OCR
Oxford Cambridge and RSA

GCE

Mathematics A

H240/03: Pure Mathematics and Mechanics

Advanced GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question includes the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for A Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 3 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			$\frac{\sin CAB}{8.3} = \frac{\sin 32}{13.5}$	M1	1.1a	Correct application of sine rule (in any form)	Or complete method using cosine rule
			CAB = 19.0	A1	1.1	Final answer – do not isw if candidates then calculate angle *ACB* – allow 19(.0)	19.014 183 98…
				[2]			
2	(a)	(i)	$(x-3)^2 - 9 + (y+2)^2 - 4 + 4 = 0 \Rightarrow (x-3)^2 + (y+2)^2 = 9$	M1	1.1	$(x \pm 3)^2$ and $(y \pm 2)^2$ seen (or implied by correct answer) or one correct coordinate	
			C (3, –2)	A1	1.1	Accept $x = 3$ and $y = -2$	
				[2]			
2	(a)	(ii)	$r = 3$	B1	1.1	Allow if stated explicitly in **(a)(i)** but not written down in **(a)(ii)** www for *r*	B0 if $r = \pm 3$ only
				[1]			
2	(b)		$(x-3)^2 + (kx-3+2)^2 = 9$ or $x^2 + (kx-3)^2 - 6x + 4(kx-3) + 4 = 0$	M1*	3.1a	Substitutes the correct equation of the line into any form of their equation of the circle	Each M is dependent on the previous Ms
			$(1+k^2)x^2 + (-6-2k)x + 1 = 0$	A1	1.1	oe (all terms on the same side – may not be factorised but should be simplified to 5 terms)	Condone lack of equal to 0
			$(-6-2k)^2 - 4(1+k^2)(1)$	M1dep*	3.1a	Correct explicit use of discriminant on their 3TQ to get an expression in *k* only	Condone equals or incorrect inequality
			$36 + 24k + 4k^2 - 4 - 4k^2 < 0 \Rightarrow 32 + 24k < 0$	M1dep*	2.1	Discriminant < 0 and simplify to the form $ak + b < 0$ (oe)	*a* and *b* non-zero
			$k < -\frac{4}{3}$	A1	2.2a	Fully correct (no additional values)	Or exact equivalent
				[5]			

Question			Answer	Marks	AO	Guidance	
3	(a)		**DR**				
			Attempt process for finding 2 critical values of $\lvert x-2\rvert \le \lvert 2x-6\rvert$	**M1**	**1.1a**	Either squaring both sides to obtain 3 terms on each side $(x^2-4x+4\le 4x^2-24x+36)$, simplifying and attempting to find two critical values (condone writing down roots from their quadratic without working)	Or attempt to solve 2 linear equations/inequalities, one with signs of x and $2x$ the same and the other with signs different (but condone other sign errors)
			Obtain 4	**A1**	**1.1**	Award whether given as $x=4$ or $x\le 4$ or $x\ge 4$ or …	
			Obtain $\frac{8}{3}$	**A1**	**1.1**	Award whether given as $x=\frac{8}{3}$ or $x\le\frac{8}{3}$ or $x\ge\frac{8}{3}$ or …	
			$x\ge 4$ or $x\le\frac{8}{3}$	**A1**	**2.5**	Correct notation and must see 'or' (do not accept 'and' or a comma) - one or more strict inequality signs is A0	Accept if in either correct set or interval notation e.g. $\{x:x\ge 4\}\cup\{x:x\le\frac{8}{3}\}$ or $\left(-\infty,\frac{8}{3}\right]\cup[4,\infty)$ SC: If no **DR** (e.g. sketch and/or answers only) so M0 then award B1 only for both correct answers $x\ge 4$ or $x\le\frac{8}{3}$ (**DR** requires a detailed and complete analytical method)
				[4]			

