OCR
Oxford Cambridge and RSA

Tuesday 7 June 2022 – Afternoon

A Level Mathematics A

H240/01 Pure Mathematics

Time allowed: 2 hours

908546055O

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **8** pages.

ADVICE
- Read each question carefully before you start your answer.

Turn over

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \frac{u}{v}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k+\tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Standard deviation

$\sqrt{\dfrac{\sum(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\dfrac{\sum fx^2}{\sum f} - \bar{x}^2}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\overline{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

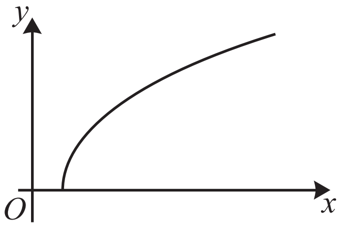
$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

1



The diagram shows part of the curve $y = \sqrt{x^2 - 1}$.

(a) Use the trapezium rule with 4 intervals to find an estimate for $\int_1^3 \sqrt{x^2 - 1}\,dx$.

Give your answer correct to **3** significant figures. **[4]**

(b) State whether the value from part **(a)** is an under-estimate or an over-estimate, giving a reason for your answer. **[1]**

(c) Explain how the trapezium rule could be used to obtain a more accurate estimate. **[1]**

2 **(a)** Given that a and b are real numbers, find a counterexample to disprove the statement that, if $a > b$, then $a^2 > b^2$. **[1]**

(b) A student writes the statement that $\sin x° = 0.5 \Longleftrightarrow x° = 30°$.

(i) Explain why this statement is incorrect. **[1]**

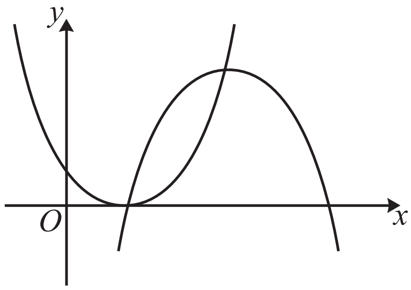
(ii) Write a corrected version of this statement. **[1]**

(c) Prove that the sum of four consecutive multiples of 4 is always a multiple of 8. **[3]**

3 (a) In this question you must show detailed reasoning.

Find the coordinates of the points of intersection of the curves with equations $y = x^2 - 2x + 1$ and $y = -x^2 + 6x - 5$. **[4]**

(b) The diagram shows the curves $y = x^2 - 2x + 1$ and $y = -x^2 + 6x - 5$.
This diagram is repeated in the Printed Answer Booklet.



On the diagram in the Printed Answer Booklet, draw the line $y = 2x - 2$. **[2]**

(c) Show on your diagram in the Printed Answer Booklet the region of the x-y plane within which all three of the following inequalities are satisfied.

$y \geqslant x^2 - 2x + 1$ $\qquad$ $y \leqslant -x^2 + 6x - 5$ $\qquad$ $y \leqslant 2x - 2$

You should indicate the region for which all the inequalities hold by labelling the region R. **[1]**

4 (a) Write $2x^2 + 6x + 7$ in the form $p(x+q)^2 + r$, where p, q and r are constants. **[3]**

(b) State the coordinates of the minimum point on the graph of $y = 2x^2 + 6x + 7$. **[2]**

(c) Hence deduce

- the minimum value of $2\tan^2\theta + 6\tan\theta + 7$,
- the smallest positive value of θ, in degrees, for which the minimum value occurs. **[3]**

5 (a) The graph of $y = 2^x$ can be transformed to the graph of $y = 2^{x+4}$ **either** by a translation **or** by a stretch.

(i) Give full details of the translation. **[2]**

(ii) Give full details of the stretch. **[2]**

(b) In this question you must show detailed reasoning.

