

GCE

Mathematics A

Unit H240/02: Pure Mathematics and Statistics

Advanced GCE

Mark Scheme for June 2018

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

| Annotation in scoris | Meaning |
|---|--|
| ✓ and × | |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0, 1 |
| SC | Special case |
| ^ | Omission sign |
| MR | Misread |
| Highlighting | |
| | |
| Other abbreviations in mark | Meaning |
| scheme | |
| E1 | Mark for explaining a result or establishing a given result |
| dep* | Mark dependent on a previous mark, indicated by * |
| cao | Correct answer only |
| | |
| oe | Or equivalent |
| oe rot | Or equivalent Rounded or truncated |
| oe rot soi | Or equivalent Rounded or truncated Seen or implied |
| oe rot soi www | Or equivalent Rounded or truncated Seen or implied Without wrong working |
| oe rot soi www AG | Or equivalent Rounded or truncated Seen or implied Without wrong working Answer given |
| oe rot soi www AG awrt | Or equivalent Rounded or truncated Seen or implied Without wrong working Answer given Anything which rounds to |
| oe rot soi www AG awrt BC | Or equivalent Rounded or truncated Seen or implied Without wrong working Answer given Anything which rounds to By Calculator |

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Subject-specific Marking Instructions for A Level Mathematics A

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

The following types of marks are available.

Μ

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

Е

Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate

d

e

f

g

h

i

j

passes through the correct answer as part of a wrong argument.

- When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
 - The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow
 - through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.
 - Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
 - For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
 - If in any case the scheme operates with considerable unfairness, consult your Team Leader.

| | Final Version | | | | | | | | | |
|---|----------------------|--|-----------|------|---|-------------------------------------|--|--|--|--|
| | Questio | n Answer | Mks | AO | Guidance | | | | | |
| 1 | (i) | $2(x^2-6x+11.5)$ | B1 | 1.1a | or $a = 2$ | | | | | |
| | | $2((x-3)^2 + 11.5 - 9)$ | B1 | 1.1 | or $b = -3$ | | | | | |
| | | | M1 | 1.1 | $23 - 2(\text{their } b)^2$ | | | | | |
| | | $2(x-3)^2+5$ | A1 | 1.1 | or $c = 5$ | | | | | |
| | | | [4] | | | | | | | |
| 1 | (ii) | $2(x+3)^2 + 5$ is always +ve | | | or $2(x+3)^2 = -5$, which is impossible | $2(x+3)^2+5=0$ | | | | |
| | | or $2(x+3)^2 + 5 > 0$ | | | or "+ve quadratic" and min on $y = 5$ | $\Rightarrow x = \sqrt{\text{neg}}$ | | | | |
| | | or $2(x+3)^2 + 5 \ge 5$ | | | or "+ve" quadratic; TP at (3, 5). Both | or $x + 3 = \sqrt{\text{neg}}$ | | | | |
| | | Hence no real roots | B1f | 1.1 | Hence no real roots | ft their (i) ($a \& c > 0$) | | | | |
| | | | [1] | | Must use (i), not use D | | | | | |
| 1 | (iii) | $2(x-3)^2 = 2(x^2 - 6x + 9)$ | M1 | 1.1a | or $12^2 - 8k = 0$ | | | | | |
| | | k = 18 | A1 | 2.2a | | | | | | |
| | | | [2] | | | | | | | |
| 2 | (i) | $(1 - (-3))^{2} + (-2 - (-1))^{2} + (5 - 2)^{2}$ (= 26) | M1 | 1.1a | Attempt. Allow with one sign error | $\sqrt{\text{not nec'y}}$ | | | | |
| | | Length = $\sqrt{26}$ or 5.10 or 5.1 (2 sf) | A1 | 1.1 | | | | | | |
| | | | [2] | | | | | | | |
| 2 | (ii) | $\begin{pmatrix} -1 \end{pmatrix}$ | | | | | | | | |
| | | -1.5 | | | | | | | | |
| | | 2.5 | B1 | 1.1 | | | | | | |
| | | (5.5) | | | | | | | | |
| | | | [1] | | | | | | | |
| 2 | (iii) | (4) | | | | SC Incorrect, but | | | | |
| | | | | | LILIN | equal, vectors BA & | | | | |
| | | $DA = \begin{bmatrix} -1 \end{bmatrix}$ | M1 | 2.1 | or quote result for BA from (ii) or (i)(a) | PQ with correct | | | | |
| | | | | | | conclusion SC B1 | | | | |
| | | (5) (1) (4) | | | | | | | | |
| | | | | | or similar methods with AQ & BP | Allow without method | | | | |
| | | $PQ = \begin{vmatrix} 1 \\ - \end{vmatrix} \begin{pmatrix} 2 \\ -1 \end{vmatrix} $ | M1 | 1.1 | or AB and QP etc | SC Lengths only seen: | | | | |
| | | (3) (0) (3) | | | Allow find eg AB and PQ | M1M0 | | | | |
| | | | | | RA = PO with arrows | | | | | |
| | | BA = PO and $BA // PO$ | | | or $D_{A} = I Q$ with allows | Iust BA = PO = A0 | | | | |
| | | $DII = I \mathcal{Q}$ and $DII // I \mathcal{Q}$ | | | or $ BA = PQ \ll BP = AQ $ snown & stated | | | | | |
| | | and han as $ABDO$ is a normalial array (AC) | A 1 | 2.25 | Deth statements needed den M1M1 | | | | | |
| | | and hence $ADPQ$ is a parahelogram (AG) | AI [2] | 2.2a | bour statements needed, dep WITWI | | | | | |
| 1 | | | [ວ] | | | | | | | |