3	**(b)**		Refers to translation and stretch	**M1**	**1.2**	In either order; ignore details here; allow any equivalent wording (such as move or shift for translation) to describe geometrical transformations but not statements such as add 4 to x (do not accept 'enlargement' or 'shear' for stretch)	**SC:** if M0 but details of one correct (including correct order if req.), award B1 for 1/3 (in Either, Or 1, Or 2 cases)
			Either State translation in (positive) x-direction by 4 (units)	**A1**	**1.1**	Or state translation by $\begin{pmatrix}4\\0\end{pmatrix}$; accept horizontal to indicate direction or parallel to the x-axis; term 'translate' or 'translation' needed for award of A1	Do not accept 'in/on/across /up/along the x axis' or 'to the right' only A0 for SF 4
			State stretch by scale factor 0.5 in x-direction	**A1** **[3]**	**1.1**	Or parallel to x-axis or horizontally; term 'stretch' needed for award of A1; these two transformations must be in this order – if details correct for M1A1A1 but order wrong, award M1A1A0	Allow 'factor' or 'SF' for 'scale factor'. Do not accept 'in/ on/ across/ up/ along the x axis', 'in the positive x-direction', 'SF 0.5 units'
			Or 1 State stretch by scale factor 0.5 in x-direction	A1		or parallel to x-axis; 'stretch' needed for A1	
			State translation in (positive) x-direction by 2 (units)	A1 [3]		Or state translation by $\begin{pmatrix}2\\0\end{pmatrix}$; these two transformations must be in this order – if details correct for M1A1A1 but order wrong, award M1A1A0	**Same conditions for Or 1 and Or 2 as for Either for acceptable terminology**
			Or 2 State translation in (positive) x-direction by 1 (unit)	A1		Or state translation by $\begin{pmatrix}1\\0\end{pmatrix}$ or parallel to x-axis	
			State stretch by scale factor 2 in y-direction	A1 [3]		Or parallel to y-axis and allow vertical; term 'stretch' needed for award of A1; these two transformations can be given in either order	Do not accept 'down' only

4	**(a)**		$\frac{\mathrm{d}y}{\mathrm{d}x} = 3\sin 2x + 6x\cos 2x$	**M1***	**2.1**	Attempt use of product rule – answer of the form $\lambda \sin 2x + \mu x \cos 2x$	$\lambda, \mu \neq 0$
			$\sin 2x + 2x\cos 2x = 0$	**M1dep***	**1.1**	Sets derivative equal to zero	
			$\left(\frac{\sin 2x}{\cos 2x}\right) + 2x\left(\frac{\cos 2x}{\cos 2x}\right) = 0 \Rightarrow \tan 2x + 2x = 0$	**A1**	**2.2a**	**AG** – at least one step of correct intermediate working (e.g. $\cos 2x \tan 2x + 2x\cos 2x = 0$) from previous M mark to given answer (If $\cos 2x = 0$ and $\tan 2x + 2x = 0$ seen from $\cos 2x(\tan 2x + 2x) = 0$ then $\cos 2x = 0$ must be rejected)	Must be convincing as AG (must be = 0) – must see division by cos 2*x* (or stating the need to divide by this term but not just divide by cos)
				[3]			
4	**(b)**		$\mathrm{f}'(x) = 2\sec^2 2x + 2$	**B1**	**1.1**	Correct derivative	
			$x_{n+1} = x_n - \frac{\tan 2x_n + 2x_n}{2\sec^2 2x_n + 2}$	**M1**	**2.1**	Substitute their derivative (of the form $\alpha \sec^2 2x + 2$) into correct N-R formula	Need not be simplified and allow in terms of *x* only
				A1	**1.1**	First two values correct $(x_1 \text{ and } x_2)$ stated to at least 4 decimal places (truncated or rounded) – the table is not exhaustive and any other values used as a starting value in the interval given in the second guidance column will need to be checked - If no evidence of using NR (e.g. correct answer with no working) then no marks in this part. Dependent on correct NR formula (so must have scored B1)	Starting value must be in the interval $\frac{\pi}{4} < x_0 < \frac{3\pi}{4}$ $(0.786 \leq x_0 \leq 2.35)$
			x-coordinate of *P* is 1.0144	**A1**	**2.2a**	Independent of previous A mark (but must have scored B1M1) – must be stated to exactly 4 decimal places	1.014 378 911 …
				[4]			

x_0	x_1	x_2
0.8	0.81389959…	0.839614100…
0.9	0.96102142…	1.00372767…
1.0	1.01365728…	1.01437714…
$\pi/3$	1.01096312…	1.01433905…
1.1	0.99373628…	1.01287398…
1.2	0.93865044…	0.99215748…
1.3	0.87695335…	0.93545928…
1.4	0.82520962…	0.85941601…
1.5	0.79282139…	0.80006720…
1.6	0.78677182…	0.78813953…
1.7	0.81484089…	0.84130177…
1.8	0.88770903…	0.94804414…

