

A-LEVEL MATHEMATICS 7357/2

Paper 2

Mark scheme

June 2019

Version: 1.0 Final



Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given
 on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
Α	mark is dependent on M or m marks and is for accuracy
В	mark is independent of M or m marks and is for method and accuracy
Е	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

A	0	Description					
	AO1.1a	Select routine procedures					
AO1	AO1.1b	Correctly carry out routine procedures					
	AO1.2	Accurately recall facts, terminology and definitions					
	AO2.1	Construct rigorous mathematical arguments (including proofs)					
	AO2.2a	Make deductions					
AO2	AO2.2b	Make inferences					
AUZ	AO2.3	Assess the validity of mathematical arguments					
	AO2.4	Explain their reasoning					
	AO2.5	Use mathematical language and notation correctly					
	AO3.1a	Translate problems in mathematical contexts into mathematical processes					
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes					
	AO3.2a	Interpret solutions to problems in their original context					
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems					
AO3	AO3.3	Translate situations in context into mathematical models					
	AO3.4	Use mathematical models					
	AO3.5a	Evaluate the outcomes of modelling in context					
	AO3.5b	Recognise the limitations of models					
	AO3.5c	Where appropriate, explain how to refine models					

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Mark	Typical solution
1	Ticks the correct response	2.2a	R1	y x
	Total		1	

Q	Marking instructions	AO	Mark	Typical solution
2	Circles the correct response	1.1b	B1	$a^{\frac{8}{15}}$
	Total		1	

Q	Marking instructions	AO	Mark	Typical solution
3	Circles the correct response	1.2	B1	$f(x) = x^2$
	Total		1	

	T		I = -	T=
Q	Marking instructions	AO	Mark	Typical solution
4 4	Explains how the factor theorem applies with reference to $f(-2) = 0$ for either function or Explains that either quadratic expression can be factorised in the form $(x + 2) (x + p)$ as $(x + 2)$ is a factor or Explains that on division by $(x + 2)$ the remainder would be zero Uses the factor theorem with $x = -2$ substituted into one	2.4 1.1a	Mark E1	Typical solution As $(x+2)$ is a factor, then when $x = -2$, $f(x) = 0$ $4-2b+c=0$ $4-2d+e=0$ $4-2b+c=4-2d+e$ $2d-2b=e-c$ $2(d-b)=e-c$
	of the expressions to obtain a correct expression ${f NB}$ It is not necessary to equate to zero for this mark or Expands one of their factorised forms and equates coefficients correctly $(x+2)(x+p)=x^2+(p+2)x+2p$ $p+2=b$ $2p=c$ or Divides one of the expressions by $(x+2)$ to obtain a correct remainder. Either one of $4-2b+c$ $4-2d+e$			
	Deduces both correct equations using factor theorem or division $4-2b+c=0$ $4-2d+e=0$ PI by $4-2b+c=4-2d+e$ or Expands both of their factorised forms and equates coefficients to deduce the correct equations – must not use p in both Forms a single equation for b , c , d and e and completes rigorous argument to show the	2.2a 2.1	A1	
	required result NB R1 can be awarded even if E1 was not awarded			
	Total		4	
	ıolai		-	

