/or y where t only appears once
			$x = \frac{1}{2}(x+y) + \frac{2}{\frac{1}{2}(x+y)}$	M1	1.2	Substitute for t into either equation	Expression for t must be correct Could be using attempt (possibly no longer correct) at a rearranged parametric equation eg $xt - t^2 = 2$
			$2x(x + y) = (x + y)^2 + 8$ $2x^2 + 2xy = x^2 + 2xy + y^2 + 8$	M1	3.1a	Attempt rearrangement	As far as requested form
			$x^2 - y^2 = 8$	A1	1.1	Correct equation	Any correct three term equivalent Allow A1 for eg $y = \pm\sqrt{x^2 - 8}$, but A0 if not ±
				[4]			

Question			Answer	Marks	AO	Guidance	
11	(i)		When $t = 0$, $M = 300$	**B1**	**2.2a**	Identify that the initial mass is 300g	Could be implied by eg $e^{-0.05t} = 0.5$
			$300e^{-0.05t} = 150$ $e^{-0.05t} = 0.5$ $-0.05t = \ln 0.5$	**M1**	**3.1a**	Equate to 150 and attempt to solve	Correct order of operations as far as attempting t If using logs on $300e^{-0.05t} = 150$ then the LHS must be dealt with correctly
			$t = 13.9$ (minutes)	**A1**	**1.1**	Obtain 13.86, or better	Allow 14 minutes www Or 13 minutes and 52 seconds
				[3]			
	(ii)		$M_2 = 400e^{kt}$	**B1**	**2.2a**	State or imply $400e^{kt}$	Could be implied by stating general form of Ae^{kt} with $A = 400$ Any unknowns permitted
			$320 = 400e^{10k}$ $k = 0.1\ln 0.8$	**M1**	**1.1a**	Attempt to find k	Substitute $M = 320$, $t = 10$ and attempt k Must be using valid method
			$M_2 = 400e^{-0.0223t}$	**A1**	**1.1**	Obtain correct expression for mass of second substance	Allow exact or decimal k (2sf or better) Must be seen or used as a complete term, not just implied by stated values of A and k
			Substance 1: $\frac{dM_1}{dt} = -15e^{-0.05t}$ Substance 2: $\frac{dM_2}{dt} = -8.93e^{-0.0223t}$	**M1**	**3.1a**	Attempt differentiation at least once	To obtain $ae^{-0.05t}$ or $be^{-0.0223t}$, where a and b are non-zero constants not 300 and 400 respectively
				A1ft	**1.1**	Both derivatives correct	Following their equation for substance 2

Question			Answer	Marks	AO	Guidance	
			$-15e^{-0.05t} = -8.93e^{-0.0223t}$ $e^{0.0277t} = 1.681$	**M1**	**3.1a**	Equate derivatives and rearrange as far as $e^{f(t)} = c$	Equation must be of the form $ae^{-0.05t} = be^{-0.0223t}$ Combining like terms to result in a two term equation – not necessarily on opposite sides If logs are introduced earlier then allow M1 only if the products are correctly split so eg ln(15) × (–0.05t) is M0 M0 if attempting to take a log of a term that is negative
			$0.0277t = 0.519$	**M1**	**1.1**	Attempt to solve equation of form $e^{f(t)} = c$	As far as attempting t Or equiv if logs have been taken earlier
			time = 18.75 minutes	**A1**	**3.2a**	Obtain correct value for t Allow 18.7, 18.8 or 19 mins	Units required Could be 18 minutes and 45 seconds Must have been working with 3sf or better throughout
				[8]			
12			**DR**				
			$\frac{dy}{dx} = \frac{(-8\sin 2x)(3-\sin 2x)-(4\cos 2x)(-2\cos 2x)}{(3-\sin 2x)^2}$	**M1**	**3.1a**	Attempt use of quotient rule	Correct structure, including subtraction in numerator Could be equivalent using the product rule
				A1	**1.1**	Obtain correct derivative	Award A1 once correct derivative seen even subsequently spoiled by simplification attempt