		ALT	$f'(x) = 2\cos 2x + 2\cos 2x - 4x\sin 2x$ or $f'(x) = 6\cos 2x + 6\cos 2x - 12x\sin 2x$	B1		Correct derivative of either $\sin 2x + 2x\cos 2x$ or $3\sin 2x + 6x\cos 2x$	
			$x_{n+1} = x_n - \dfrac{\sin 2x_n + 2x_n\cos 2x_n}{4\cos 2x_n - 4x_n\sin 2x_n}$	M1		Substitute their derivative (of the form $\alpha\cos 2x + \beta x\sin 2x$) into correct formula for N-R	Need not be simplified and allow in terms of x only
			(table below)	A1		First two values correct $(x_1$ and $x_2)$ stated to at least 4 decimal places – the table is not exhaustive and any other values used as a starting value in the interval given in the second guidance column will need to be checked - If no evidence of using NR (e.g. correct answer with no working) then no marks in this part. Dependent on correct NR formula (so must have scored B1)	Starting values must be in interval $0.65 \le x_0 \le 1.61$
			x-coordinate of P is 1.0144	A1 [4]		Independent of previous A mark (but must have scored B1M1) – must be stated to exactly 4 decimal places	1.014 378 911 …

x_0	x_1	x_2
0.65	1.56363958…	0.80243617…
0.7	1.28834722…	1.02095799…
0.75	1.15730247…	1.02135573…
0.8	1.08739962…	1.01697831…
0.9	1.02795637…	1.01449472…
1.0	1.01452414…	1.01437893…
$\pi/3$	1.01500404…	1.01437917…
1.1	1.01775095…	1.01438636…
1.2	1.02326915…	1.01442951…
1.3	1.01965227…	1.01439698…
1.4	0.99197402…	1.01474280…
1.5	0.91147495…	1.02488159…
1.6	0.70130089…	1.28368799…

4	(c)		$h=\frac{\pi}{8}$	B1	1.1	For using $\frac{1}{2}\times\frac{\pi}{8}$ or $\frac{\pi}{16}$ or exact equivalent or for stating h	Not just for $\frac{\pi}{8}$ seen
			$\frac{1}{2}h\left[0+2\left(3\left(\frac{\pi}{8}\right)\sin\left(\frac{\pi}{4}\right)+3\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{2}\right)+3\left(\frac{3\pi}{8}\right)\sin\left(\frac{3\pi}{4}\right)\right)+0\right]$ $\left(=\frac{1}{2}h\left[0+2\left(\frac{3}{16}\pi\sqrt{2}+\frac{3}{4}\pi+\frac{9}{16}\pi\sqrt{2}\right)+0\right]\right)$	M1	2.1	Correct […] structure including multiplying the middle terms by 2. The zeros may be omitted. Allow one incorrect y value only. Any additional values or repeated values is M0. M0 if using x values or if only non-exact values seen but allow for the M mark if left in terms of sin	Ignore $\frac{1}{2}h$ term for this mark Note first 0 might be $3(0)\sin(2(0))$ and second 0 might be $3\left(\frac{\pi}{2}\right)\sin\pi$
			$\frac{1}{16}\pi\left(\frac{3}{8}\pi\sqrt{2}+\frac{3}{2}\pi+\frac{9}{8}\pi\sqrt{2}\right)$	A1	1.1	Correct (possibly un-simplified) exact expression for integral	Not in terms of sin and correct value of h used
			$\frac{3}{32}\pi^2\left(\sqrt{2}+1\right)$	A1 [4]	2.2a	$k=\frac{3}{32}$ www	
4	(d)	(i)	$\int_0^{\frac{1}{2}\pi}3x\sin 2x\,dx=\frac{3}{4}\pi$	B1 [1]	1.1	**BC** – ignore any working and mark final answer only (allow awrt 2.36)	oe, e.g. 2.356…
4	(d)	(ii)	$\frac{3}{32}\pi^2\left(\sqrt{2}+1\right)\approx 2.23<2.356...$ so trapezium rule gives an under-estimate of the area	B1 [1]	2.2a	Dependent on correct value in **(c)** (but may not be exact) and correct value for integral in **(d)(i)** – must state in this part correct decimal values (to at least 2 sf) for comparison (or 2.36 seen in **(d)(i)**)	B0 if only 'under-estimate' stated with no reasoning
4	(d)	(iii)	LH trapezium above curve, but others below curve, so overall approximation not clear	B1	2.4	oe e.g. trapezia/strips not all below the curve e.g. the curve changes from being convex to concave (concave up to concave down) e.g. the rate of change of the gradient changes from positive to negative	Condone mention of the curve being both concave and convex in the interval