	I a.a.		. .	I =
Q 5	Marking instructions	AO	Mark	Typical solution
5	Separates the variables – one side correct Condone missing integral signs	3.1a	M1	$\int \frac{1}{x^2} \ln x \mathrm{d}x = \int t \mathrm{d}t$
	PI by correct integration Integrates their $\int t dt$ correctly	1.1b	A1F	$\int t \mathrm{d}t = \frac{t^2}{2} + c$
	Obtains $u' = \frac{1}{v}$ and $v = -\frac{1}{v}$ OE	1.1b	B1	$u = \ln x$
	Integrates $\int \frac{1}{x^2} \ln x dx$	1.1a	M1	$u' = \frac{1}{x}$
	$\int x^2$ Substitutes their u , u ', v and v '			$v' = x^{-2}$ $v = -x^{-1}$
	into the correct formula for integration by parts			$-\frac{1}{x}\ln x - \int \frac{1}{x} \left(-x^{-1}\right) \mathrm{d}x$
	Condone sign errors in formula			$\int -\frac{1}{r} \ln x + \int \frac{1}{r^2} dx$
	Obtains $-\frac{1}{x} \ln x - \frac{1}{x}$	1.1b	A1	$\begin{bmatrix} x & \mathbf{J} & x^2 \\ -\frac{1}{x} \ln x - \frac{1}{x} \end{bmatrix}$
	Substitutes $t = 2$ and $x = 1$ into their integrated equation to find their + c	1.1a	M1	$-\frac{1}{x}\ln x - \frac{1}{x} = \frac{t^2}{2} + c$
	Obtains correct solution must have $t^2 =$ ACF	2.5	A1	$t = 2, x = 1 \Rightarrow -1 = 2 + c$ $c = -3$ $(1 + \ln x)$
				$t^2 = 6 - 2\left(\frac{1 + \ln x}{x}\right)$
	Total		7	
	Total	<u> </u>		

Q	Marking instructions	AO	Mark	Typical solution
6	Compares with $R\cos(x\pm\alpha)$ or	3.1a	M1	$R\sin(x+\alpha) = a\sin x + b\cos x$
	` '			$A \sin(x + a) = a \sin x + b \cos x$
	$R\sin(x\pm\alpha)$			
	by forming an identity e.g.			R=4
	$R\sin(x+\alpha) \equiv a\sin x + b\cos x$			
	OE			(=) =
	or			$4\sin\left(\frac{\pi}{3} + \alpha\right) = 2\sqrt{3}$
	Differentiates correctly and			(3)
	equates to zero CAO PI by			$\alpha = \frac{\pi}{3}$
	$a\cos x = b\sin x$			3
	PI by			
				π
	$R = 4 \text{ or } a^2 + b^2 = 16$ Deduces $R = 4$	2.2a	A1	$a = 4\cos\frac{\pi}{3} = 2$
	or			
	$a^2 + b^2 = 16$			$b = 4\sin\frac{\pi}{3} = 2\sqrt{3}$
	Forms a correct equation for α	1.1b	B1]
	PI by			
	correct α			
	or Forms the equation shown			
	below			
	$2\sqrt{3} = \frac{a\sqrt{3}}{2} + \frac{b}{2}$ OE			
	Must substitute correct exact			
	values for the trig functions			
	Solves their equation to obtain	1.1a	M1	
	any correct value of α			
	Correct values are shown below			
	$\alpha = \frac{\pi}{3} \text{ or 0 for } R \sin(x \pm \alpha)$			
	$\alpha = \pm \frac{\pi}{6} \text{ for } R \cos(x \pm \alpha)$			
	or			
	Eliminates a variable correctly			
	from their two equations – must			
	obtain a correct simplified equation			
	Deduces $a = 2$	2.2a	R1	
	Deduces $b = 2\sqrt{3}$	2.2a	R1	
	Doddoes 0 - 2 y 3			
	Total		6	

