GCE

Mathematics A

Unit **H230/01**: Pure Mathematics and Statistics

Advanced Subsidiary GCE

Mark Scheme for June 2018

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for A Level Mathematics A

a   Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c   The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d	When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
e	The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
	Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
f	Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.
g	Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
h	For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
i	If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
j	If in any case the scheme operates with considerable unfairness consult your Team Leader.

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DR means the question requires detailed reasoning

Question			Answer	Mks	AO	Guidance	
1			**DR**				
1	(i)		$(\sqrt{3})^7$ or $\sqrt{3^7}$ or $3^3 \times \sqrt{3}$ or $3\sqrt{243}$	M1	1.1a	or any **correct intermediate step** using $\sqrt{}$ or $3^3 \times 3^{\frac{1}{2}}$	**If this step is not seen, M0A0**
			$27\sqrt{3}$	A1	1.1	or $a = 27$, $b = 3$	
				[2]			
1	(ii)		**DR**				
			$\frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$	M1	1.1a		**If this step is not seen, M0A0**
			$= \frac{\sqrt{2}+2}{1-2}$ or $\frac{\sqrt{2}+2}{-1}$ or $\frac{\sqrt{2}+2}{1+\sqrt{2}-\sqrt{2}-2}$	A1	1.1	A1 for correct num OR denom	
			$= -2 - \sqrt{2}$ ISW	A1	1.1	or $-2 + (-1\sqrt{2})$ or $c = -2$, $d = -1$ and $e = 2$	Allow $-(2 + \sqrt{2})$
				[3]			
2	(i)		$3^2 - 4k = 0$	M1	1.2	$x^2 + 3x + k = (x + a)^2 = x^2 + 2ax + a^2$ $\Rightarrow a = 1.5 \Rightarrow k = 1.5^2$	or $(x + 1.5)^2 - 2.25 + k = 0$
			$k = \frac{9}{4}$ or 2.25	A1	1.1		
				[2]			
2	(ii)		$(3 - x)(2 + x) > 0$ or $(x - 3)(x + 2) < 0$	M1	1.1a	oe Allow $(3 - x)(2 + x)$ or $(x - 3)(x + 2)$	or -2 and 3 seen
			$-2 < x < 3$ or $3 > x > -2$ ISW	A1	2.2a	Allow $x > -2$, $x < 3$ or $x > -2$ and $x < 3$	
			or $x \in (-2, 3)$				$x > -2$ **or** $x < 3$ M1A0
						Correct ans: BOD M1A1	unless followed by ans
				[2]			

	Question		Answer	Mks	AO	Guidance	
3	(i)		$\sin\theta = 0.5$ and -0.5 or $\sin\theta = \pm\sqrt{0.25}$ both	B1	1.1a	"–0.5" may be implied by **all 4 answers**	$\sin\theta = 0.5$, $\theta = 30$ and 210
			$\theta = 30^o$ and 150°	B1	1.1	Ignore other answers for this B1	B0B0B0
							$\sin\theta = \pm0.5$, $\theta = 30$ and 210
							B1B0B0
			$\theta = 210^0$ and 330^0	B1	1.1	NB Correct ans with no wking: B1B1B1	
				[3]			
3	(ii)		**DR**				
			60° and 240° seen or implied	B1	1.1a	Both needed, but ignore other values	
			20° seen	B1	1.1		
			$\phi = 20^o, 80^o$ With no other sol'ns	B1	1.1	SC: correct ans with no wking: B0B1B0	
				[3]			
4	(i)	(a)	$2x + 3$	B1	1.1	B1 for $2x$ or $2x^1$	
				B1	1.1	B1 for $+3$ or $+3x^0$	
				[2]			
4	(i)	(b)	$2x + 3 > 0$	M1	1.1	ft their (i)(a) Allow $x = -\frac{3}{2}$ is min, stated or shown	
			$x > -\frac{3}{2}$	A1f	2.2a	ft their **(i)(a)** so long as two terms	
				[2]			
4	(ii)		$3x$	B1	1.1		
			$-4\,x^{\frac{1}{2}}$	M1	1.1	M1 for $x^{\frac{1}{2}}$ seen before integration	May be implied by next line
			$-\frac{4x^{\frac{3}{2}}}{\frac{3}{2}}$	M1	1.2	M1 for $x^{\frac{3}{2}}$ or equiv seen after integ or increase their fractional power by 1	
			$-\frac{8}{3}x^{\frac{3}{2}}$ or equivalent	A1	1.1	ISW	
			$3x - \frac{8}{3}x^{\frac{3}{2}} + c$	B1f	2.5	Their integral $+ c$ in final ans ISW eg "$y =$" or attempt find c B0 if include integral sign or dx.	Correct ans, no working: full mks
				[5]			