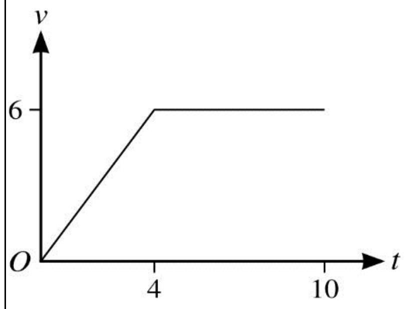
				[1]			

5		(a)	**DR** $(\cot\theta+\operatorname{cosec}\theta)^2=\left(\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}\right)^2$ or $\cot^2\theta+2\cot\theta\operatorname{cosec}\theta+\operatorname{cosec}^2\theta=$ $\frac{\cos^2\theta}{\sin^2\theta}+2\left(\frac{\cos\theta}{\sin\theta}\right)\left(\frac{1}{\sin\theta}\right)+\frac{1}{\sin^2\theta}$	**M1***	**2.1**	Replace both cot and cosec correctly in terms of cos and sin **or** expands brackets and replaces all terms with correct expressions in terms of sin and cos	Allow omission of 2 if brackets expanded but must contain a $\cot\theta\operatorname{cosec}\theta$ term
			$=\left(\frac{1+\cos\theta}{\sin\theta}\right)^2=\frac{(1+\cos\theta)^2}{\sin^2\theta}=\frac{(1+\cos\theta)^2}{1-\cos^2\theta}$ or $\frac{1+2\cos\theta+\cos^2\theta}{\sin^2\theta}=\frac{1+2\cos\theta+\cos^2\theta}{1-\cos^2\theta}$	**M1dep***	**2.1**	Combine terms and using $\sin^2\theta=1-\cos^2\theta$ correctly in denominator	Ignore terms in numerator for this mark
			$=\frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$ or $\frac{1+2\cos\theta+\cos^2\theta}{(1-\cos\theta)(1+\cos\theta)}$	**M1dep***	**1.1**	Re-writes $1-\cos^2\theta=(1+\cos\theta)(1-\cos\theta)$	Dependent on both previous M marks
			$=\frac{1+\cos\theta}{1-\cos\theta}$	**A1** **[4]**	**2.2a**	**AG -** correct proof - no notational or other errors such as missing $\theta's$ or inconsistent variables – must see $(1+\cos\theta)(1+\cos\theta)$ or $(1+\cos\theta)^2$ in numerator before **AG**	
		ALT	$\frac{1+\cos\theta}{1-\cos\theta}=\frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$	M1*		Multiplying numerator and denominator by $(1+\cos\theta)$	
			$\frac{(1+\cos\theta)^2}{1-\cos^2\theta}=\frac{(1+\cos\theta)^2}{\sin^2\theta}$	M1dep*		Expanding and using $\sin^2\theta=1-\cos^2\theta$ correctly in denominator	Ignore numerator for this mark
			$\left(\frac{1+\cos\theta}{\sin\theta}\right)^2=\left(\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}\right)^2$	M1dep*		Rewrite as a single squared term and split up into two terms	Dependent on both previous M marks
			$=(\cot\theta+\operatorname{cosec}\theta)^2$	A1 [4]		**AG –** allow candidates to 'meet in the middle' but for the A1 mark they must give a conclusion (e.g. 'LHS = RHS' or 'proved')	

5	(b)		DR			**If 3 and/or 2 missing then M1M1M1A0B0 max. (so do not treat as a MR)**	
			$3(\cos\theta+\operatorname{cosec}\theta)^2=2\sec\theta \Rightarrow 3\left(\dfrac{1+\cos\theta}{1-\cos\theta}\right)=\dfrac{2}{\cos\theta}$	M1*	2.1	Using the result from part **(i)** and re-placing sec with 1/cos	Allow omission or errors with the placement of the 3 and or the 2
			$3\cos\theta(1+\cos\theta)=2(1-\cos\theta)\Rightarrow 3\cos^2\theta+5\cos\theta-2=0$	M1dep*	1.1	Removing fractions and form three term quadratic in cos	Condone not = 0
			$(3\cos\theta-1)(\cos\theta+2)=0$	M1dep*	1.1	Solving their three-term quadratic in cosine provided discriminant is non-negative. Use of correct quadratic equation formula (if formula is quoted correctly then only one sign slip is permitted, if the formula is quoted incorrectly M0, if not quoted at all substitution must be completely correct to earn the M1) or factorising (giving their $\cos^2\theta$ term and one other term when factors multiplied out) or comp. the square (must get to the square root stage involving ± and arithmetical errors may be condoned provided their $3\left(\cos\theta+\frac{5}{6}\right)^2$ seen or implied)	Dependent on both previous M marks – as DR required if answers stated with no working then this mark cannot be awarded
			$\cos\theta\neq-2 \because \lvert\cos\theta\rvert\leq 1$	A1	2.3	Explicit rejection of -2 seen - allow $\cos\theta\neq-2$	No reason for rejection required
			$\cos\theta=\frac{1}{3}\Rightarrow\theta=1.23,\ 5.05$	B1	2.2a	awrt 1.23 and 5.05 (ignore any extra answers outside of the range of $0<\theta<2\pi$) but withhold this mark if there are any extra values in range (B0 if given in degrees)	1.230 959 … 5.052 225 …
				[5]			

