



Oxford Cambridge and RSA

Wednesday 8 June 2022 – Afternoon

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Kinematics

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u+v)t$$

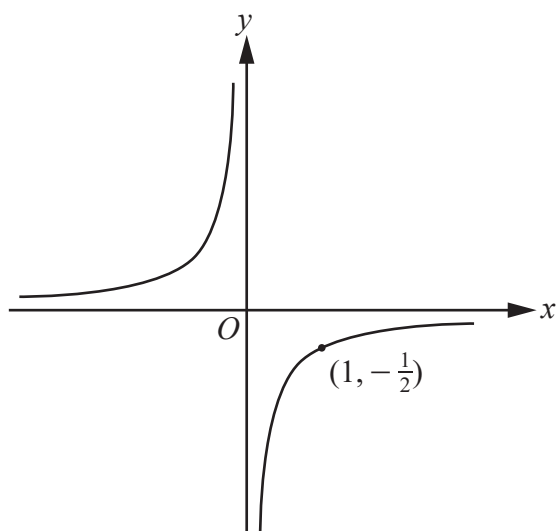
$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Section A: Pure Mathematics

Answer **all** the questions.

- 1** Write the solution of the inequality $(x - 2)(x + 3) > 0$ using set notation. [2]
- 2** **In this question you must show detailed reasoning.**
Solve the equation $3x + 1 = 4\sqrt{x}$. [4]
- 3** Give a counter example to disprove the following statement.
If x and y are both irrational then $x + y$ is irrational. [2]
- 4** The circle $x^2 + y^2 - 6x + 4y + k = 0$ has radius 5.
Determine the value of k . [3]



The diagram shows a curve C for which y is inversely proportional to x . The curve passes through the point $(1, -\frac{1}{2})$.

(a) (i) Determine the equation of the **gradient function** for the curve C . [3]

(ii) Sketch this **gradient function** on the axes in the Printed Answer Booklet. [1]

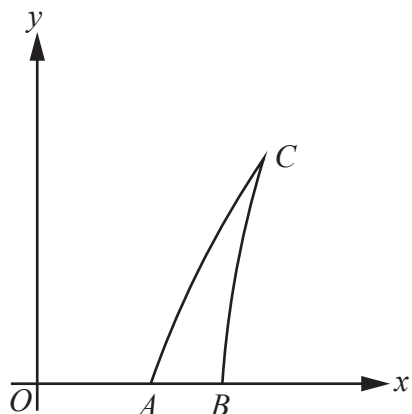
(b) The diagram indicates that the curve C has no stationary points.

State what feature of your sketch in part (a)(ii) corresponds to this. [1]

(c) The curve C is translated by the vector $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$.

Find the equation of the curve after it has been translated. [2]

6



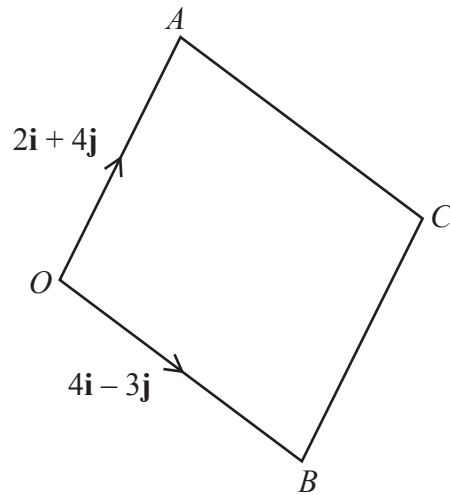
The shape ABC shown in the diagram is a student's design for the sail of a small boat.

The curve AC has equation $y = 2 \log_2 x$ and the curve BC has equation $y = \log_2 \left(x - \frac{3}{2}\right) + 3$.

- (a) State the x -coordinate of point A . [1]
- (b) Determine the x -coordinate of point B . [3]
- (c) By solving an equation involving logarithms, show that the x -coordinate of point C is 2. [4]

It is given that, correct to 3 significant figures, the area of the sail is 0.656 units^2 .

- (d) Calculate by how much the area is over-estimated or under-estimated when the curved edges of the sail are modelled as straight lines. [4]



