

GCE

Mathematics A

Unit **H230/02**: Pure Mathematics and Mechanics

Advanced Subsidiary GCE

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for AS Level Mathematics A

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. E marks will be lost except when results agree to the accuracy required in the question.
- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.
- k **Crossed Out Responses**
Where a candidate has crossed out a response and provided a clear alternative then the crossed out response is not marked. Where no alternative response has been provided, examiners may give candidates the benefit of the doubt and mark the crossed out response where legible.

Rubric Error Responses – Optional Questions

Where candidates have a choice of question across a whole paper or a whole section and have provided more answers than required, then all responses are marked and the highest mark allowable within the rubric is given. Enter a mark for each question answered into RM assessor, which will select the highest mark from those awarded. *(The underlying assumption is that the candidate has penalised themselves by attempting more questions than necessary in the time allowed.)*

Multiple Choice Question Responses

When a multiple choice question has only a single, correct response and a candidate provides two responses (even if one of these responses is correct), then no mark should be awarded (as it is not possible to determine which was the first response selected by the candidate). *When a question requires candidates to select more than one option/multiple options, then local marking arrangements need to ensure consistency of approach.*

Contradictory Responses

When a candidate provides contradictory responses, then no mark should be awarded, even if one of the answers is correct.

Short Answer Questions (requiring only a list by way of a response, usually worth only **one mark per response**)

Where candidates are required to provide a set number of short answer responses then only the set number of responses should be marked. The response space should be marked from left to right on each line and then line by line until the required number of responses have been considered. The remaining responses should not then be marked. Examiners will have to apply judgement as to whether a 'second

response' on a line is a development of the 'first response', rather than a separate, discrete response. *(The underlying assumption is that the candidate is attempting to hedge their bets and therefore getting undue benefit rather than engaging with the question and giving the most relevant/correct responses.)*

Short Answer Questions (requiring a more developed response, worth **two or more marks**)

If the candidates are required to provide a description of, say, three items or factors and four items or factors are provided, then mark on a similar basis – that is downwards (as it is unlikely in this situation that a candidate will provide more than one response in each section of the response space.)

Longer Answer Questions (requiring a developed response)

Where candidates have provided two (or more) responses to a medium or high tariff question which only required a single (developed) response and not crossed out the first response, then only the first response should be marked. Examiners will need to apply professional judgement as to whether the second (or a subsequent) response is a 'new start' or simply a poorly expressed continuation of the first response.

I Award No Response (NR) if:

- there is nothing written in the answer space.

Award Zero '0' if:

- anything is written in the answer space and is not worthy of credit (this includes text and symbols).

Team Leaders must confirm the correct use of the NR button with their markers before live marking commences and should check this when reviewing scripts.

Question			Answer	Marks	AOs	Guidance	
1	(i)		$\frac{\sin x}{20} = \frac{\sin 45}{16}$	M1*	1.1a	Use sine formula correctly in any form	SC B1 $\frac{\sin x}{16} = \frac{\sin 45}{20}$ If the previous A mark was not awarded then award for evidence of using inverse sin on their value of $\sin x$
			$\sin x = \frac{20 \sin 45}{16} \left(= \frac{5\sqrt{2}}{8} \right)$	A1	1.1	Correct expression for $\sin x$ or 0.883...	
				Dep*M1	1.1	Correct work leading to a value for x – if previous A mark awarded then this mark is for getting to either 62.1 or 117.9	
			62.1 and 117.9	A1 [4]	1.1	Cao	
1	(ii)		$\frac{1}{2}(BC)(20)\sin(45) = 75\sqrt{2}$	M1	1.1a	Use $\frac{1}{2}ab \sin C$ correctly and equate to $75\sqrt{2}$	
			$(BC \Rightarrow) 15 \text{ (cm)}$	A1 [2]	1.1	Accept 15.0 from correct working	

