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I declare this is my own work.

# A-level MATHEMATICS

## Paper 1

Tuesday 6 June 2023

Afternoon

Time allowed: 2 hours

### Materials

- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

### Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

### Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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Answer **all** questions in the spaces provided.

**1** Find the coefficient of  $x^7$  in the expansion of  $(2x - 3)^7$

Circle your answer.

[1 mark]

-2187

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2

128

**2** Given that  $y = 2x^3$  find  $\frac{dy}{dx}$

Circle your answer.

[1 mark]

$$\frac{dy}{dx} = 5x^2$$

$$\frac{dy}{dx} = 6x^2$$

$$\frac{dy}{dx} = \frac{x^4}{2}$$

$$\frac{dy}{dx} = 6x^3$$



- 3 The curve with equation  $y = \ln x$  is transformed by a stretch parallel to the  $x$ -axis with scale factor 2

Find the equation of the transformed curve.

Circle your answer.

[1 mark]

$$y = \frac{1}{2} \ln x$$

$$y = 2 \ln x$$

$$y = \ln \frac{x}{2}$$

$$y = \ln 2x$$

- 4 Given that  $\theta$  is a small angle, find an approximation for  $\cos 2\theta$

Circle your answer.

[1 mark]

$$1 - \frac{\theta^2}{2}$$

$$2 - 2\theta^2$$

$$1 - 2\theta^2$$

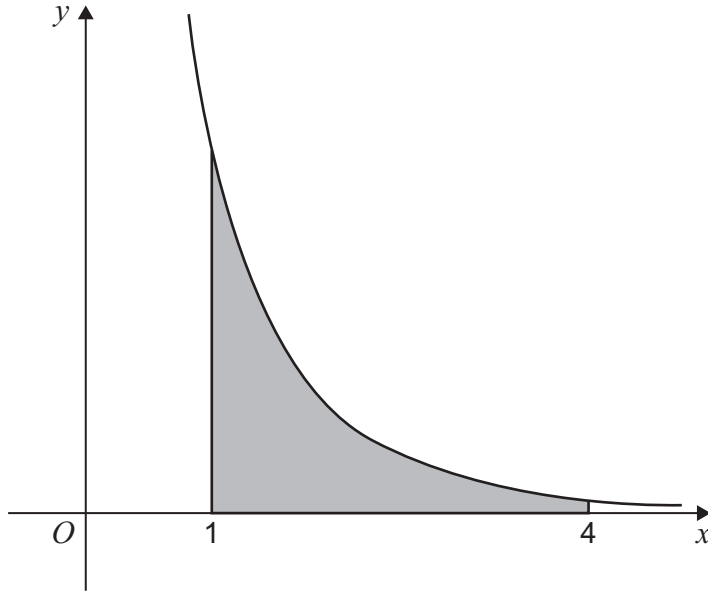
$$1 - \theta^2$$

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5 The graph of  $y = \frac{5}{e^x - 1}$  is shown in the diagram below.



The trapezium rule with 6 ordinates (5 strips) is to be used to find an approximation for the shaded area.

The values required to obtain this approximation are shown in the table below.

$x$	1	1.6	2.2	2.8	3.4	4
$y$	2.90988	1.26485	0.62305	0.32374	0.17263	0.09329

5 (a) Use the trapezium rule with 6 ordinates (5 strips) to find an approximate value for the shaded area.

Give your answer to four decimal places.

[3 marks]

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5 (b) Using your answer to part (a) deduce an estimate for  $\int_1^4 \frac{20}{e^x - 1} dx$

[1 mark]

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**6**

Show that the equation

$$2 \log_{10} x = \log_{10} 4 + \log_{10} (x + 8)$$

has exactly one solution.

Fully justify your answer.

**[5 marks]**

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**7 (a)** Given that  $n$  is a positive integer, express

$$\frac{7}{3 + 5\sqrt{n}} - \frac{7}{5\sqrt{n} - 3}$$

as a single fraction not involving surds.