| | Question | | Answer | Mks | AO | Guidance | |
|---|---------------|---|---|------------|-------------|--|--|
| 3 | (i) | | 7 hours | B1 | 2.2b | Allow between 6 and 8 hours. | |
| | | | | [1] | | | |
| 3 | (ii) | | Dave will gain no marks even if he does no | B1 | 3.5a | oe | |
| | () | | revision | | 0.00 | | |
| | | | | | | | |
| 2 | (;;;) | | (Bob believes) too much revision leads to the dness | D1 | 3 50 | a Must suggest reason for drop in marks | |
| 3 | (III) | | lower mark (Avesha does not) | DI | 3.5a | be Must suggest reason for drop in marks | |
| | | | Tower mark. (Ayesha does not.) | [1] | | | |
| | | | (She believes that) | <u> </u> | | or (She thinks) | |
| 3 | (iv) | | however much or little revision she does it will | B 1 | 2.2b | revision will not increase her mark | revision is unhelpful |
| | , í | | make no difference to the mark she obtains | | | or revision is unnec'y to obtain high mark | oe |
| | | | | [1] | | | |
| 4 | | | Summary of method | | | | |
| | | | Use of $\cos(A + B)$ or $\sin(A + B)$ or $\cos 2\theta$ formula | M1 | 3.1a | Correct formula | |
| | | | Correct result | Al | 2.1 | | |
| | | | Use of one of the above or sin2A formula | М1 | 11 | Correct formula | |
| | | | Correctly obtain result | | 1.1 | Confect formula | |
| | | | Concerty obtain result | AI | 1.1 | | |
| | | | Example of method | | | | |
| | | | $\overline{\sin^2(\theta + 45)} - \cos^2(\theta + 45) \equiv -\cos 2(\theta + 45)$ | M1 | | <u>Use</u> of correct $\cos 2\theta$ formula | |
| | | | | A1 | | Correct result | |
| | | | $\equiv -\cos\left(2\theta + 90\right)$ | M1 | | <u>Use</u> of correct $\cos(A + B)$ formula | |
| | | | $\equiv -\left[\cos 2\theta \cos 90 - \sin 2\theta \sin 90\right] \equiv \sin 2\theta \text{ AG}$ | A1 | | Must see this step and final answer | |
| | | | | [4] | | | |
| 5 | (i) | | eg $1 + 3 = 4$ or $4 + 5 = 9$ or $9 + 7 = 16$ | B1 | 1.1 | or $25 + 11 = 36$ or any correct example | |
| | | | | [1] | | | |
| 5 | (ii) | | $ \lim_{n \to \infty} m - n = 1 \qquad (\text{or } -1) $ | E1 | 2.3 | | (or if $m + n = 1$) |
| | | | then $(m-n)(m+n)$ could be prime | [1] | | or One of the factors of p could be 1 | |
| 5 | (iii) | | $\int dt S - n^2$ | | + | | $853-m^2$ n^2 & m $n-1$ |
| 3 | (11) | | rightarrow 0 Let $S = nrightarrow 0 Other square number is (n + 1)^2$ | М1 | 319 | or Other square number is $(\sqrt{S} + 1)^2$ | $ 0.55 - m - n \propto m - n - 1 $ $ \Rightarrow 853 - m + n $ |
| | | | $\Rightarrow 853 = (n+1)^2 - n^2 = 2n+1$ | M1 | 2.29 | $\Rightarrow 853 = (\sqrt{S} + 1)^2 - S = 2\sqrt{S} + 1$ | $\Rightarrow 853 = 2n + 1$ |
| | | | $ \Rightarrow n = 426 $ | A1 | 1 1 | $\Rightarrow \sqrt{S} = 426$ | $\Rightarrow 0.05 = 2n + 1$ $\Rightarrow n = 426$ |
| | 1 | 1 | / // 120 | | 101 | · 15 = 120 | n = 120 |