Question			Answer	Marks	AOs	Guidance	
2	(i)		$\frac{2}{3+x-4}$ or $\frac{2}{3+x+4}$	M1	1.1	Translates curve by $+/- 4$ parallel to the x -axis	
			$y = \frac{2}{x-1}$	A1	1.1	Fully correct, must have $y =$	
				[2]			
	(ii)		Stretch	B1	1.2	Must use stretch/stretched/stretching...	B0B1 is possible for e.g. 'enlarge by scale factor...' etc. but not for (e.g.) 'translate by scale factor...' etc. More than one transformation B0B0
			Scale factor $\frac{5}{2}$ parallel to the y -axis	B1	2.5	Allow "factor" or "SF" for "scale factor". Allow "vertically", "in the y direction". Do not accept "in/on/across/up/along the y axis", "in the positive y direction", "SF 5/2 units"	
				[2]			
3	(i)		$P \Rightarrow Q$	B1 [1]	1.1		
	(ii)		$P \Leftrightarrow Q$	B1 [1]	1.1		
	(iii)		$P \Rightarrow Q$	B1 [1]	1.1		

Question			Answer	Marks	AOs	Guidance	
4	(i)		$4[x^2 - 3x] + 11$			No marks until attempt to complete the square	
			$4\left[\left(x - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 11$	B1	1.1	Must be of the form $4(x \pm \alpha)^2 \pm \dots$	
			$(x - 3/2)^2$	B1	1.1		
			$4\left(x - \frac{3}{2}\right)^2 + 2$	B1	1.1		
				[3]			
	(ii)		No real roots	B1	2.2a	Zero, none, 0, ... if not 'no real roots' must be consistent with their (i)	
				[1]			
	(iii)		$r = 0 \Rightarrow 1$ real root or 1 repeated root	M1	2.4	Attempt to relate the value of r to the number of real roots (this can be implied with at least one correct statement)	
			$r < 0 \Rightarrow 2$ real roots				
			$r > 0 \Rightarrow$ no real roots	A1	2.4	All three statements correct	
				[2]			

Question			Answer	Marks	AOs	Guidance	
5			Equations are $x^2 - 4y = 10, x + 5y = k$				
			DR				
			$(k - 5y)^2 - 4y = 10$	M1*	3.1a	Substitute for x/y to eliminate one of the variables	If y eliminated
			$25y^2 + (-4 - 10k)y + (k^2 - 10)(= 0)$	A1	1.1	Obtain correct (unsimplified) quadratic	$5x^2 + 4x - 4k - 50 (= 0)$
			Tangent $\Rightarrow b^2 - 4ac = 0$	Dep*M1	2.1	Uses $b^2 - 4ac$ correctly for their quadratic	
			$(-4 - 10k)^2 - 4(25)(k^2 - 10) = 0$	A1	1.1	Fully correct substitution must equal 0	$16 - 4(5)(-4k - 50) = 0$
			$k = -\frac{127}{10}$ (-12.7)	A1	2.2a	k correct – with sufficient working	
		OR DR	Gradient of line = $-\frac{1}{5}$	B1			
			$\frac{dy}{dx} = \frac{1}{2}x$	B1		Correct differentiation	$2x - 4\frac{dy}{dx} = 0$
			$\frac{1}{2}x = -\frac{1}{5}$	M1		Equates their derivative with their gradient of line	
			$x = -\frac{2}{5}$	A1		x from correct working only	
			$y = -\frac{123}{50}(-2.46) \Rightarrow k = -\frac{127}{10}$ (-12.7)	A1		k from correct working only	
				[5]			

Question			Answer	Marks	AOs	Guidance	
6	(i)		$(a =) 75$	B1 [1]	3.3		
	(ii)		25 is the value that T approaches after a long time So therefore it is the ambient temperature	B1 [1]	2.2a	oe e.g. room temperature, minimum, lowest, etc.	Not e.g. initial, etc.
	(iii)		$-ake^{-kt}$ $-ak = -15$ $k = \frac{1}{5}$	B1 M1 A1ft [3]	3.1a 3.4 1.1	Correct rate of change of T Substitute $t = 0$ into their rate of change and equate with $+/-15$ oe FT their $\frac{15}{a}$	
	(iv)		$45 = 25 + 75e^{\frac{1}{5}t} \Rightarrow 75e^{\frac{1}{5}t} = 20$ (eg) $-\frac{1}{5}t = \ln\left(\frac{4}{15}\right) \Rightarrow t = \dots$ After 6.6 mins	M1 M1 A1 [3]	1.1 1.1 3.2a	Substitute $T = 45$ and subtract 25 from both sides Take logs correctly and attempt to solve for t Cao (no FT on this mark) with units	Their a and k 6.6087792...
	(v)		Decrease the value of a	B1 [1]	3.5c	Ignore mention of changes to k and/or 25	

