



Oxford Cambridge and RSA

**Tuesday 21 June 2022 – Afternoon**

**A Level Mathematics B (MEI)**

**H640/03 Pure Mathematics and Comprehension**

**Insert**

**Time allowed: 2 hours**



**INSTRUCTIONS**

- Do **not** send this Insert for marking. Keep it in the centre or recycle it.

**INFORMATION**

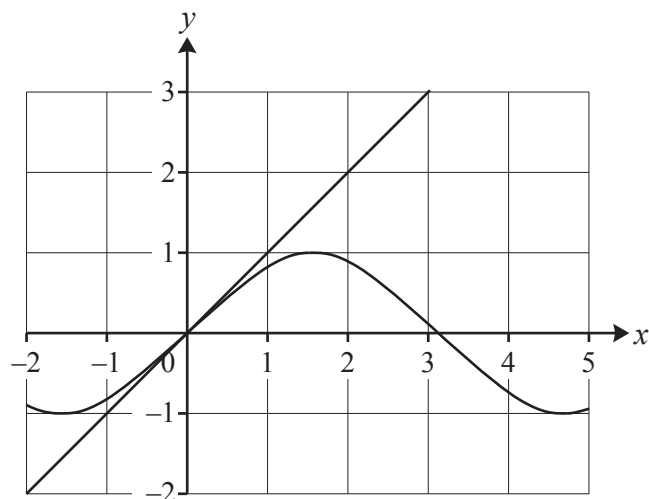
- This Insert contains the article for Section B.
- This document has **4** pages.

## Approximating the sine function

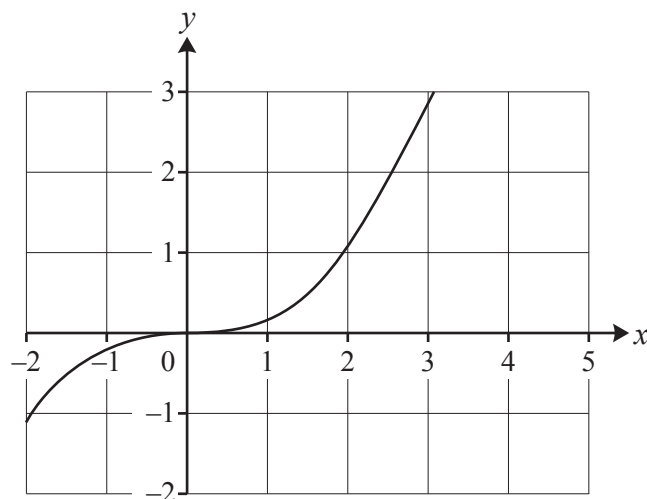
### Small angles

For a small angle  $x$  radians, the approximation  $\sin x \approx x$  is valid. The curve  $y = \sin x$  and the straight line  $y = x$  are shown in **Fig. C1.1**. **Fig. C1.2** shows the curve  $y = x - \sin x$ . Inspection of the graphs suggests that  $x$  is a reasonable approximation for  $\sin x$  for  $-0.5 \leq x \leq 0.5$  and also that  $y = x$  has the same gradient as  $y = \sin x$  when  $x = 0$ .

5



**Fig. C1.1**



**Fig. C1.2**

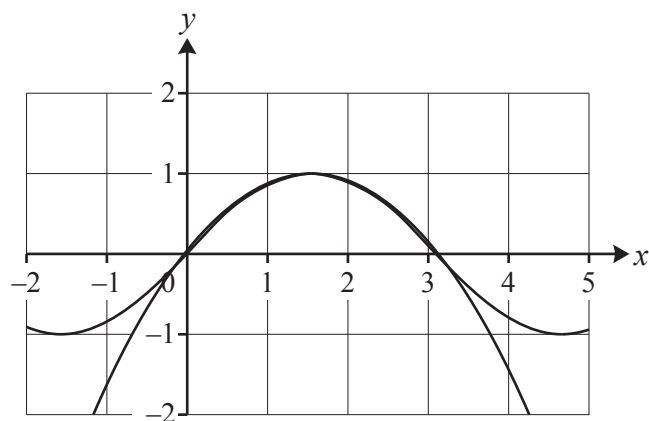
### Calculating $\sin x$

Trigonometric functions, including  $\sin x$ , are widely used so it is useful to be able to calculate the value of the sine of any angle accurately and quickly. This is easily done nowadays using a calculator but this was not possible in the past. The linear function,  $y = x$ , is only a reasonable approximation for  $y = \sin x$  for values of  $x$  close to zero. Perhaps using a higher degree polynomial would give a reasonable approximation for a wider range of values of  $x$ .