| | Questio | n | Answer | Mks | AO | Guidance | |
|---|---------------|---|--|--------|--------------|--|--|
| | | | $\Rightarrow S = 181476$ | A1 | 3.2a | $\Rightarrow S = 181476$ | \Rightarrow S = 181476 |
| | | | | | | m - n = 1, m + n = 853 M1 | T & I: |
| | | | | | | 2m = 854 M1 | 426 seen M1M1A1 |
| | | | | | | m = 427 $n = 426$ A1 | S = 181476 A1 |
| | | | | | | $n^2 = 181476$ A1 | |
| | | | | [4] | | | |
| 6 | | | DR | | | | |
| 6 | (i) | | $\frac{\ln x}{2} = 0$ | | | May not be seen | |
| U | (1) | | x = 0 | | | Whay not be seen | |
| | | | $\Rightarrow \ln x = 0 \qquad \text{or } \frac{\ln 1}{1} = 0$ | M1 | 1.1 a | May be implied | |
| | | | $\Rightarrow x = 1$ | A1 | 1.1 | | |
| | | | | [2] | | | |
| 6 | (ii) | | y-coordinates are $\frac{\ln 2}{2}$ and $\frac{\ln 4}{4}$ | | | | |
| | | | 2 4 | | | 1 | |
| | | | $\frac{\ln 4}{4} = \frac{2\ln 2}{4} = \frac{\ln 2}{2}$ oe | B1* | 1.1 | Allow $\frac{\ln 4}{4} = \ln 4^{\frac{1}{4}} = \ln \sqrt{2} = \frac{\ln 2}{2}$ | Both = 0.346 B0B0 |
| | | | | | | $\frac{1}{2}$ | use of $\frac{\ln 4}{\ln 4} - \frac{\ln 2}{\ln 4} = 0$ |
| | | | $\Rightarrow AB \text{ is // to } x \text{-axis} \mathbf{AG}$ | B1dep* | 3.1a | Show that $\frac{m+2}{4} = \frac{2m2}{4}$ and conclusion | 4 2 univertified DODO |
| | | | | [2] | | | ulijustilled bobo |
| 6 | (iii) | | $\frac{dy}{dx} = \frac{x \times \frac{1}{x} - 1 \times \ln x}{x}$ or $\frac{1}{x} \times \frac{1}{x} + \ln x \times (-\frac{1}{x})$ or | M1 | 3.1a | Attempt diff > one term correct | |
| Ū | (111) | | $dx = x^2$ of $x + x + mx + (x^2) = 0$ | | - Cilu | | |
| | | | $\frac{1}{12} - \frac{\ln x}{2} = 0$ or $\frac{1 - \ln x}{2} = 0$ | M1 | 11 | oe their $\frac{dy}{dy} = 0$ | |
| | | | x^2 x^2 x^2 | | 1.1 | $\frac{dx}{dx} = 0$ | |
| | | | $1 - \ln x = 0$ oe | . 1 | 11 | | |
| | | | x = e or 2.72 or 2.7 (2 sf) | AI | 1.1 | | |
| | | | Coordinates are (e, $\frac{1}{e}$) | A1 | 1.1 | Allow (e, 0.368) or (e, 0.37) | or (2.7, 0.37) (2 sf) |
| | | | 6 | | | | |
| | | | | [4] | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |
| | | | | | | | |

| Question | | n | Answer | Mks | AO | Guidance | | |
|----------|------|---|---|---|---|--|--|--|
| 6 | (iv) | | | | | | Example of grad method | |
| | | | Attempt $\frac{d^2 y}{dx^2}$ | M1 | 2.1 | Attempt diff their $\frac{dy}{dx}$ | Sub 2.7 & 2.8 in $\frac{dy}{dx}$ M1 | |
| | | | $= \frac{x^2(-\frac{1}{x}) - 2x(1-\ln x)}{x^4} \text{ or } \frac{-3 + 2\ln x}{x^3} \text{ oe}$ | A1 | 1.2 | All correct, not necessarily simplified cao | 0.00093, -0.0038 A1A1 | |
| | | | Substitute $x = e$ (or 2.72) into $\frac{d^2 y}{dx^2}$ | M1 | 1.1 | Sub their x from (iii) into their $\frac{d^2 y}{dx^2}$ | State grad +ve & -ve or show on diag dep A1A1 M1 | |
| | | | $\frac{d^2 y}{dx^2} = -\frac{1}{e^3}$ oe or -0.0498 | A1 | 1.1 | cao Allow or – 0.0497 or –0.05 | | |
| | | | $\frac{d^2 y}{dx^2} < 0$, hence maximum | B1f | 3.2a | ft their result of sub their x into their $\frac{d^2 y}{dx^2}$ dep see result | Hence max B1f dep M1A1A1 | |
| | | | | [5] | | | No proof, no marks | |
| 7 | | | Summary of marks:Attempt find x at intersection of curves $x = 1$ Correct integral, any limitsCorrect numerical resultAttempt area of part or all of 2×2 squareWholly correct method $\frac{44}{3}$ | M1 A1 M1 A1 M1 M1 A1 [7] | 3.1a 1.1 3.1a 1.1 1.1 2.1 1.1 | Can be implied from correct limits | | |
| | | | | | | | | |