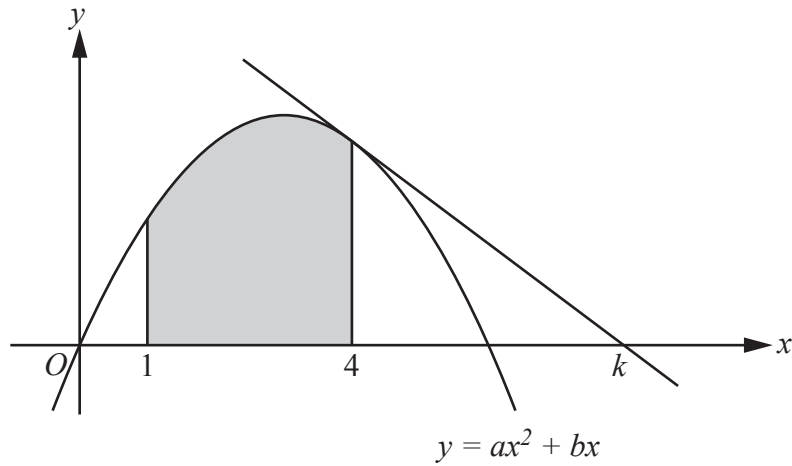
The diagram shows the parallelogram $OACB$ where $\overrightarrow{OA} = 2\mathbf{i} + 4\mathbf{j}$ and $\overrightarrow{OB} = 4\mathbf{i} - 3\mathbf{j}$.

- (a) Show that $\cos AOB = -\frac{2\sqrt{5}}{25}$. [5]
- (b) Hence find the exact value of $\sin AOB$. [2]
- (c) Determine the area of $OACB$. [2]

- 8 (a) The quadratic polynomial $ax^2 + bx$, where a and b are constants, is denoted by $f(x)$.

Use differentiation from first principles to determine, in terms of a , b and x , an expression for $f'(x)$. [4]

(b)



The diagram shows the quadratic curve $y = ax^2 + bx$, where a and b are constants. The shaded region is enclosed by the curve, the x -axis and the lines $x = 1$ and $x = 4$.

The tangent to the curve at $x = 4$ intersects the x -axis at the point with coordinates $(k, 0)$.

Given that the area of the shaded region is 9 units², and the gradient of this tangent is $-\frac{3}{4}$, determine the value of k . [7]

Section B: Mechanics

Answer **all** the questions.

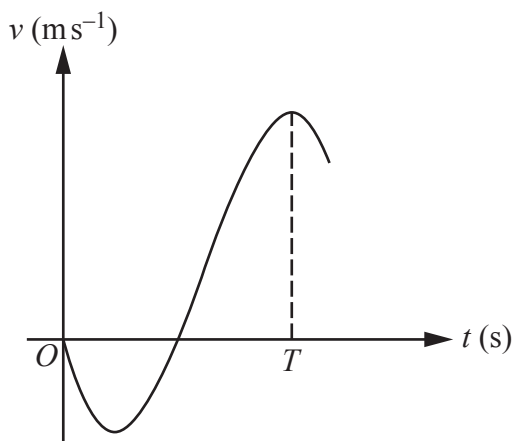
- 9 Two forces $(3\mathbf{i} + 2\mathbf{j})\text{ N}$ and $\mathbf{F}\text{ N}$ act on a particle P of mass 4 kg .

Given that the acceleration of P is $(-2\mathbf{i} + 3\mathbf{j})\text{ m s}^{-2}$, calculate $\mathbf{F}$. [2]

- 10 A small ball B is projected vertically upwards from a point 2 m above horizontal ground. B is projected with initial speed 3.5 m s^{-1} , and takes t seconds to reach the ground.

Find the value of t . [3]

11



A particle P moves along the x -axis. At time t seconds, where $t \geq 0$, the velocity of P in the positive x -direction is $v\text{ m s}^{-1}$. It is given that $v = t(t-3)(8-t)$.

P attains its maximum velocity at time T seconds. The diagram shows part of the velocity-time graph for the motion of P .

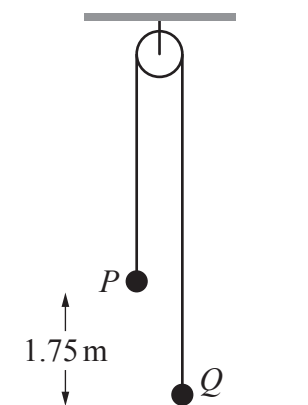
- (a) State the acceleration of P at time T . [1]

- (b) **In this question you must show detailed reasoning.**

Determine the value of T . [5]

- (c) Find the total distance that P travels between times $t = 0$ and $t = T$. [3]

12



Particles P and Q , of masses 4 kg and 6 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The system is in equilibrium with P hanging 1.75 m above a horizontal plane and Q resting on the plane. Both parts of the string below the pulley are vertical (see diagram).

- (a) Find the magnitude of the normal reaction force acting on Q . [1]

The mass of P is doubled, and the system is released from rest. You may assume that in the subsequent motion Q does not reach the pulley.

- (b) Determine the magnitude of the force exerted on the pulley by the string before P strikes the plane. [5]
- (c) Determine the total distance travelled by Q between the instant when the system is released and the instant when Q first comes momentarily to rest. [4]

When this motion is observed in practice, it is found that the total distance travelled by Q between the instant when the system is released and the instant when Q first comes momentarily to rest is less than the answer calculated in part (c).

- (d) State **one** factor that could account for this difference. [1]

END OF QUESTION PAPER

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