6			$\frac{2x-1}{(2x+3)(x+1)^2} \equiv \frac{A}{2x+3}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$	**B1**	**3.1a**	Correct form for partial fractions – may be awarded later or implied by later working	Check carefully for their labelling of their *A*, *B* and *C*
			$2x-1 \equiv A(x+1)^2 + B(2x+3)(x+1) + C(2x+3)$	**M1***	**1.1a**	Allow sign errors only – this mark can be implied by at least one correct value www	
			$x=-1 \Rightarrow C=-3$	**A1**	**1.1**	www	
			$x=-\frac{3}{2} \Rightarrow A=-16$	**A1**	**1.1**	www	
			$x=0 \Rightarrow B=8$	**A1**	**1.1**	www $-\frac{16}{2x+3}+\frac{8}{x+1}-\frac{3}{(x+1)^2}$	
			$\int\left(\frac{-16}{2x+3}+\frac{8}{x+1}-\frac{3}{(x+1)^2}\right)\mathrm{d}x$ $= a\ln(2x+3)+b\ln(x+1)+c(x+1)^{-1}$	**M1dep***	**2.1**	Any non-zero values for *a*, *b* and *c* (from correct form of pf – no additional terms) – allow use of modulus instead of brackets throughout – condone omission of brackets throughout if recovered later	All signs may have been swapped (in advance of calculating area)
			$=-8\ln(2x+3)+8\ln(x+1)+3(x+1)^{-1}$	**A1**	**1.1**	All correct, may be un-simplified	Limits not required for this or previous mark
			$=\left(-8\ln 4+8\ln\frac{3}{2}+2\right)-\left(-8\ln 3+3\right)$	**M1dep***	**3.1a**	Correct use of the correct limits of 0 and $\frac{1}{2}$ Allow $\pm\left(\mathrm{F}\left(\frac{1}{2}\right)-\mathrm{F}(0)\right)$	Dependent on all previous M marks
			$=8\ln\frac{3}{2}+8\ln 3-8\ln 4-1=8\ln\left(\frac{\frac{3}{2}\times 3}{4}\right)-1$	**M1**	**2.1**	Correctly combining their log terms to a single log term– dependent on correct use of the correct limits and two log terms only (of the form $a\ln(2x+3)+b\ln(x+1)$)	Must be using 0.5 and 0 as limits
			Integral is $8\ln\frac{9}{8}-1 \Rightarrow$ Area $=1+8\ln\frac{8}{9}$	**A1**	**3.2a**	Final answer must be positive (as it is an area) www	$p=1,\ q=8,\ r=\frac{8}{9}$
				[10]			