Question			Answer	Marks	AOs	Guidance	
7	(i)		$2\sin x\left(\frac{\sin x}{\cos x}\right)=\cos x+5$	M1	3.1a	Uses $\tan x=\sin x/\cos x$	Must show sufficient working to justify the given answer
			$2\sin^2 x=\cos^2 x+5\cos x$	M1	3.1a	Uses $\sin^2 x=1-\cos^2 x$	
			$2(1-\cos^2 x)=\cos^2 x+5\cos x$				
			$2-2\cos^2 x=\cos^2 x+5\cos x$	A1	2.1	AG – correct working throughout	
			$3\cos^2 x+5\cos x-2=0$	[3]			
	(ii)		$(3\cos 2\theta-1)(\cos 2\theta+2)=0$	M1	1.1a	Attempt to solve 3-term quadratic	$(2\theta=)70.52877\dots,$ 289.471...
			$\cos 2\theta=\frac{1}{3}$ (and $\cos 2\theta=-2$)	A1	1.1	Condone $\cos x=\frac{1}{3}$	
			$\theta=\frac{1}{2}\arccos\left(\frac{1}{3}\right)$	M1	1.1	Correct order of operation to find one value of θ (or both values of 2θ correct)	
			$\theta=35.3^\circ$	A1	1.1	One correct value to the nearest integer or better	
			$\theta=144.7^\circ$	A1	1.1	Cao (35.3 and 144.7)	
				[5]			Any additional values in the range loses final A mark if earned

Question			Answer	Marks	AOs	Guidance	
8			DR			If $a = 27$ with no working then 0/9	
			$\int_8^a 2x^{\frac{1}{3}} - 7x^{-\frac{1}{3}} dx = 45$				
			$\left[\frac{2x^{\frac{4}{3}}}{\left(\frac{4}{3}\right)} - \frac{7x^{\frac{2}{3}}}{\left(\frac{2}{3}\right)} \right]_8^a (= 45)$	M1*	3.1a	M1 – attempt integration (increase in power by 1 for at least 1 term)	
				A1	1.1	A1 – 1 term correct (accept unsimplified)	
				A1	1.1	A1 – both correct (accept unsimplified)	
			$\frac{3}{2}a^{\frac{4}{3}} - \frac{21}{2}a^{\frac{2}{3}} - \left(\frac{3}{2}(8)^{\frac{4}{3}} - \frac{21}{2}(8)^{\frac{2}{3}} \right) (= 45)$	Dep*M1	1.1	$F(a) - F(8)$	
			$\frac{3}{2}a^{\frac{4}{3}} - \frac{21}{2}a^{\frac{2}{3}} - (24 - 42) (= 45)$	A1	1.1	oe	
				M1	1.1	Equate integrated expression to 45 – dependent on both previous M marks	
			$a^{\frac{4}{3}} - 7a^{\frac{2}{3}} - 18 = 0$				
			$\left(a^{\frac{2}{3}} - 9 \right) \left(a^{\frac{2}{3}} + 2 \right) = 0$	M1	3.1a	Attempt to solve quadratic in $a^{\frac{2}{3}}$	SC if M0 for fourth M mark then award
			$a^{\frac{2}{3}} = 9 \left(\text{and } a^{\frac{2}{3}} = -2 \right)$	A1	1.1		B1 for $a^{\frac{2}{3}} = 9$
			$a = 27$ only	A1	2.2a		B1 $a = 27$ only
				[9]			