**[3 marks]**

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**7 (b)** Hence, deduce that

$$\frac{7}{3 + 5\sqrt{n}} - \frac{7}{5\sqrt{n} - 3}$$

is a rational number for all positive integer values of  $n$

**[1 mark]**

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**9** The points  $P$  and  $Q$  have coordinates  $(-6, 15)$  and  $(12, 19)$  respectively.

**9 (a) (i)** Find the coordinates of the midpoint of  $PQ$

**[1 mark]**

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**9 (a) (ii)** Find the equation of the perpendicular bisector of  $PQ$

Give your answer in the form  $ax + by = c$  where  $a$ ,  $b$  and  $c$  are integers.

**[4 marks]**

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**9 (b) (i)** A circle passes through the points  $P$  and  $Q$

The centre of the circle lies on the line with equation  $2x - 5y = -30$

Find the equation of the circle.

[3 marks]

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**9 (b) (ii)** The circle intersects the coordinate axes at  $n$  points.

State the value of  $n$

[1 mark]

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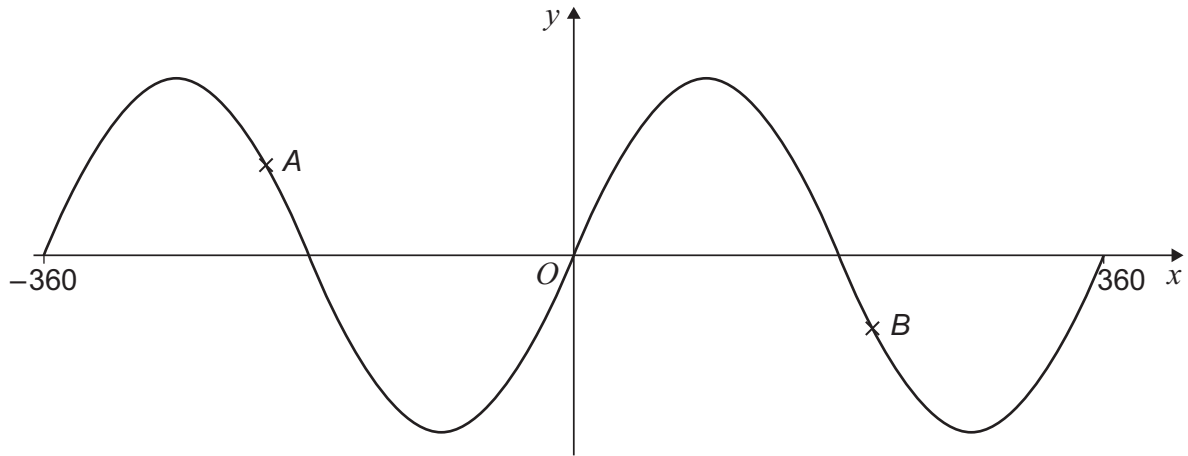
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**10** The curve with equation

$$y = \sin x^\circ$$

for  $-360 \leq x \leq 360$  is shown below.



**10 (a)** Point  $A$  on the curve has coordinates  $(a, 0.5)$

**10 (a) (i)** Find the value of  $a$

**[2 marks]**

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**10 (a) (ii)** State the value of  $\sin(180^\circ - a^\circ)$

**[1 mark]**

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**10 (b)** Point  $B$  on the curve has coordinates  $\left(b, -\frac{3}{7}\right)$

**10 (b) (i)** Find the exact value of  $\sin(b^\circ - 180^\circ)$

**[2 marks]**

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**10 (b) (ii)** Find the exact value of  $\cos b^\circ$

**[3 marks]**

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**11** The  $n$ th term of a sequence is  $u_n$

The sequence is defined by

$$u_{n+1} = pu_n + 70$$

where  $u_1 = 400$  and  $p$  is a constant.

**11 (a)** Find an expression, in terms of  $p$ , for  $u_2$

**[1 mark]**

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**11 (b)** It is given that  $u_3 = 382$

**11 (b) (i)** Show that  $p$  satisfies the equation

$$200p^2 + 35p - 156 = 0$$

**[3 marks]**

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**11 (b) (ii)** It is given that the sequence is a decreasing sequence.