Q	Marking instructions	AO	Mark	Typical solution
7(a)	Sketches any cubic graph,	1.2	B1	1 Jerour Sorucion
()	crossing the x-axis in three			1
	places	4.0	D4	
	Sketches any cubic graph with a positive coefficient of x^3	1.2	B1	
- (1)(1)				
7(b)(i)	Differentiates to obtain $f'(x)$	1.1a	M1	For a turning point $f'(x) = 0$
	Two terms with at least one			$f(x) = x^3 + 3px^2 + q$
	correct - either $3x^2$ or $6px$			$f'(x) = 3x^2 + 6px$
	Solves $3x^2 + 6px = 0$ to obtain	1.1b	A1	
	x = 0 or x = -2p			$3x^2 + 6px = 0$
	or Substitutes $x = 0$ in			3x(x+2p)=0
	$f'(x) = 3x^2 + 6px \text{ and obtains 0}$			x = 0
	Obtains the correct two roots	2.4	R1	
	x = 0 and $x = -2p$ OE	۷.٦		x = -2p
	and states why there must be a			Since one of the roots is $x = 0$ there
	turning point referring to root			must be a turning point on the y axis
7(b)(ii)	x = 0 Deduces that turning point at	2.2a	B1	Since $p > 0$
/ (D)(II)	x = -2p is a maximum or	Σ.Σα		x = -2p is the maximum
	deduces that turning point $x = 0$			x = 0 is the minimum
	is a minimum			
	May have been seen in part (b)(i)			f(0) = q
	Accept a sketch showing correct			$f(-2p) = (-2p)^3 + 3p(-2p)^2 + q$
	relative positions of turning			
	points Substitutes their $x = -2p$ into	1.1a	M1	$=4p^3+q$
	f(x)			
	Obtains correct $f(0) = q$ and	1.1b	A1	4 - 3 0
	$f\left(-2p\right) = 4p^3 + q$			$-4p^3 < q < 0$
	Deduces	2.2a	R1	
	either $q < 0$ or $-4p^3 < q$			
	Condone ≤	0.0	F.4	
	Deduces $-4p^3 < q < 0$ CAO	2.2a	R1	
	Total		10	

Q	Marking instructions	AO	Mark	Typical solution
8(a)	Takes logs of both sides of the	1.1a	M1	$\log_{10} V = \log_{10} p \ q^t$
	equation and applies addition			310 3101 1
	rule			$\log_{10} V = \log_{10} p + \log_{10} q^t$
	Completes rigorous argument to	2.1	R1	
	show required result			$\log_{10} V = \log_{10} p + t \log_{10} q$
0/1-1	Condone missing base	0.4	N 4 4	
8(b)	Equates $\log_{10} p$ to 3.90	3.4	M1	$\log_{10} p = 3.90$
	Or			7040
	Forms two simultaneous			p = 7940
	equations using points from the line of best fit only			5 29 2 00
	Calculates gradient and	3.4	M1	$\log q = \frac{5.28 - 3.90}{40 - 0} = 0.0345$
	equates to $\log_{10} q$	0.1	1011	40-0
	or			4.00
	Solves their pair of simultaneous			<i>q</i> = 1.08
	equations to obtain p and q			
	Obtains correct	1.1b	A1	
	AWRT 8000			
	CSO			
	Obtains correct q	1.1b	A1	
	AWRT 1.1			
	CSO			
8(c)	Substitutes <i>V</i> = 500000 into their	3.4	M1	$500000 = 7940 \times 1.08^t$
	$V = 7940 \times 1.08^t$			t = 53.82
	or into their			
	$\log_{10} V = \log_{10} 7940 + t \log_{10} 1.08$			
	to form an equation for t			
	PI by correct t value Solves their equation for t	1.1a	M1	
	Must have $t > 40$	ı.ıa	IVII	
	States their correct year using	3.2a	A1F	The house will first be worth half a
	1970+ their integer part of t	0.24	/ / / /	million pounds during 2023
	Must be later than 2010			9 1 1
8(d)	Explains that their 2023	3.5b	E1F	The model is only based on data
	(FT later than 2010) is outside			between 1970 and 2010
	the range of data collected			
	Explains that house prices may	3.2b	E1	House prices may not continue to
	not continue to grow in the same			grow in the same way indefinitely
	way			
	Must refer to context not just to			
	Must refer to context not just to extrapolation/pattern			
	Can be implied by comments			
	such as:			
	Theresa may have made			
	improvements by adding a new			
	room			
	Prices could fall in a market			
	crash			
	Total		11	