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Question			Answer	Mks	AO	Guidance	
5			$N = 3k + 1$ or $N = 3k + 2$ (where k is an integer)	M1	3.1a	**One** of these. Allow without "N = "	Any letter other than p
			$(3k + 1)^2$ $(3k + 2)^2$	M1	1.1	Attempt **one** of these	Allow p
			$= 9k^2 + 6k + 1$ $= 9k^2 + 12k + 4$	A1	2.1	**Both** correct	Allow p
			$= 3(3k^2 + 2k) + 1$ or $= 3(3k^2 + 4k + 1) + 1$	A1	2.4	Or $9k^2+6k$ div by 3 or $9k^2+12k+3$ div by3 **One** of these	or similar in words. Allow p
			Both these are of form $3p + 1$, p an integer	E1	2.2a	Must say p is integer or $3k^2 + 2k$ and $3k^2 + 4k + 1$ are integers	Dep on M1M1A1A1 $N = 3p + 1$: max M0M1A1A1E0
						Similar marks for method using $N = 3k + 1$ & $N = 3k - 1$	
				[5]			
6	(i)		Roughly correct shape, both parts, no extra Clearly approaching axes in all four places	B1* dep*B1 **[2]**	1.1a 1.1b	not nec'y in correct posn on axes Allow thick or "double"	SC: half graph alone, correct shape, approaching axes, B1
	(ii)		$x(x^2 - 6x + 9)$				$\dfrac{dy}{dx} = 3x^2 - 12x + 9 = 0$
			$= x(x - 3)(x - 3)$ or $x = 0, x = 3, x = 3$ oe Curve drawn, with correct orientation: shape $\bigwedge$ roughly correct	M1 M1	1.1a 2.1	may be implied by diag Allow thick or "double" or wobbly or straight or vertical sections.	Min at (3, 0) stated or drawn
			thro' (0,0) min on x-axis min labelled (3, 0) or 3	B1 A1 A1	1.1 2.2a 1.1	Indep not at (0, 0) NB A-mks dep on M1 SC: All correct but upside down or all correct but stops at O B4	Correct graph: full marks regardless of wking
				[5]			
7			Allow without arrows or squiggles throughout				
7	(i)	(a)	$\mathbf{c} - \mathbf{a}$ oe	B1 **[1]**	1.2		

	Question		Answer	Mks	AO	Guidance	
7	(i)	(b)	$\mathbf{a} + \frac{1}{2}(\mathbf{c} - \mathbf{a})$ or $\mathbf{c} + \frac{1}{2}(\mathbf{a} - \mathbf{c})$	M1	3.1a	$\mathbf{a} + \frac{1}{2}$ their (a) or $\mathbf{c} - \frac{1}{2}$ their (a)	
			$= \frac{1}{2}(\mathbf{a} + \mathbf{c})$ or $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}$	A1	1.1b	Correct ans without wking: M1A1	
				[2]			
	(ii)		$\overrightarrow{OB} = (\mathbf{a} + \mathbf{c})$	M1	3.1a	$\overrightarrow{PB} = \mathbf{a} + \frac{1}{2}(\mathbf{c} - \mathbf{a})$ or $\mathbf{a} + \frac{1}{2}$ their (i)(a) or $\mathbf{c} + \frac{1}{2}(\mathbf{a} - \mathbf{c})$ $(= \frac{1}{2}(\mathbf{a}+\mathbf{c})$ oe), ft their (i)(a) **NB** $\overrightarrow{PB} = \frac{1}{2}(\mathbf{a} + \mathbf{c})$ without justification: M0A0A0E0	or $\overrightarrow{PB} = \mathbf{c} - \frac{1}{2}$ their (i)(a)
			$\Rightarrow \overrightarrow{OP} = \frac{1}{2}\overrightarrow{OB}$ Must see previous line	A1*	1.1	$\Rightarrow \overrightarrow{PB} = \overrightarrow{OP}$	or similar with $\overrightarrow{BP}$ or $\overrightarrow{BO}$
			$\Rightarrow P$ is midpt of OB or OPB is a straight line and $OP = PB$	dep* A1	2.1		
			Hence diagonals of //m bisect one another	E1	2.2a	dep M1A1A1	
				[4]			