| Qu | iestion | Answer | Mks | AO | Guidance | |
|----|---------|--|----------|----|--|---|
| | | Examples of methods: Method 1 | | | | |
| | | $3-2x^2 = x$ $2x^2 + x = 3 = 0$ | M1 | | | or $3 - 2x^2 = -x$ $2x^2 - x - 3 = 0$ |
| | | $\frac{2x + x - 5 = 0}{x = 1}$ | A1 | | Ignore other root | $\begin{array}{c} 2x - x - 5 = 0 \\ x = -1 \end{array}$ |
| | | $\int_{0}^{1} (3 - 2x^{2}) dx \qquad \text{or } \int_{-1}^{1} (3 - 2x^{2}) dx$ | M1 | | Correct integrand with any limits | |
| | | $= \left[3x - \frac{2x^3}{3}\right]_0^1 \qquad \text{or } \left[3x - \frac{2x^3}{3}\right]_{-1}^1$ | | | | |
| | | $=\frac{7}{3}$ or $\frac{14}{3}$ | A1 | | | |
| | | " $\frac{7}{3}$ " - 1 (= $\frac{4}{3}$) or " $\frac{14}{3}$ " - 2 (= $\frac{8}{3}$) | M1 | | Attempt area above $y = 1$ or above $y = x$ | or " $\frac{14}{3}$ " – 1 (= $\frac{11}{3}$) |
| | | $8 \times "\frac{4}{3}" + 4$ or $4 \times "\frac{8}{3}" + 4$ | M1 | | Complete correct method | $4 \times \frac{11}{3}$ |
| | | $=\frac{44}{3}$ | A1 | | | |
| | | $\frac{\text{Method } 2}{3 - 2x^2 = x}$ x = 1 | M1 A1 | | $\frac{\text{Method } 3}{3 - 2x^2 = x}$ x = 1 | |
| | | $\int_{1}^{3} (\frac{y-3}{2})^{\frac{1}{2}} dy$ | M1 | | $\int_0^1 (3 - 2x^2 - 1) dx$ | |
| | | = | | | $= \left[2x - \frac{2x^3}{3}\right]_0^1$ | |
| | | $=\frac{4}{3}$ | A1 | | $=\frac{4}{3}$ | |
| | | $\frac{4}{3} + \frac{1}{2} \ (= \frac{11}{6})$ | M1 | | $\frac{4}{3} + \frac{1}{2} \ (= \frac{11}{6})$ | |
| | | $8 	imes rac{11}{6}$ | M1 | | $8 \times \frac{11}{6}$ | |
| | | $=\frac{44}{3}$ | A1 | | $=\frac{44}{3}$ | Other correct methods may be seen |
| | | | | | | |

| | Question | | Answer | Mks | AO | Guidance | | |
|---|----------|------------|--|-----------|------|---|---|--|
| 8 | (i) | (a) | 0.0478 or 0.048 (2 sf) | B1 | 1.1 | BC | | |
| | | | | [1] | | | | |
| | (i) | (b) | 22.5 or 23 (2 sf) | B1 | 1.1 | BC | | |
| | | | | [1] | | | | |
| | (i) | (c) | P(X < 20 + b) = 0.75 or $P(X > 20 + b) = 0.25$ | M1 | 1.1a | P(X < 20 - b) = 0.25 | | |
| | | | 20 + b = 22.02 or 22.0 or 22 | Al | 1.1 | 20 - b = 17.98 or 18 | | |
| | | | b = 2.02 or 2.0 (2 st) Allow b = 2 | Al | 1.1 | | | |
| | | | | [3] | | b = 22(.02) M1A1A0 | | |
| | | | | | | T & Langth a di | | |
| | | | | | | 1 & 1 method: T_{me} 2 yelves one ≈ 2 M1 | | |
| | | | | | | $\begin{array}{c} 11y \ 2 \ \text{values, one} \sim 2 \\ Correct \ \text{probe for two values in [2, 2, 1] A1} \end{array}$ | (0.405 & 0.516) | |
| | | | | | | Correct probe for two values in [2, 2.1] AT | $(0.495 \approx 0.510)$ | |
| | | | | | | & ans 2.0 or 2 | | |
| | | | | | | | SC (eg) | |
| 8 | (ii) | | $1.5\mu-\mu$ | M1 | 1.1a | $4.5\sigma - 3\sigma$ | | |
| U | (11) | | $\mu/3$ | | 1.14 | σ | Let $\mu = 1$; N(1, $\frac{1}{9}$) M1 | |
| | | | _ 3 | 4.1 | 11 | | $\mathbf{v} = 3$ A0 | |
| | | | $-\frac{1}{2}$ | AI | 1.1 | | $A = \frac{1}{2}$ A0 | |
| | | | $P(X > 1.5\mu) = 0.0668 \text{ or } 0.67 (2.sf)$ | A1 | 1.1 | | $P(X > \frac{3}{2}) = 0.067 \text{ A1}$ | |
| | | | | | | | 2 | |
| | | | | [3] | | | | |
| 9 | | | $H_0: p = \frac{1}{6}$ | B1 | 1.1 | | | |
| | | | H ₀ : $p > \frac{1}{6}$ where $p = P(2 \text{ on one throw})$ | B1 | 2.5 | B1B0 one error eg undefined p or two-tail | | |
| | | | $B(35, \frac{1}{6})$ | M1 | 3.3 | stated or implied unless clearly using N() | | |
| | | | $P(X \ge 10) = 1 - P(X \le 9)$ | | | | | |
| | | | or $P(X \ge 11) = 1 - P(X \le 10)$ | MI | 1.1a | \geq 1 of these probabilities stated | or $P(X \le 9)$, $P(X \le 10)$ | |
| | | | $P(X \ge 10) = 0.055$ | A1 | 2.1 | BC | $P(X \le 9) = 0.945$ | |
| | | | $P(X \ge 11) = 0.023$ | A1 | 3.4 | BC | $P(X \le 10) = 0.977$ | |
| | | | (0.04 lies between these hence) | | | | (0.96 between these) | |
| | | | rejection region is $X \ge 11$ Allow eg $a > 11$ | A1 | 2.2a | dep \geq one of above probs seen & correct | rej'n region is $X \ge 11$ | |
| | | | Special case, using N~Bin; Method A | | | | | |
| | | | $H: u = \frac{35}{2}$ | D1 | 11 | | | |
| | | | $11_0. \mu = \frac{1}{6}$ | DI | 1.1 | | | |