Question			Answer	Marks	AOs	Guidance	
9			$F = \sqrt{36(2T-5)^2 + 64}$	M1*	3.3	Correct use of F = ma and Pythagoras	Allow t throughout
			$36(2T-5)^2 = 36$	A1	1.1	Correct equation(s) for both values of T e.g. $10 = 2\sqrt{9(2T-5)^2 + 16}$	
			$2T-5 = \pm 1 \Rightarrow T = \dots$	Dep*M1	1.1	Attempt to solve a quadratic leading to at least one value for T	
			$T = 2$ and $T = 3$	A1 [4]	2.2a		
10	(i)		$T - 3g = 3a$	M1*	3.3	Attempt N2L for P and Q – three terms, mass required, condone sign errors	M0 for $a = 0$ or $\pm g$
			$5g - T = 5a$	A1	1.1		
			$5g - T = 5\left(\frac{T-3g}{3}\right) \Rightarrow T = \dots$	Dep*M1	1.1	Eliminate a and attempt to solve for T	
			$T = 36.75 \text{ (N)}$	A1 [4]	1.1	Accept $\frac{15}{4}g$, 36.8	
	(ii)		$a = 2.45 \text{ ms}^{-2}$	B1	3.4	0.25g	M0 for $a = 0$ or $\pm g$ $a = \pm g$
			$v^2 = 0 + 2(2.45)(2.5)$	M1*	3.3	Use of $v^2 = u^2 + 2as$ for P with $u = 0$	
			$0 = 12.25 + 2h(-g)$	Dep*M1	3.3	Use of $v^2 = u^2 + 2as$ for P with $v = 0$	
			$(2h \Rightarrow) 1.25 \text{ (m)}$	A1 [4]	1.1	oe	

Question			Answer	Marks	AOs	Guidance	
11	(i)	(a)	$18 = \left(\frac{8+u}{2} \right)(9)$ $u = -4$ therefore the speed of P is $4 \text{ (ms}^{-1}\text{)}$	M1 A1 [2]	3.4 1.1	Use of $s = \left(\frac{u+v}{2} \right)t$ AG	
	(i)	(b)	eg $8 = -4 + 9a$ $a = \frac{4}{3} \text{ (ms}^{-2}\text{)}$	M1 A1 [2]	3.4 1.1	Use of $v = u + at$ with their u or $s = vt - \frac{1}{2}at^2$ or $v^2 = u^2 + 2as$ with their u or $s = ut + \frac{1}{2}at^2$ with their u Accept 1.33 or better	

Question			Answer	Marks	AOs	Guidance	
	(ii)		$0 = -4 + \frac{4}{3}t$ $t = 3$ $-s_{\max} = -4t + \frac{1}{2}\left(\frac{4}{3}\right)t^2$ $s_{\max} = 6 < 10$ so P is never at B	M1 A1 M1	3.1b 1.1 3.4	Use of $v = u + at$ with $v = 0$ and their a and u Use of $s = ut + \frac{1}{2}at^2$ with their a, u & t	
		OR		A1 [4] M1	2.2a	Compare with 10 or suitable comment Use of $s = ut + \frac{1}{2}at^2$ with their u and a and suitable s	
			$-10 = -4t + \frac{1}{2}\left(\frac{4}{3}\right)t^2$ e.g. $\det = -24$ therefore not possible	A1 M1 A1		Consider $b^2 - 4ac$ or attempt to solve three term quadratic in t Or $36 - 60 < 0$ therefore not possible	
		OR	$0 = (\pm 4)^2 + 2\left(\frac{4}{3}\right)s$ or $v^2 = (\pm 4)^2 + 2\left(\frac{4}{3}\right)(-10)$ $s = -6$ or $v^2 = -\frac{32}{3}$ Suitable conclusion	M2 A1 A1		Use of $v^2 = u^2 + 2as$ with their a and u and either $v = 0$ or $s = \pm 10$ Dependent on previous A mark	

Question		Answer	Marks	AOs	Guidance	
	(iii)	$x = at^3 + bt^2 + ct$ $\dot{x} = 3at^2 + 2bt + c$ $\ddot{x} = 6at + 2b$ $c = -4$ and $b = \frac{2}{3}$ $18 = a(6)^3 + \frac{2}{3}(6)^2 - 4(6) \Rightarrow a = \frac{1}{12}$ Velocity of $Q = \left(\frac{1}{4}(6)^2 + \frac{4}{3}(6) - 4 \right) = 13(\text{ms}^{-1})$	M1 M1 A1ft A1ft A1 [5]	1.1 2.1 1.1 1.1 1.1	Attempt to differentiate once Attempt to differentiate again and substitute $t = 0$ into both equations and substitute their acceleration in their second derivative and their u in their first derivative $b = 0.5 \times$ their accel. and $c = \pm 4$ Allow $a = -\frac{5}{36}, -\frac{7}{108}, \frac{1}{108}$ which come from $u = 4$ cao	Two terms differentiated correctly Two terms differentiated correctly following through from their first derivative Allow $b = 0.665$ from accel. = 1.33

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