Q	Marking instructions	AO	Mark	Typical solution
9(a)	Write in a form to which the	3.1a	M1	1
J(a)	binomial expansion can be applied	J.1a	1011	$\sqrt{4 - 2x^2} = 2\left(1 - \frac{x^2}{2}\right)^{\frac{1}{2}}$
	Must be of form $a \left(1 - \frac{x^2}{2}\right)^{\frac{1}{2}}$			$\approx 2\left(1 + \frac{1}{2}\left(-\frac{x^2}{2}\right)\right)$
	Completes rigorous argument to obtain correct expansion AG	2.1	R1	$\approx 2 - \frac{x^2}{2}$
9(b)	Compares their $\frac{x^2}{2}$ to 1	1.1a	M1	$\left -\frac{x^2}{2} \right < 1$ $\Rightarrow x < \sqrt{2}$
	Condone incorrect inequality PI by $\left -2x^2\right < 4$			$\Rightarrow x < \sqrt{2}$
	Obtains correct range of values ACF	1.1b	A1	
9(c)	Explains that as 0.4 radians is	2.4	E1	As 0.4 is small
	small therefore $\cos x \approx 1 - \frac{x^2}{2}$			$\cos x \approx 1 - \frac{x^2}{2}$
	Must refer to 0.4 and small angle approximation for cos <i>x</i>			$\int_{0}^{0.4} \sqrt{\cos x} dx \approx \int_{0}^{0.4} \sqrt{1 - \frac{x^2}{2}} dx$
	Uses half of their expansion	1.1a	M1	
	from 9(a) as the integrand	4.4-	N 4 4	$\approx \frac{1}{2} \int_{0}^{0.4} 2 - \frac{x^2}{2} dx$
	Integrates their expression with at least one term correct	1.1a	M1	
	Obtains correct value must be at least five decimal places	1.1b	A1	$\approx \int_0^{0.4} 1 - \frac{x^2}{4} \mathrm{d}x$
	Condone $\frac{148}{275}$			$\approx \left[x - \frac{x^3}{12}\right]_0^{0.4}$
	375 CAO			
	CAO			$\approx 0.4 - \frac{0.4^3}{12}$
0(4)	States that 1.4 radians is not a	2.4	E1	≈ 0.39467
9(d)	small angle so the approximation is not valid Must refer to small angle approximation and 1.4	2.4		Since 1.4 is not a small angle the approximation is not suitable
	or State invalid as 1.4 is bigger than 0.664 NB 0.664 is the limiting value for			
	approximation to be valid Total		9	
	lotai		9	

Q	Marking Instructions	AO	Marks	Typical Solution
10	Ticks correct box	2.2a	B1	The particle was decelerating for $12 \le t \le 20$
	Total		1	

Q	Marking Instructions	AO	Marks	Typical Solution
11	Circles correct answer	1.1b	B1	1000 N
	Total		1	

Q	Marking Instructions	AO	Marks	Typical Solution
12	Circles correct answer	1.1b	B1	-400
	Total		1	

Q	Marking Instructions	AO	Marks	Typical Solution
13(a)	States appropriate $suvat$ equation and clearly identifies $s=h, a=g$ and $u=0$ PI by $v^2=0^2+2gh$ OE	1.1a	M1	$v^{2} = u^{2} + 2as$ $u = 0 a = g s = h$ $v^{2} = 0^{2} + 2gh$ $v = \sqrt{2gh}$
	Completes rigorous argument by substituting key values and rearranging correctly for v Must have used consistent signs for s and a AG	2.1	R1	$v = \sqrt{2gn}$
13(b)	Substitutes two values in $v = \sqrt{2gh}$ to find the third value OE	3.1b	M1	When $g = 9.8$ and $h = 18$
	Obtains correct third value If finding v then accept AWRT 19 If finding g then accept AWRT 11 If finding h then accept AWRT 20	1.1b	A1	$v = \sqrt{2 \times 9.8 \times 18} = 18.8$ 18.8 < 20 Machine is faulty
	Makes an appropriate comparison for correct v , g or h and infers that the teacher's claim is correct. The comparison can be implied in their comment, eg the value of v is less than 20 or Makes an appropriate comparison using $g = 10$ and infers that the teacher's claim is incorrect. Their answer must be rounded to 20. The comparison can be implied in their comment, eg the value matches the given value of v	2.2b	R1	
	Total		5	