		ALT	$\frac{2x-1}{(2x+3)(x+1)^2} \equiv \frac{A}{2x+3} + \frac{Bx+C}{(x+1)^2}$	B1		Correct form for partial fractions – may be awarded later or implied by later working	
			$2x-1 \equiv A(x+1)^2 + (2x+3)(Bx+C)$	M1*		Allow sign errors only – this mark can be implied by at least one correct value www	
			$A = -16$	A1		www	
			$B = 8$	A1		www	
			$C = 5$	A1		www $\frac{2x-1}{(2x+3)(x+1)^2} \equiv -\frac{16}{2x+3} + \frac{8x+5}{(x+1)^2}$	
			$\int \frac{8x+5}{(x+1)^2}\,dx$ e.g. using the substitution $u = x+1$ gives $\int \frac{8(u-1)+5}{u^2}\,du = \int \frac{8}{u} - \frac{3}{u^2}\,du = 8\ln u + \frac{3}{u}$	M1dep*		Correct method for integrating $\frac{8x+5}{(x+1)^2}$ leading to an expression of the form $\pm\alpha \ln u \pm \frac{\beta}{u}$ (oe for their correct method)	
			$= -8\ln(2x+3) + 8\ln(x+1) + 3(x+1)^{-1}$ or $= -8\ln(2x+3) + 8\ln u + \frac{3}{u}$	A1		All correct, may be un-simplified	Limits not required for this or previous mark
			$= \left(-8\ln 4 + 8\ln\frac{3}{2} + 2\right) - \left(-8\ln 3 + 3\right)$	M1dep*		Correct use of the correct limits of 0 and $\frac{1}{2}$ for x and 1 and $\frac{3}{2}$ if integrated expression still in terms of u (oe) (as in the main scheme allow limits applied either way round)	Dependent on all previous M marks
			$= 8\ln\frac{3}{2} + 8\ln 3 - 8\ln 4 - 1 = 8\ln\left(\frac{\frac{3}{2}\times 3}{4}\right) - 1$	M1		Correctly combining their log terms to a single log term– dependent on correct use of the correct limits and two log terms only (of the form $a\ln(2x+3) + b\ln(x+1)$)	Must be using 0.5 and 0 as limits
			Integral is $8\ln\frac{9}{8} - 1 \Rightarrow$ Area $= 1 + 8\ln\frac{8}{9}$	A1 [10]		Final answer must be positive (as it is an area) www	$p = 1,\ q = 8,\ r = \frac{8}{9}$

7	(a)			**B1** **B1** **[2]**	**1.1** **1.1**	Correct shape – if *t* axes not labelled then assume sketch is from 0 to 10 (ignore sketch after 10 if labelled) Correct values labelled on axes	Condone axes not labelled with *v* and/or *t* Condone origin not labelled
7	(b)		$\frac{1}{2}\times 4\times '6' + 6\times '6'$ or $\frac{1}{2}(10+6)('6')$ 48 m	**M1** **A1** **[2]**	**1.1a** **1.1**	Triangle & rectangle or trapezium (oe) with their velocity (which must be positive) – allow if 36 and 12 seen (provided these values are not subtracted) Units not required and condone incorrect units throughout mechanics section	or *suvat* twice

8	**(a)**		$s = 4d$	**B1** **[1]**	**1.1**		
8	**(b)**		$2.4U - \frac{1}{2}g \times 2.4^2 = 0$	**M1**	**3.3**	Use of $s = ut + \frac{1}{2}at^2$ correctly with $s = 0$, $a = \pm g$ and time equal to 2.4 **or** $v = u + at$ with $t = 1.2$, $v = 0$ and $a = \pm g$	oe use of other *suvat* equation(s) but must be a complete method to find U
			$U = 11.76 \text{ m s}^{-1}$	**A1** **[2]**	**1.1**	Accept $1.2g$ or awrt 11.8 (accept -11.76)	From $-g$
8	**(c)**		$d = \left(\frac{5}{3}d\right)t$	**B1**	**1.1**	oe - where t is the time for P to reach the wall	e.g. $t = 0.6$
			$h = Ut - \frac{1}{2}gt^2$	**M1**	**3.3**	Use of $s = ut + \frac{1}{2}at^2$ correctly with $a = \pm g$ and their U, t $\left(h = 11.76(0.6) - 0.5(9.8)(0.6)^2\right)$	M0 if their t is 2.4 or 1.2 or if their U is $5d/3$ (or in terms of d), M0 if $t = f(d)$
			$h = 5.292$ m	**A1** **[3]**	**1.1**	Accept 5.29	Not 5.30
8	**(d)**		$v_1 = U - gt$ or $v_1^2 = U^2 - 2gh$	**M1***	**3.3**	Use of a correct *suvat* equation(s) to find the vertical speed v_1 at the top of the wall using either their U and t or their U and h For reference: ($v_1 = 11.76 - g(0.6)$ or $v_1{}^2 = 11.76^2 - 2g(5.292)$) - may be seen in expression for speed	For reference $v_1 = 5.88$ M0 if vertical speed or U is in terms of d
			$\sqrt{\left(\frac{5}{3}d\right)^2 + ('U' - 9.8 \times 't')^2}\ (= 16)$	**M1dep***	**3.4**	Setting up an expression of the correct form for the speed or the speed squared in terms of d only (using their v_1)	
			$\sqrt{\left(\frac{5}{3}d\right)^2 + ('11.76' - 9.8 \times 0.6)^2} = 16$	**A1ft**	**1.1**	'Correct' equation in d following through their value for U (and h) only	All other terms must be correct
			$d = 8.93$	**A1** **[4]**	**1.1**	Accept 8.92 (from using 11.8) www	8.928 225 80…