| (| Questio | n Answer | Mks | AO | Guidance | | |
|----|---------|--|-----------|-------------|--|--|--|
| | | H ₀ : $\mu > \frac{35}{6}$ where $\mu = \text{pop mean no. of 2's}$ | B1 | 2.5 | B1B0 one error eg undefined μ or two-tail | | |
| | | $N(\frac{35}{6}, \frac{175}{36})$ or N(5.833, 4.861) soi | M1 | 3.3 | Allow incorrect variance | | |
| | | $P(X \ge 10) = 1 - P(X < 9.5)$ or $P(X \ge 11) = 1 - P(X < 10.5)$ | M1 | 1.1a | \geq 1 of these probabilities attempted | P(X < 9.5) or $P(X < 10.5)$ | |
| | | P(X > 10) - 0.048 | 40 | 21 | BC | P(X < 9.5) = 0.952 | |
| | | $P(X \ge 10) = 0.048$ $P(X \ge 11) = 0.017$ | AU A1 | 3.4 | BC | P(X < 10.5) = 0.952 P(X < 10.5) = 0.983 | |
| | | (0.04 lies between these hence) | | | | (0.96 between these) | |
| | | rejection region is $X \ge 11$ | A1 | 2.2a | dep \geq one of above probs seen & correct | rej'n region is $X \ge 11$ | |
| | | Special case, using N~Bin; Method B | | | 1 | 5 0 | |
| | | $H_0: \mu = \frac{35}{6}$ | B1 | 1.1 | | | |
| | | $H_0: \mu > \frac{35}{6}$ where $\mu = pop$ mean no. of 2's | B1 | 2.5 | B1B0 one error eg undefined μ or two-tail | | |
| | | $N(\frac{35}{6}, \frac{175}{36})$ or N(5.833, 4.861) soi | M1 | 3.3 | Allow incorrect variance | | |
| | | P(X > a) = 0.04 soi | M1 | 1.1a | $z = \phi^{-1}(0.96) \qquad (= 1.751)$ | | |
| | | $\frac{35}{6} + 1.751 \times \sqrt{\frac{175}{36}}$ | A1 | 2.1 | dep $\phi^{-1}(0.96)$ attempt. May be implied BC | | |
| | | = 9.69 or 9.7 | AO | 3.4 | | | |
| | | rejection region is $X \ge 11$ | A1 | 2.2a | | | |
| | | | [7] | | | | |
| 10 | (i) | Only 784 trees and $810 > 784$ | E1 | 2.4 | or other similar | | |
| 10 | (ii) | eg Each no. not independent of previous no. Each no. is related to the next | E1 | 2.3 | Allow 2nd digit of each no. is 1st of next Consecutive nos share two digits | or similar correct Digits are re-used | |
| | | | [1] | | | | |
| | | | | | Ignore all else | | |
| | | | | | | | |
| | | | | | | | |