	Maulina Instruction	40	Maulia	Typical Calution
Q 44(5)	Marking Instructions	AO	Marks	Typical Solution
14(a)	Finds a moment of a force about	1.1b	B1	Take moments about A
	any point. Must have the form			m = × 0.04 = 0.20 = × 0.02
	force x distance			$mg \times 0.04 = 0.28g \times 0.03$
	Can be awarded for 6 <i>R</i>			
	PI by fully correct equation			- 0.04
	Forms a fully correct moments	3.3	M1	m = 0.21
	equation using the correct model			
	Must have included a an both sides			
	Must have included g on both sides			
	Moments about <i>B</i> gives (in metres)			
	0.28g(0.03) + 0.1mg =			
	0.06(0.28g + mg)			
	Solves equation to show $m = 0.21$	1.1b	A1	
	AG		,	
14(b)	Forms a moments equation for	3.1b	M1	Take moments about A
	equilibrium of rod with correct			
	number of terms – can use m , 0.21			$0.21g \times 0.04 = 0.048g \times 0.05 \times n$
	or their value for m from part 14(a)			
	Condone omission of g			
	throughout part 14(b)			n = 3.5
	Forms a moments equation for	3.4	A1F	
	equilibrium of rod with term			
	involving n correct – can use m ,			Maximum $n=3$
	0.21 or their <i>m</i> value from part			
	14(a)			
	FT their incorrect m			
	Moments about B gives			
	0.06R = 0.00048ng + 0.1mg			
	Obtains a fully correct moments	1.1b	A1	
	equation with $m = 0.21$ substituted			
	Moments about B gives			
	0.06(0.21g + 0.048ng) =			
	0.00048ng + 0.1(0.21)g			
	Must have substituted correct			
	expression for <i>R</i>			
	States $n = 3$	1.1b	A1	+
	CSO	1.10	Λ1	
14(c)	States an assumption about the	3.5b	E1	The rod is uniform
(•)	rod	2.00		
	Accept			
	The mass/weight of the rod acts in			
	the middle			
	The rod is in limiting equilibrium OE			
	The rod is rigid			
	Total		8	
	iotai		•	1

Q	Marking Instructions	AO	Marks	Typical Solution
15(a)	Finds \overrightarrow{AB} or \overrightarrow{CD} or \overrightarrow{BC} or \overrightarrow{DA}	3.1a	M1	
10(4)	correctly	0.14	1411	$\overrightarrow{AB} = \begin{bmatrix} -620 \\ -180 \end{bmatrix} \overrightarrow{CD} = \begin{bmatrix} 930 \\ 270 \end{bmatrix}$
	Condone a direction error in the			
	label			
	$\overrightarrow{BC} = \begin{bmatrix} -130 \\ -840 \end{bmatrix} \overrightarrow{DA} = \begin{bmatrix} -180 \\ 750 \end{bmatrix}$ OE			$\overrightarrow{CD} = -1.5 \times \overrightarrow{AB}$
	Finds gradient of AB or CD or BC or DA correctly			Thus AB and CD are parallel but not equal in length
	Gradient $AB = CD = \frac{9}{31}$ OE			ABCD is a trapezium but not a parallelogram
	Gradient $BC = \frac{84}{13}$ OE			p manus agrams
	Gradient $DA = -\frac{25}{6}$ OE			
	Accept ratios $\frac{31}{9}, \frac{13}{84}, -\frac{6}{25}$ OE			
	Ignore any incorrect labelling of ratios here			
	Finds \overrightarrow{AB} and \overrightarrow{CD} correctly OE	1.1b	A1	
	or Finds gradients of AB and CD correctly			
	or Finds a corresponding pair of ratios correctly – Do not award if			
	reciprocals of gradients are labelled as gradients or vectors			
	Shows/states $\overrightarrow{CD} = \pm 1.5 \times \overrightarrow{AB}$ OE	4 4 h	۸.4	
	or Shows/states that $\overrightarrow{BC} \neq k \times \overrightarrow{DA}$	1.1b	A1	
	or Finds \overrightarrow{BC} and \overrightarrow{DA} correctly			
	or Finds gradients of BC and DA			
	correctly or			
	Finds a second corresponding pair of ratios correctly– Do not award if			
	reciprocals of gradients are			
	labelled as gradients or vectors			
	If incorrect labelling used for ratios then maximum mark is M1 A0 A0 E1 R0			