Question			Answer	Marks	AO	Guidance	
			EITHER when $x=\frac{1}{4}\pi$, gradient $=\frac{-16-0}{4}=-4$ **OR** $\frac{(-8\sin\frac{\pi}{2})(3-\sin\frac{\pi}{2})-(4\cos\frac{\pi}{2})(-2\cos\frac{\pi}{2})}{(3-\sin\frac{\pi}{2})^2}=-4$	**M1**	**2.4**	**DR** Attempt to find gradient at $\frac{1}{4}\pi$	**EITHER** State that $x=\frac{1}{4}\pi$ is being used, and show their fraction with each term (including 0) explicitly evaluated before being simplified ie $x=\frac{1}{4}\pi$, gradient $=-4$ is M0 **OR** Substitute $\frac{1}{4}\pi$ into their derivative and evaluate
			gradient of normal is $\frac{1}{4}$	**B1ft**	**2.1**	Correct gradient of normal	ft their gradient of tangent
			area of triangle is $\frac{1}{2}\times\frac{1}{16}\pi\times\frac{1}{4}\pi(=\frac{1}{128}\pi^2)$	**M1**	**2.1**	Attempt area of triangle ie $\frac{1}{2}\times\frac{1}{4}\pi\times(\text{their } y)$	*y* coordinate could come from using equation of normal, $y=\frac{1}{4}(x-\frac{1}{4}\pi)$, or from using gradient of normal Could integrate equation of normal
			$\int\frac{4\cos 2x}{3-\sin 2x}\,dx=-2\ln\lvert 3-\sin 2x\rvert$	**M1***	**3.1a**	Obtain integral of form $k\ln\lvert 3-\sin 2x\rvert$	Condone brackets not modulus Allow any method, including substitution, as long as integral of correct form
				A1	**1.1**	Obtain correct integral	Possibly with unsimplified coefficient
			$\int_0^{\frac{1}{4}\pi}\frac{4\cos 2x}{3-\sin 2x}\,dx=(-2\ln 2)-(-2\ln 3)$	**M1d***	**2.1**	Attempt use of limits	Using $\frac{1}{4}\pi$ and 0; correct order and subtraction (oe if substitution used) Must see a minimum of $-2\ln 2+2\ln 3$
			$2\ln 3-2\ln 2=\ln 9-\ln 4=\ln\frac{9}{4}$ OR $2\ln 3-2\ln 2=2\ln\frac{3}{2}=\ln\frac{9}{4}$	**A1**	**1.1**	Correct area under curve	Must be exact At least one log law seen to be used before final answer

Question			Answer	Marks	AO	Guidance	
			hence total area is $\ln\frac{9}{4}+\frac{1}{128}\pi^2$ **A.G.**	**A1**	**2.1**	Obtain correct total area	Any equivalent exact form AG so method must be fully correct A0 if the gradient of – 4 results from an incorrect derivative having been used A0 if negative area of triangle not dealt with convincingly
				[10]			
13	**(i)**	**(a)**	$\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{k}{N}$	**B1**	**3.3**	State correct differential equation	Or $\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{1}{k'N}$ or equiv with k on LHS
				[1]			
		(b)	$kt=\int N\mathrm{d}N$ $kt=\frac{1}{2}N^2+c$	**M1***	**2.1**	Attempt integration	Obtain equation of form $at = bN^2 + c$ Condone no + c
			$0=80000+c\Rightarrow c=-80000$	**M1d***	**3.4**	Attempt c from (0, 400)	Substitute (0, 400) into their equation containing c and k Could give value for c, or could result in an equation involving both c and k depending on structure
			$k=96800-80000=16800$	**M1d***	**3.4**	Attempt k from (1, 440)	Substitute (1, 440) into their equation containing c (possibly now numerical) and k If c is numerical then value of k must be attempted If this gives second equation in c and k then the equations need to be solved simultaneously for c and k to award M1
			$N=\sqrt{(33600t+160000)}$	**A1**	**1.1**	Correct equation for N	N must be the subject of the equation

Question			Answer	Marks	AO	Guidance	
				[4]			
	(ii)		$\int 3988N^{-2}\mathrm{d}N = \int \mathrm{e}^{-0.2t}\mathrm{d}t$	M1	3.1a	Separate variables and attempt integration	Must be valid method to separate variables so allow coefficient slips only Some attempt to integrate, but may not be correct BOD if no integral signs, as long as integration is actually attempted
			$-3988N^{-1} = -5\mathrm{e}^{-0.2t} + c$ **OR** $-N^{-1} = -\frac{5}{3988}\mathrm{e}^{-0.2t} + c$	M1*	1.1a	Integrate to obtain answer of correct form	Obtain integral of the form $aN^{-1} = b\mathrm{e}^{-0.2t} + c$ or equiv
				A1	1.1	Obtain correct integral	Condone no + c Any equivalent form
			$-9.97 = -5 + c \Rightarrow c = -4.97$ **OR** $-\frac{1}{400} = -\frac{5}{3988} + c \Rightarrow c = -\frac{497}{398800}$	M1d*	2.2a	Attempt c from (0, 400) or (1, 440)	As far as attempting numerical value for c NB (0, 400) gives –4.97, (1, 440) gives an answer which rounds to –4.97 Equation may no longer be correct
			$\frac{3988}{N} = 5e^{-0.2t} + 4.97$ **OR** $\frac{1}{N} = \frac{5}{3988}e^{-0.2t} + \frac{497}{398800}$	M1d*	1.1	Attempt to make N the subject	Using correct algebraic processes throughout, but allow sign slips – this includes any rearrangement attempt made prior to attempting c Must involve a c, either numerical or still as c
			$N = \frac{3988}{5\mathrm{e}^{-0.2t}+4.97}$	A1	1.1	Correct equation for N	Any correct equation of form $N = \ldots$
				[6]			

Question			Answer	Marks	AO	Guidance	
	(iii)		Model in **(i)** predicts that population will continue to increase	**E1**	**3.5a**	Comment about continuing to increase	Allow comments such as tending to infinity Any additional comments must also be correct so E0 for eg ‘will always increase at a steady rate’ but E1 for ‘will always increase but the rate of increase will decrease’
			Model in **(ii)** predicts that population will tend towards a limit of 802	**E1**	**3.5a**	Comment about tending towards a limit of 802	Allow a limit of 803 or 802.4 Must come from a fully correct function Any additional comments must also be correct
				[2]			

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