| Question | | Answer | Mks | AO | Guidance | | |
|---------------|---|---|--|---|---|--|--|
| (iii) | | $H_0: \mu = 4.2$ | B1 | 1.1 | Allow other letters except X or \overline{X} | | |
| | | H ₁ : $\mu < 4.2$ where μ is mean height of trees (in the wood) | B1 | 2.5 | One error, eg undefined μ or 2-tail: B0B1 | | |
| | | $\overline{X} \sim N(4.2, \frac{0.8^2}{50}) \text{ and } \overline{X} < 4.0 \text{ or } \overline{X} \le 4.0$ | M1 | 3.3 | Stated or implied Allow $\overline{X} > 4.0$ or $\overline{X} = 4.0$ | $\phi^{-1}(0.98)$ (= 2.054) | |
| | | $P(\overline{X} < 4.0) = 0.038549$ or 0.039 | A1 | 3.4 | BC Allow 0.038 NB 0.038 implies M1A1 | $4.2 - 2.054 \times \frac{0.8}{\sqrt{50}}$ (= 3.968) | |
| | | Compare 0.02 | A1 | 1.1 | dep P(\overline{X} < 4.0) attempted | comp their 3.968 with 4.0 | |
| | | Do not reject H ₀ | M1 | 2.2b | Allow Accept H ₀ dep P(\overline{X} < 4.0) attempted | Can be implied by conclusion | |
| | | There is insufficient evidence that mean height of these trees in the wood is less than 4.2m. | A1f | 3.5a | In context, not definite; eg "Mean height not less than 4.2m": A0 | | |
| | | | [7] | | | | |
| (i) | (a) | Both the number of employees using public transport and the number of employees using private vehicles depend on the LA population. | E1 | 2.1 | or similar, but must be in context. Ignore all else | NOT No. using pt is prop to no. using pv | |
| | | | [1] | | | | |
| (i) | (b) | Negative If a large prop use public transport then a smaller | E1ind | 2.2b | Ignore "strong" or "slight" etc | NOT Inverse prop'n NOT "as <i>a</i> increases | |
| | | prop drive (and vice versa) | E1ind | 2.4 | or similar in context | <i>b</i> decreases" unless in context | |
| | | | [2] | | | | |
| (ii) | (a) | Decrease the size of r or Make r less negative | E1 | 2.2b | Make (value of) <i>r</i> increase | NOT Make <i>r</i> decrease | |
| | | | | | r closer to 0 | NOT Weaken the corr'n | |
| | | | [1] | | Ignore eg "greatly" | NOT Make corr'n less | |
| | | Little affact (happing the population of the LA is | | | Ignore all else | | |
| (ii) | (b) | small compared with the whole population) | E1 | 2.2b | Ignore all else | | |
| | | | [1] | | | | |
| (ii) | (c) | Ignore all reference to public transport | | | Type 2 analyzers | | |
| | | <u>1 ype 1 answers</u> | | | <u>1 ype \angle answers</u> | NOT inst Four drive | |
| | | Lobs are close | F1 | 24 | Any suggested <u>reason</u> why lew drive | NOT just rew drive | |
| | | High proportion walk (or cycle) | [1] | 2.4 | or similar in context | | |
| | <u>Questic</u> (iii) (i) (i) (ii) (ii) | Question (iii) (iii) (i) (i) (i) (i) (i) (ii) (iii) (iii) (ii) (ii) (iii) (iii) (iii) | QuestionAnswer(iii) $H_0: \mu = 4.2$ $H_1: \mu < 4.2$ where μ is mean height of trees (in the wood) $\overline{X} \sim N(4.2, \frac{0.8^2}{50})$ and $\overline{X} < 4.0$ or $\overline{X} \leq 4.0$ $P(\overline{X} < 4.0) = 0.038549$ or 0.039 $P(\overline{X} < 4.0) = 0.038549$ or 0.039 Compare 0.02 Do not reject H_0 There is insufficient evidence that mean height of these trees in the wood is less than 4.2m.(i)(a)Both the number of employees using public transport and the number of employees using private vehicles depend on the LA population.(ii)(b)Negative If a large prop use public transport then a smaller prop drive (and vice versa)(iii)(b)Little effect (because the population of the LA is small compared with the whole population)(iii)(c)Ignore all reference to public transport Type 1 answers People don't travel far to work Jobs are close High proportion walk (or cycle) | QuestionAnswerMks(iii) $H_0: \mu = 4.2$ B1 $H_1: \mu < 4.2$ where μ is mean height of trees (in the wood)B1 $\overline{X} \sim N(4.2, \frac{0.8^2}{50})$ and $\overline{X} < 4.0$ or $\overline{X} \leq 4.0$ M1 $\overline{X} \sim N(4.2, \frac{0.8^2}{50})$ and $\overline{X} < 4.0$ or $\overline{X} \leq 4.0$ M1 $P(\overline{X} < 4.0) = 0.038549$ or 0.039 A1Do not reject H_0 M1There is insufficient evidence that mean height of these trees in the wood is less than 4.2m.A1f(i)(a)Both the number of employees using public transport and the number of employees using private vehicles depend on the LA population.E1(i)(b)Negative If a large prop use public transport then a smaller prop drive (and vice versa)E1(ii)(a)Decrease the size of r or Make r less negativeE1(iii)(b)Little effect (because the population of the LA is small compared with the whole population)E1(iii)(c)Ignore all reference to public transport small compared with the whole populationE1(iii)(c)Ignore all reference to public transport High proportion walk (or cycle)E1 | QuestionAnswerMksAO(iii) $H_0: \mu = 4.2$ $H_1: \mu < 4.2$ where μ is mean height of trees (in the wood)B11.1 $\overline{X} \sim N(4.2, \frac{0.8^2}{50})$ and $\overline{X} < 4.0$ or $\overline{X} \le 4.0$ M13.3 $P(\overline{X} < 4.0) = 0.038549$ or 0.039 A13.4 $P(\overline{X} < 4.0) = 0.038549$ or 0.039 A11.1 Do not reject H_0 M12.2bThere is insufficient evidence that mean height of these trees in the wood is less than 4.2m.A1f (i) Both the number of employees using public transport and the number of employees using public for any private vehicles depend on the LA population.E1 (i) (b)Negative If a large prop use public transport then a smaller prop drive (and vice versa)E12.4 (ii) (a)Decrease the size of r or Make r less negativeE12.2b (ii) (b)Little effect (because the population of the LA is small compared with the whole population)E12.2b (ii) (c)Ignore all reference to public transport $Type 1$ answers People don't travel far to work Jobs are closeE12.4 (ii) (c)Ignore all reference to public transport $Type 1$ answers People don't travel far to work Jobs are closeE12.4 (ii) (c)Ignore all reference to public transport $Type 1$ answers People don't travel far to work Jobs are closeE12.4 (ii) (c)Ignore all r | QuestionAnswerMksAOGuidance(iii) $H_0: \mu = 4.2$ $H_1: \mu < 4.2$ where μ is mean height of trees (in the wood)B11.1Allow other letters except X or \overline{X} $\overline{X} \sim N(4, 2, \frac{0.8^2}{50})$ and $\overline{X} < 4.0$ or $\overline{X} \le 4.0$ M13.3Stated or implied Allow $\overline{X} > 4.0$ or $\overline{X} = 4.0$ $P(\overline{X} < 4.0) = 0.038549$ Do not reject H_0 A13.4BC Allow 0.038 NB 0.038 implies M1A1Compare 0.02A11.1dep P($\overline{X} < 4.0$) attemptedDo not reject H_0 M12.2b $Allow Accept H_0$ dno these trees in the wood is less than 4.2m.Allow(i)(a)Both the number of employees using public transport and the number of employees using private vehicles depend on the LA population.E12.4(ii)(b)Negative If a large prop use public transport then a smaller prop drive (and vice versa)E12.4bor similar in context.(iii)(a)Decrease the size of r or Make r less negative small compared with the whole populationE12.4bMake (value of) r increase r closer to 0 Ignore all else(iii)(b)Little effect (because the population of the LA is small compared with the whole populationE12.2bMake (value of) r increase r closer to 0 Ignore all else(iii)(b)Little effect (because the population of the LA is small compared with the whole populationE12.2bMake (value of) r increase r closer to 0 Ignore all else(iii)(b)Little effect (because the population of the LA is small compared with | |