Question			Answer	Mks	AO	Guidance	
8			**DR** $y-1=-2(x-2)$ | or $y=-2x+c$ & sub (2, 1)	M1	3.1a	**If no wking seen, no marks** or $y-1=2(x-(-2))$ or solve $y=-2x+5$ & $y=2x+5$	Alt method using proportion: Centre is on y-axis, not (0, 1) (may be implied) M1 $\frac{c-1}{2}=2$ or $c=1+2\times2$
			$y=-2x+5$ | $c=5$ Centre is (0, 5)	A1 A1	1.1 3.2a	$y=2x+5$ or $c=5$ stated or implied	$c=5$ A1 Centre is (0, 5) A1
			$r=\sqrt{2^2+4^2}$ $=\sqrt{20}$	M1	1.1a	or $r^2=2^2+4^2$ or ft their centre $=20$ or $a=0,\ b=-10,\ c=5$	
			$x^2+(y-5)^2=20$ oe	M1	1.2	ft their centre and rad^2 ($\neq 0$), however found	
			$x^2+y^2-10y+5=0$	A1 **[6]**	1.1	cao	
9	(i)		**Any mention of diff categories or types, eg** Includes students in all years (or classes) More representative of diff ages Prevents obtaining too many in one year gp	E1 **[1]**	2.4	**or any mention of proportions, eg** Uses the right proportions of students Ignore all else	**NOT** eg: Wider variety results More representative of pop
9	(ii)		**Must include <u>reason</u> why not rep've, eg** Her school may be biased Students' friends may be in local band Opinions at one school not indep of one another Ignore all else	E1 **[1]**	2.3	**except** allow Small sample or Only 100 students in sample Student opinion elsewhere may differ Diff conditions, or diff types of school, in diff areas, Diff bands liked in diff areas	**NOT**: Not representative of UK "No" may be implied

Question			Answer	Mks	AO	Guidance	
10	(i)		$\frac{3}{8} + \frac{5}{16} + 4p + p = 1$	M1	1.1a	oe eg $5p = 1 - (\frac{3}{8} + \frac{5}{16})$	
			$p = \frac{1}{16}$ or 0.0625	A1 [2]	1.1		
10	(ii)		$\frac{3}{8} \times \frac{5}{8}$ or $\frac{3}{8} \times \frac{3}{8}$ seen oe	M1	1.1a	or eg $\frac{3}{8} \times \frac{5}{16} + \frac{3}{8} \times \frac{4}{16} + \frac{3}{8} \times \frac{1}{16}$ ft their p	or $1 - (\frac{5}{16} + \frac{1}{4} + \frac{1}{16})^2$ M2
			$\frac{3}{8} \times \frac{5}{8} + \frac{5}{8} \times \frac{3}{8} + \frac{3}{8} \times \frac{3}{8}$ oe	M1	2.1	ft their p	or $1 - (\frac{5}{8})^2$ M2
			$= \frac{39}{64}$ or 0.609 (3 sf))	A1 [3]	1.1	Allow 0.61	
11	(i)		Prob of seeing a kingfisher is the same each day OR Seeing a kingfisher on one day is independent of other days	B1 [1]	3.5b	oe In context	Not: Prob of seeing kingfisher is indep
11	(ii)		0.318 (3 sf)	B1 [1]	3.4	**BC** Allow 0.32 or 0.317	
11	(iii)		0.318 or their (ii) used in a calculation	M1	3.1b	or B(6, their (ii)) stated	
			$1 - P(X \leq 3)$ using $p = 0.318$ or their (ii)	M1	1.1	or $P(X = 4, 5, 6)$ attempted using $p = 0.318$ or their (ii)	
			$= 0.0854$	A1 [3]	1.1	Allow 0.0845 to 0.0875 Allow 2 sf **BC**	