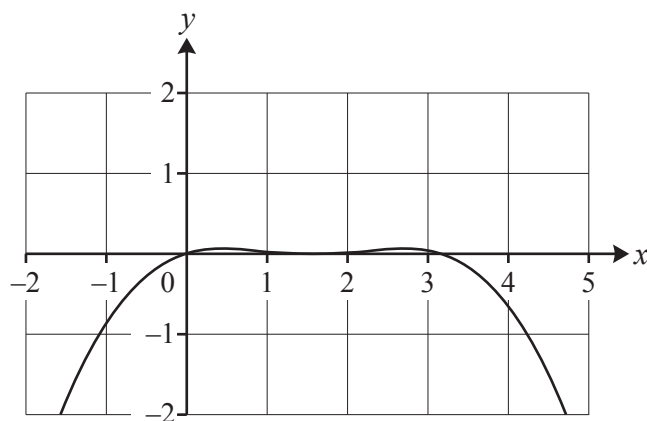
**Fig. C2.1** shows the curve  $y = \sin x$  and the quadratic curve which goes through the points  $(0, 0)$ ,

$(\frac{\pi}{2}, 1)$  and  $(\pi, 0)$ . The equation of this curve is  $y = \frac{4x(\pi - x)}{\pi^2}$ . **Fig. C2.2** shows the curve

$$y = \frac{4x(\pi - x)}{\pi^2} - \sin x.$$



**Fig. C2.1**



**Fig. C2.2**

The quadratic function seems to be a reasonably good approximation for  $\sin x$  in the interval  $0 \leq x \leq \pi$ . However, calculating percentage errors for selected values of  $x$  shows that the percentage errors made by using the quadratic function as an approximation to  $\sin x$  are quite high for values of  $x$  close to zero or  $\pi$ . 15

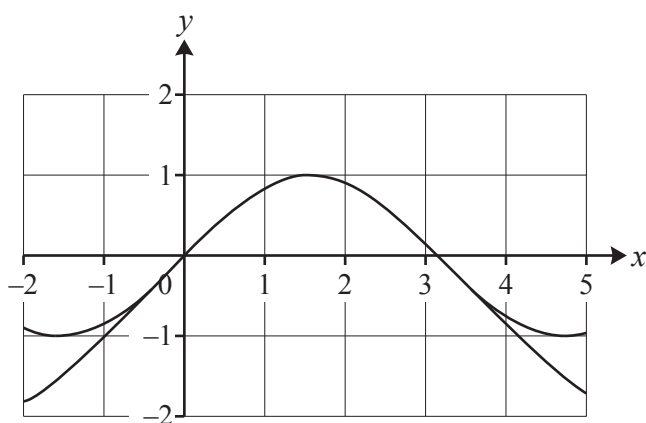
The spreadsheet in **Fig. C3** shows values of  $x$  in column A, with the corresponding values of  $\sin x$  and the quadratic function  $\frac{4x(\pi-x)}{\pi^2}$  in columns B and C. Columns D and E show the percentage 20 errors in using  $x$  and the quadratic as approximations for  $\sin x$ .

	A	B	C	D	E
1	x	sin(x)	quadratic	% error for x	% error for quadratic
2	0	0	0		
3	0.1	0.099833	0.123271	0.166861	23.476799
4	0.2	0.198669	0.238437	0.669791	20.016773
5	0.3	0.295520	0.345496	1.515901	16.911206
6	0.4	0.389418	0.444450	2.717298	14.131825

**Fig. C3**

### A better approximation

The approximation  $\sin x \approx \frac{16x(\pi-x)}{5\pi^2 - 4x(\pi-x)}$  was discovered by an Indian mathematician named Bhaskara in the 7th century. It is not known how Bhaskara derived the formula but it can be seen that the curve  $y = \frac{16x(\pi-x)}{5\pi^2 - 4x(\pi-x)}$  is symmetrical about  $x = \frac{\pi}{2}$  and goes through the points  $(0, 0)$ ,  $(\frac{\pi}{2}, 1)$  and  $(\pi, 0)$ . **Fig. C4** shows the curves  $y = \sin x$  and  $y = \frac{16x(\pi-x)}{5\pi^2 - 4x(\pi-x)}$ . Radians were not in use until the 18th century; Bhaskara gave the formula for an angle  $\theta$  degrees as  $\sin \theta \approx \frac{4\theta(180-\theta)}{40500 - \theta(180-\theta)}$ . 25



**Fig. C4**

The percentage error in approximating  $\sin x$  by  $\frac{16x(\pi-x)}{5\pi^2 - 4x(\pi-x)}$  is less than 2% throughout the interval  $0 \leq x \leq \pi$ . The Bhaskara approximation for  $\sin x$  can be used to derive the following 30 approximation for  $\cos x$ ;  $\cos x \approx \frac{\pi^2 - 4x^2}{\pi^2 + x^2}$ .

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