| | Questio | n | Answer | Mks | AO | Guidance | | |
|----|---------------|---|---|-----------|--------------|--|-----------------------------|--|
| 12 | (i) | | $a(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}) = 1$ soi | M1 | 3.1 a | or $\frac{16}{31}(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}) = 1$ oe seen | | |
| | | | $a = \frac{16}{31}$ | A1 | 1.1 | correctly obtained | | |
| | | | 51 | [2] | | | | |
| 12 | (ii) | | $P(X = 1, 3 \text{ or } 5) = \frac{21}{31}$ 0r 0.677 or 0.68 (2 sf) | B1 | 1.1a | | | |
| | | | 51 | [1] | | | | |
| 12 | (iii) | | P(sum odd) = P(OE) + P(EO) | | | | | |
| | | | $= 2 \times \frac{21}{31} \times (1 - \frac{21}{31})$ | M1 | 2.1 | or correct "long" method | Allow without "2 \times " | |
| | | | $=\frac{420}{961}$ or 0.437 or 0.44 (2 sf) | A1 | 1.1 | | | |
| | | | | [2] | | | | |
| 12 | (i v) | | $P(Sum > 8 \ r \text{ odd}) = P(Sum = 0)$ | | | | | |
| 14 | (1) | | = P(4, 5) + P(5, 4) | | | or $P(> 8) \times P(O > 8)$ | | |
| | | | $= \frac{2}{31} \times \frac{1}{31} + \frac{1}{31} \times \frac{2}{31} \qquad (=\frac{4}{961})$ | M1 | 1.1 a | $=\frac{5}{961}\times\frac{4}{5}$ | Correct method | |
| | | | $\frac{P(\text{Sum} > 8 \& \text{odd})}{P(\text{Sum odd})}$ | M1 | 2.4 | Attempt ft their (iii) and their P(Sum > 8 & odd) | | |
| | | | $= \frac{4}{961} \div \frac{420}{961}$ | | | | | |
| | | | $=\frac{1}{105}$ or 0.00952 or 0.0095 (2 sf) | A1 | 1.1 | cao NB $\frac{2}{961} \div \frac{210}{961} = \frac{1}{105}$ M0M1A0 | | |
| | | | 105 | [3] | | | | |
| 12 | (v) | | $S_{\infty} = \frac{p}{1 - 0.5} = 1$ | M1 | 3.4 | | | |
| | | | P(X = 1) = 0.5 | A1 [2] | 3.4 | Correct ans, no working M1A1 | | |
| 12 | (vi) | | Eg Y. (Y takes all values, but) X cannot be > 5 Eg X because > 5 is very unlikely | B1 [1] | 3.5b | oe, eg <i>Y</i> . It may take more than 5 attempts or "limited no." oe instead of 5 | | |
| | | | | | | | | |