9	(a)		$0.3 = \frac{1}{2}a \times 0.4^2 \ (\Rightarrow a = 3.75)$	B1	3.1b	Cao seen or implied by correct T	
			$0.5g\sin\theta - T = 0.5a$	M1	3.3	Use of N II parallel to the plane (correct number of terms) – weight must be resolved (but allow sin/cos confusion in component of the weight – may still be in terms of θ and a (or their incorrect values of θ and a))	Allow sign errors but M0 if using 0.5g for the mass on the rhs or if only resolving the mass (e.g. $0.5\sin\theta$) on the lhs
			$0.5g \times \frac{3}{5} - T = 0.5 \times 3.75$	A1ft	1.1	'correct' equation in terms of T with their a Allow $0.5g \times \sin 36.9 - T = 0.5 \times '3.75'$ or $0.5g \times \sin 0.644 - T = 0.5 \times '3.75'$ (in radians)	A0 if a = 0 or $\pm g$ (or a component of g)
			T = 1.065 N	A1	1.1	Allow 1.07 www	
				[4]			
9	(b)		$R = 0.2g$	B1	1.1	Where R is the normal contact force of surface acting on B – may be implied in N II applied horizontally	
			$T - F = 0.2a$ or $T - \mu R = 0.2a$	M1*	3.3	Use of N II horizontally for B or $0.5g\sin\theta - F = 0.7a$ for the whole system (M0 if any other mass used for whole system)	Correct number of terms (M0 if mass is 0.2g) – allow sign errors
			$1.065 - F = 0.2(3.75)$	A1	1.1	Or for a correct statement without explicitly seeing F e.g. $1.065 - 0.2g\mu = 0.2(3.75)$ (which would imply the next M mark as well)	$F = 0.315$ or $\mu R = 0.315$ or $F = 0.32$
			$0.315 = \mu \times 0.2 \times 9.8$	M1dep*	3.4	Use of $F = \mu R$ where $R = 0.2g$ - may be implied in N II applied horizontally	
			$\mu = 0.161$	A1	1.1	Allow awrt 0.161 or 0.163 from using T = 1.07 www	$\mu = \frac{9}{56}$
				[5]			

10	(a)			M1*	3.1b	At least two terms differentiated correctly (but not for e.g. $\mathbf{a} = (pt-3)\mathbf{i} + \left(8 + qt^{-1}\right)\mathbf{j}$ which is just dividing each term in t by t)	
			$\mathbf{a} = (2pt-3)\mathbf{i} + 8\mathbf{j}$	A1	1.1	Allow stated as a column vector	A0 if +**c**
			$(\lvert\mathbf{F}\rvert =) m\sqrt{(2pt-3)^2 + 8^2}$ Or $\left(\lvert\mathbf{F}\rvert^2 =\right) m^2\left\{(2pt-3)^2 + 8^2\right\}$	M1dep*	3.3	Correct use of |**F**| = m|**a**| with their **a** – allow in terms of m (and with or without t = 0.5 substituted) – must multiply both terms by m	M0 if m = 1implied
			$(p-3)^2 + 64 = 100$ or $(2p-6)^2 + 16^2 = 400$	A1	1.1	A correct equation in p only eg $20 = 2\sqrt{\left(2p(0.5)-3\right)^2 + 64}$	Allow unsimplified
				M1dep*	1.1	Attempt to solve their 3TQ in p (see 5(b) for awarding this M mark if working shown). If no method seen this mark can be implied by either the correct value of p or both 9 and –3 seen. As this part is not DR then the correct real roots of their 3TQ (with or without working) or their negative root of their 3TQ (with or without working) can score this mark	Dependent on both previous M marks
			$p = -3$ only	A1	2.2a	Do not award this mark if p = 9 is also stated without being rejected	
				[6]			
10	(b)			M1*	3.1b	At least two terms integrated correctly	
			$\mathbf{s} = \left(\frac{1}{3}pt^3 - \frac{3}{2}t^2\right)\mathbf{i} + \left(4t^2 + qt\right)\mathbf{j} + \mathbf{c}$	A1ft	1.1	Condone lack of + **c** and allow in terms of p or their value for p found/stated in **(a)**	Allow unsimplified
			$t = 0,\ \mathbf{s} = 2\mathbf{i} - 3\mathbf{j} \Rightarrow \mathbf{c} = 2\mathbf{i} - 3\mathbf{j}$	M1dep*	3.4	Using correct initial conditions to find **c**	
			$\mathbf{s} = \left(-t^3 - \frac{3}{2}t^2 + 2\right)\mathbf{i} + \left(4t^2 + qt - 3\right)\mathbf{j}$	A1	1.1	cao (oe)	Accept any vector form
				[4]			