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Question	Answer	Mks	AO	Guidance	
12	$H_0: p = 0.2$ where p = P(A plant gets disease)	**B1**	**1.1**	Allow "possibility" or "proportion". Not p = %age having disease	**If 2-tail test:**
	$H_1: p < 0.2$ (not $p \leq 0.2$)	**B1**	**2.5**	Undefined p: B1B0	$H_o: p = 0.2$ (defined p) B1
	$X \sim$ B(250, 0.2) and $X = 36$ (allow 35)	**M1**	**3.3**	Stated or implied eg by 0.0139 (or 0.00884)	$H_1: p \neq 0.2$ B0
	$P(X \leq 36) = 0.0139$ or 0.014	**A1**	**3.4**	cao **BC**	M1
	$0.0139 < 0.02$	**A1f**	**1.1**	NB dep attempt $P(X \leq 36)$ ft their $P(X \leq 36)$ (< 0.02)	A1
	Reject H_0 (Allow Accept H_1)	**M1**	**2.2b**	Must see this statement NB dep attempt $P(X \leq 36)$ or $P(X < 36)$ and dep comp 0.02, ft their $P(X \leq 36)$, possibly not reject H_0	$0.0139 > 0.01$ A1 No more marks
	There is evidence that new method reduces prop of diseased plants	**A1f**	**3.5a**	In context, not definite ft only their $P(X \leq 36)$ or $P(X < 36)$ possibly "no evidence.." Ignore all else $P(X < 36)$: max B1B1M1A0A0M1A1	
		[7]			

Question			Answer	Mks	AO	Guidance	
13			In all parts, once mark gained, ignore all else			Allow eg "Group 1" for 0-17s etc.	Allow "children" for 0-17s
13	(i)		**Advantage: Type 1 answers**: **State or imply compare proportions (or distributions or structure or profile or pattern)** Examples: Can comp proportions (or distributions or structure or profile) Allow can see props Can compare areas' age groups relative to size of area Easier to see age groupp distributions	E1	1.1	**Advantage: Type 2 answers**: **State or imply with same scale, sizes of diags wd be very different** Examples: Prevents diag from becoming too big or too small to use effectively If one set of values is a lot lower than the other, it will be hard to compare them on the same scale.	**NOT** e.g: Easy to compare large area with small Easier to see results Easy to compare populations Because L is bigger than R Can compare age in small & large areas
			Disadvantage: Type 1 answers: **State or imply pop sizes not <u>easy</u> to compare** Examples: Diag does not show relative sizes of the authorities R'd appears to have more in 0-17, but actually L'l has more in this group Hard to compare because diff nos rep by same size on diags Can't compare numbers (or results or pops or sizes) easily Can't compare numbers (or results or pops or sizes) without calculation	E1 [2]	1.1	**Disadvantage: Type 2 answers**: **State or imply mismatch between diag size and pop size** Examples: Confusing because same size diag for diff size populations Looks as if same no. of people in each Might miss the fact that scales are diff, looks as if more 0-17s in R'd than L'l	**NOT eg** Can't compare results Can't compare numbers Easy to be mistaken when comparing
13	(ii)		90000. Allow between 75000 & 95000 incl.	B1 [1]	2.2b	Allow reasonable ans given as range eg "Much more than 50000 but < 100000"	

Question			Answer	Mks	AO	Guidance	
13	(iii)		**"L" = Liverpool. "R" = Rutland**				
			NB: Must be about 60-74s and/or 18-29s and/or 0-17s **Answer type 1** **Compare <u>proportions</u> in two age groups.** Examples: **Any two of eg:** L has smaller prop of 60-74 (than R'd) L has smaller prop of 0-17s (than R'd) L has larger prop of 18-29s (than R) eg, L prop of 18-29s is 4 × R prop 18-29s R has smaller prop of 18-29s R has hier prop of 0-17s	**E1** **E1**	2.2b 2.2b	**Answer type 2** **Compare gps with largest (or smallest) props.** **Allow "number" instead of prop only for this type of answer** Examples: L's hiest no. (or mode) is 18-29s AND R's hiest no. (or mode) is 0-17s E1 only L'ls smallest is 75+ AND R's smallest is 18-29 E1 only (75+ allowed in this case only) NOT "number" except in ans about modes or smallest. Ignore all else.	**Answer type 3** **Comp <u>props</u> in same age gps** Examples: L has high prop 18-29s AND R has low prop 18-29s E1 only R has high prop 60-74s AND L has low prop 60-74s E1 only **NOT** eg L has **more** 18-29s than R
				[2]			
13	(iv)		**Must state gp who are likely to have babies ie 18-29s or 30-44s or 18-44s. (Allow 0-29s or "young")** This gp is large in L, AND is small in R	**E1 ind** **E1dep**	2.4 2.4	**Inadequate ans:** L high prop of young, who will have babies E1 R high prop of old E0	Allow "number" instead of "proportion" NOT just This gp is larger in L
				[2]			
				75			