| Question | | n | Answer | Mks | AO | Guidance | | |
|----------|---------------|------------|---|------|------|--|--|--|
| 13 | | | DR | | | | | |
| 13 | (i) | | N(450×0.15, 450×0.15×0.85) | M1 | 3.1b | seen or implied | B(450, 0.15) | |
| | | | or N(67.5, 57.375) oe | | | | with T & I method | |
| | | | $P(Y > \mu + \sigma) \approx \frac{1}{6}$ or $\phi^{-1}(\frac{1}{6}) = 0.9674$ | | | P($Y < a$) = $\frac{5}{6}$ | $using \ge one of 74, 75, 76, 61, 60, 59$ | |
| | | | $67.5' + \sqrt{57.375}$ or $67.5' + 0.9674 \times \sqrt{57.375}$ | M1 | 1.2 | or 74.83 seen; ft their $\mu \& \sigma$ for M1 only | P(X > 74) = 0.177 P(X > 75) = 0.145 both | |
| | | | = 74 or 75 or 76 | A1 | 1.1 | Integer. No ft Dep M1M1 Correct ans, inadequate wking: M0M0A0 | <i>a</i> = 74 or 75 or 76 | |
| | | | | | | NB 450/6 = 75 M0M0A0 | | |
| | | | 501 | [3] | | | | |
| | | | $\frac{30!}{r!(50-r)!}$ × 0.15' × 0.85' × 0.85' | | | ${}^{50}C_r \times 0.15^r \times 0.85^{50-r}$ | | |
| 13 | (ii) | | $\frac{50!}{(r+1)!(50-(r+1))!} \times 0.15^{r+1} \times 0.85^{50-(r+1)} \qquad \text{oe}$ | M1 | 1.1a | $50 C_{r+1} \times 0.15^{r+1} \times 0.85^{50-(r+1)}$ | Fully correct | |
| | | | eg $\frac{\frac{1}{50-r} \times 0.85}{\frac{1}{r+1} \times 0.15}$ or $\frac{0.85}{50-r} \times \frac{r+1}{0.15}$ oe | A1 | 2.1 | Any correct simplification without factorials OR without indices | or $\frac{17}{20} \times \frac{20}{3} \times \frac{r+1}{50-r}$ | |
| | | | $=\frac{17(r+1)}{3(50-r)}$ AG | A1 | 1.1 | Any correct simplification without factorials AND without indices and correctly obtain result | | |
| | | | 17(r+1) | [2] | | | No factorials or | |
| 13 | (iii) | (a) | $\frac{17(7+1)}{3(50-r)} \le 1$ oe | M1 | 3.1b | $\frac{1}{50-r} \times 0.85 \le \frac{1}{r+1} \times 0.15$ oe M1 | indices | |
| | | | 17r + 17 < 150 - 3r | | | $0.85(r+1) \le 0.15(50 - r)$ | malees | |
| | | | $20\pi < 122$ | 4.1 | 11 | - 50×0.15 0.85 | Correct, in form $ar \leq b$ | |
| | | | $20r \le 155$ 0e | AI | 1.1 | $7 \le 50 \times 0.15 - 0.85$ AI | or $r < \text{correct expr'n}$ | |
| | | | $r \le 6.65$ | A1 | 1.1 | | | |
| | | | <i>r</i> is an integer so $r \le 6$ | A1 | 1.1 | | | |
| | | | | | | SC: | | |
| | | | | | | P(X=6)=0.142, P(X=7)=0.157, P(X=8)=0.149 | | |
| | | | | | | B1 (must be these three) bence $r < 6$ B1den | No wking BOBO | |
| | | | | [4] | | (must be these three) hence $r \ge 0$ bruch | THO WRITE DODO | |
| | | | | [1] | | | | |
| | | | | | | | | |

Mark Scheme

| | Question | | Answer Mks AO | | Guidance | | |
|----|----------|-----|---|-----------------|-------------|--|---|
| 13 | (iii) | (b) | $P(X = r) \le P(X = r + 1) \text{ for } r \le 6$ Hence most likely value is r is 6 or 7 $\frac{P(X = 6)}{P(X = 7)} = \frac{17(6+1)}{3(50-6)} = 0.902 < 1$ Most likely value is 7 | B1 B1 [2] | 2.1 3.2a | or $P(X = 6) = 0.142$ & $P(X = 7) = 0.157$ indep, but dep on some reasonable explanation | NOT 6.65 rounds to 7 B0B0 No expl'n: B0B0 |
| | | | | 100 | | | |

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