10	(c)		$t = 1,\ \mathbf{s} = -\frac{1}{2}\mathbf{i} + (1+q)\mathbf{j}$	M1*	3.4	Substitute t = 1 into their **s**	Dependent on first M mark in (b)
			$k\left(-\frac{1}{2}\mathbf{i} + (1+q)\mathbf{j}\right) = 2\mathbf{i} - 8\mathbf{j} \Rightarrow k = \ldots$	M1dep*	3.1b	Correct method in an attempt to find q (e.g. equating a scalar multiple of their **s** (evaluated at t = 1) to 2**i** – 3**j** and solving for the scalar)	
			$k = -4 \Rightarrow q = 1$	A1 [3]	2.2a	www	

11	**(a)**		Dist. from A to the wall along the ladder: $(l=)\frac{h}{\sin 30}$	**B1**	**3.1a**		$(l=)2h$
				M1	**3.3**	Moments about A (each term must therefore by a force × distance) – three terms, both weights resolved (but allow sin/cos confusion), allow sign errors (M0 if only using masses)	M0 if the contact force at the wall appears as a component in their equation
			$2mgd\cos 30 + mga\cos 30 = R_w l$	**A1**	**1.1**	Correct LHS and RHS with their $l=\alpha h$ - A0 if $l=2a$	$\alpha \neq 0, 1$
			$R_w = \frac{1}{2h}\left(2mgd\frac{\sqrt{3}}{2} + mga\frac{\sqrt{3}}{2}\right) = \frac{1}{4h}(2mgd+mga)\sqrt{3}$ $R_w = \dfrac{mg(a+2d)\sqrt{3}}{4h}$	**A1**	**2.2a**	**AG** www – note that using $l = 2a$ but then stating $a = h$ is M1 only	Enough working must be shown as AG
				[4]			
11	**(b)**			**M1***	**3.3**	Resolve vertically (three or four terms (if both weights not combined)) – reaction at the wall resolved – allow sign errors and sin/cos confusion (must be using mg not m)	R_A is the normal contact force at A
			$R_w\cos 30 + R_A = 2mg + mg$	**A1**	**1.1**	oe	
				M1*	**3.3**	Resolve horizontally (allow sin/cos confusion) – two terms only	F_A is the frictional contact force at A
			$F_A = R_w \sin 30$	**A1**	**1.1**	oe	
			$R_w\sin 30 = \dfrac{\sqrt{3}}{8}(3mg - R_w\cos 30)$	**M1dep***	**3.4**	Use of $F=\frac{\sqrt{3}}{8}R$ with their expressions for F_A and R_A – dependent on previous M marks	Either in terms of R_w, m and g or a, d and h (and m, g)
			$\dfrac{mg(a+2d)\sqrt{3}}{8h} = \dfrac{\sqrt{3}}{8}\left[3mg - \dfrac{3mg(a+2d)}{8h}\right]$	**A1**	**3.4**	Correct equation in terms of (m, g) a, d and h	Condone non-exact values for this A mark
			$h = \frac{11}{24}(a+2d)$	**A1**	**2.2a**	$k=\frac{11}{24}$ oe www (must be exact value of k)	From exact working
				[7]		For reference: $k = 0.45833\ldots$	

11	(c)		$\frac{11}{24}(a+2d) \le a$	M1	3.1b	Uses the condition that h cannot exceed $2a\sin30$ $(= a)$ – allow if in terms of k or their incorrect k (e.g. $k\left(a+2d\right)=a$ is M1)	Allow any inequality sign or equals
			$d \le \frac{13}{22}a$ so greatest possible value of d is $\frac{13}{22}a$	A1 [2]	2.2a	Allow $d \le \frac{13}{22}a$ or $d = \frac{13}{22}a$	A0 if exact answer not seen
11	(d)		e.g. model the ladder as non-uniform e.g. include a frictional component for the contact of the ladder with the wall e.g. consider the size of the object at C e.g. consider the thickness of the ladder e.g. consider the fact that the ladder could bend	B1 [1]	3.5c	B0 if suggestion is to model the ground as smooth B0 for using a more accurate value for g	

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