	Deduces that \overrightarrow{AB} and \overrightarrow{CD} are parallel - implied by reference to equal gradients or Deduces correctly that \overrightarrow{BC} and \overrightarrow{DA} are not parallel NB E1 is Independent of any other marks Completes rigorous proof by deducing correctly that the scalar multiple of ± 1.5 OE means the parallel sides are not equal in length or Completes rigorous proof by deducing correctly that \overrightarrow{AB} and \overrightarrow{CD} are parallel giving justification and	3.2a 2.1	E1	
	that \overrightarrow{BC} and \overrightarrow{DA} are not parallel giving justification			
	Must include a statement that <i>ABCD</i> is not a parallelogram at some point			
	NB R1 can be awarded even if E1 was not awarded CSO			
15(b)	Uses velocity/displacement/time relationship	3.1b	M1	$\boldsymbol{v} = \frac{1}{50} \times \begin{bmatrix} -130 \\ -840 \end{bmatrix}$
	Evidenced by dividing any vector /distance from part 15(a) by 50			$\boldsymbol{v} = \begin{bmatrix} -2.6 \\ -16.8 \end{bmatrix}$
	Finds the magnitude of their \overrightarrow{BC} or ν	1.1a	M1	$Speed = v = \sqrt{2.6^2 + 16.8^2}$
	Obtains 17	1.1a	A1	$Speed = 17 \text{ m s}^{-1}$
	States correct speed with correct units	3.2a	A1	5,000 17 III 0
	Total		9	

Q	Marking Instructions	AO	Marks	Typical Solution
16(a)	Differentiates to obtain $\frac{dv}{dt}$ with at least one exponent term correct	3.4	M1	$\frac{dv}{dt} = 10.512e^{-0.9t} - 0.009e^{0.3t}$
	Obtains fully correct expression for $\frac{dv}{dt}$	1.1b	A1	Maximum v occurs when $\frac{dv}{dt} = 0$
	Explains that maximum v occurs when $\frac{dv}{dt} = 0$	2.4	E1	10.510 =0.9t 0.000 0.3t 0
	Accept reference to stationary point			$10.512e^{-0.9t} - 0.009e^{0.3t} = 0$
	Forms equation $\frac{dv}{dt} = 0$ and solves to find a value for t	1.1a	M1	
	PI by correct t			t = 5.886
	Obtains correct value of <i>t</i> AWRT 5.9	1.1b	A1	11 71 11 CO 0 9×5 886
	Substitutes their <i>t</i> into the given model PI by correct <i>v</i>	1.1b	M1	$v = 11.71 - 11.68e^{-0.9 \times 5.886}$ $-0.03e^{0.3 \times 5.886}$
	Finds value for maximum v	1.1b	A1	v = 11.5
	 Justifies final answer as being a maximum value eg: This is the maximum value as it is the only value which relates to dv/dt = 0 Evaluates second derivative at t = 5.9 where dv/dt² = -9.4608e^{-0.9t} - 0.0027e^{0.3t} obtaining correct value of -0.063 or explains both terms are negative so it is less than 0 Tests first derivative considering gradient either side of t=5.9 Sketches curve with maximum identified at (5.9, 11.5) CSO NB R1 can be awarded even if E1 was 	2.4	R1	This is the maximum value as it is the only value which relates to $\frac{dv}{dt} = 0$
	not awarded			