Solve the equation $\log_2(8x) = 1 - \log_2(1-x)$. **[4]**

6 **(a)** Find the first four terms in the expansion of $(3+2x)^5$ in ascending powers of x. **[4]**

(b) Hence determine the coefficient of y^3 in the expansion of $(3+2y+4y^2)^5$. **[4]**

7 A curve has equation $2x^3+6xy-3y^2=2$.

Show that there are no points on this curve where the tangent is parallel to $y=x$. **[8]**

8 **(a)** Substance A is decaying exponentially such that its mass is m grams at time t minutes. Find the missing values of m and t in the following table.

t	0	10		50
m	1250	750	450	

[2]

(b) Substance B is also decaying exponentially, according to the model $m=160\mathrm{e}^{-0.055t}$, where m grams is its mass after t minutes.

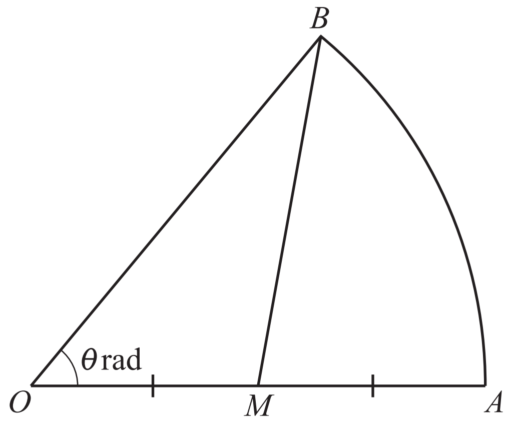
(i) Determine the value of t for which the mass of substance B is half of its original mass. **[3]**

(ii) Determine the rate of decay of substance B when $t=15$. **[3]**

(c) State whether substance A or substance B is decaying at a faster rate, giving a reason for your answer. **[1]**

9 Use the substitution $x=2\sin\theta$ to show that $\int_1^{\sqrt{3}}\sqrt{4-x^2}\,\mathrm{d}x=\frac{1}{3}\pi$. **[7]**

10



The diagram shows a sector OAB of a circle with centre O and radius OA. The angle AOB is θ radians. M is the mid-point of OA. The ratio of areas $OMB : MAB$ is 2:3.

(a) Show that $\theta = 1.25\sin\theta$. **[4]**

The equation $\theta = 1.25\sin\theta$ has only one root for $\theta > 0$.

(b) This root can be found by using the iterative formula $\theta_{n+1} = 1.25\sin\theta_n$ with a starting value of $\theta_1 = 0.5$.

- Write down the values of θ_2, θ_3 and θ_4.
- Hence find the value of this root correct to **3** significant figures. **[3]**

(c) The diagram in the Printed Answer Booklet shows the graph of $y = 1.25\sin\theta$, for $0 \leqslant \theta \leqslant \pi$.

- Use this diagram to show how the iterative process used in **(b)** converges to this root.
- State the type of convergence. **[3]**

(d) Draw a suitable diagram to show why using an iterative process with the formula $\theta_{n+1} = \sin^{-1}(0.8\theta_n)$ does not converge to the root found in **(b)**. **[2]**

11 The gradient function of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2\ln x}{\mathrm{e}^{3y}}$.

The curve passes through the point (e, 1).

(a) Find the equation of this curve, giving your answer in the form $\mathrm{e}^{3y} = \mathrm{f}(x)$. **[6]**

(b) Show that, when $x = \mathrm{e}^2$, the y-coordinate of this curve can be written as $y = a + \frac{1}{3}\ln\left(b\mathrm{e}^3 + c\right)$, where a, b and c are constants to be determined. **[3]**

12 A curve has parametric equations $x = \frac{1}{t}$, $y = 2t$. The point P is $\left(\frac{1}{p}, 2p\right)$.

(a) Show that the equation of the tangent at P can be written as $y = -2p^2x + 4p$. **[4]**

The tangent to this curve at P crosses the x-axis at the point A and the normal to this curve at P crosses the x-axis at the point B.

(b) Show that the ratio $PA : PB$ is $1 : 2p^2$. **[8]**

END OF QUESTION PAPER