16(b)	Integrates at least one term correct	3.4	M1	- (4 <i>t</i>
	Integrates at least two terms correct	1.1a	M1	$s = \int v dt$
	Obtains a fully correct integrated expression including a constant	1.1b	A1	$s = 11.71t + 12.978e^{-0.9t} - 0.1e^{0.3t} + c$
	Interprets initial conditions - states $s = 0$ when $t = 0$ PI by substitution of correct values	3.4	B1	s = 0 when $t = 0$
	Substitutes $s = 0$ and $t = 0$ to find their constant – must be clear evidence of substitution seen if incorrect c obtained	1.1a	M1	c = -12.878 Diator of 11.71 + 12.070 = 0.9t
	Obtains fully correct expression for distance – coefficients can be in any form and do not have to be evaluated as a single decimal ACF	3.2a	A1	Distance = $11.71t + 12.978e^{-0.9t}$ $-0.1e^{0.3t} - 12.878$
16(c)	Substitutes $t = 9.8$ into their expression for distance to find s PI by sight of 99.99 m for s or Substitutes $s = 100$ into their expression for distance to find t PI by sight of 9.801 for t	1.1a	M1	$s=99.99~\mathrm{m}$ Model predicts distance to be 99.99 which is very near to 100 Accurate
	Compares s value with 100 metres or t value with 9.8 and concludes that it is a good model CAO	3.5a	A1	
	Total		16	

Q	Marking Instructions	AO	Marks	Typical Solution
	Marking Instructions	3.1b	Marks M1	Typical Solution
17(a)	Resolves vertically to form a three	3.10	IVI I	
	term equation			$D + T \sin \theta - M \pi$
	Condone sign error or sin/cos error			$R + T\sin\theta = Mg$
	Obtains fully correct equation for	1.1b	A1	
	resolving vertically			$T\cos\theta - F = Ma$
	Uses Newton's second law	3.1b	M1	$I \cos \theta - F = M\alpha$
	horizontally to form a three term			
	equation			F=µR
	Condone sign error or consistent			$\Gamma = \mu \Gamma$
	cos/sin error			$T\cos\theta - \mu R = Ma$
	Obtains fully correct equation for	1.1b	A1	$I \cos \theta - \mu K = M a$
	resolving horizontally			$T\cos\theta - \mu(Mg - T\sin\theta) = Ma$
	Uses $F = \mu R$ to replace F with μR in	3.3	B1	$1\cos\theta - \mu(Mg - 1\sin\theta) = Mu$
	their horizontal equation			$T(\cos\theta + \mu\sin\theta) = Ma + \mu Mg$
	Eliminates R to form a single	1.1a	M1	$\Gamma(\cos \theta + \mu \sin \theta) = M\alpha + \mu Mg$
	equation			$M(a + \mu g)$
	Completes rigorous argument to find	2.1	R1	$T = \frac{M(a + \mu g)}{\cos \theta + \mu \sin \theta}$
	required expression.	2.1	1 1 1	$\cos \theta + \mu \sin \theta$
	roquired expression.			
	Must see <i>T</i> as a factor before			
	division e.g. $T(\cos\theta + \mu\sin\theta)$			
	AG			
	Explains that the relationship may	2.4	B1	The sledge is at rest so the
17(b)	not be valid because the sledge is at			relationship may not be valid as
(()	rest			friction may not be acting at its
	Identifies that friction may not be at	3.5b	B1	limiting value
	its limiting value			
	Accept reference to $F \le \mu R$			
	Sledge may not be on the point of			
	slipping			
	Total		9	
	Total			