Find the value of  $u_4$  and the value of  $u_5$

**[3 marks]**

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**11 (c)** The limit of  $u_n$  as  $n$  tends to infinity is  $L$

**11 (c) (i)** Write down an equation for  $L$

**[1 mark]**

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**11 (c) (ii)** Find the value of  $L$

**[1 mark]**

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One of the rides at a theme park is a room where the floor and ceiling both move up and down for  $10\pi$  seconds.

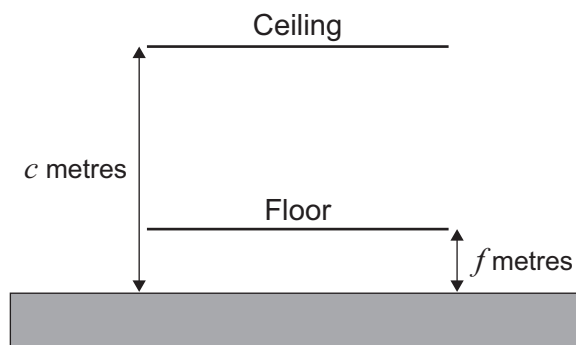
At time  $t$  seconds after the ride begins, the distance  $f$  metres of the floor above the ground is

$$f = 1 - \cos t$$

At time  $t$  seconds after the ride begins, the distance  $c$  metres of the ceiling above the ground is

$$c = 8 - 4 \sin t$$

The ride is shown in the diagram below.



12 (a)

Show that the initial distance between the floor and ceiling is 8 metres.

[1 mark]

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**12 (b)** Show that the distance  $d$  metres between the floor and ceiling at time  $t$  is given by

$$d = 7 + R \cos(t + \alpha)$$

where  $R$  and  $\alpha$  are positive constants to be found.

**[5 marks]**

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**12 (c)** Hence, find the minimum distance between the ceiling and the floor.

Give your answer to the nearest centimetre.

**[2 marks]**

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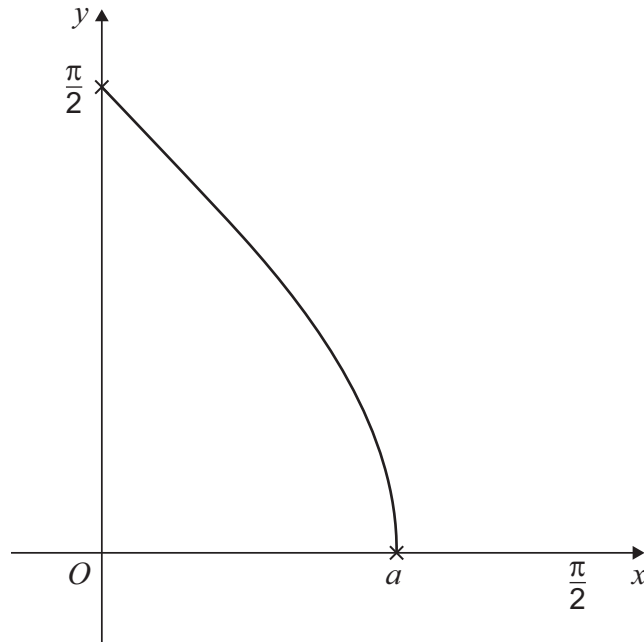
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13 The function  $f$  is defined by

$$f(x) = \arccos x \text{ for } 0 \leq x \leq a$$

The curve with equation  $y = f(x)$  is shown below.



13 (a) State the value of  $a$

[1 mark]

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13 (b) (i) On the diagram above, sketch the curve with equation

$$y = \cos x \text{ for } 0 \leq x \leq \frac{\pi}{2}$$

and

sketch the line with equation

$$y = x \text{ for } 0 \leq x \leq \frac{\pi}{2}$$

[4 marks]



**13 (b) (ii)** Explain why the solution to the equation

$$x - \cos x = 0$$

must also be a solution to the equation

$$\cos x = \arccos x$$

**[1 mark]**

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2 1

14 (a) (i) Given that

$$y = 2^x$$

write down  $\frac{dy}{dx}$

[1 mark]

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14 (a) (ii) Hence find

$$\int 2^x dx$$

[2 marks]

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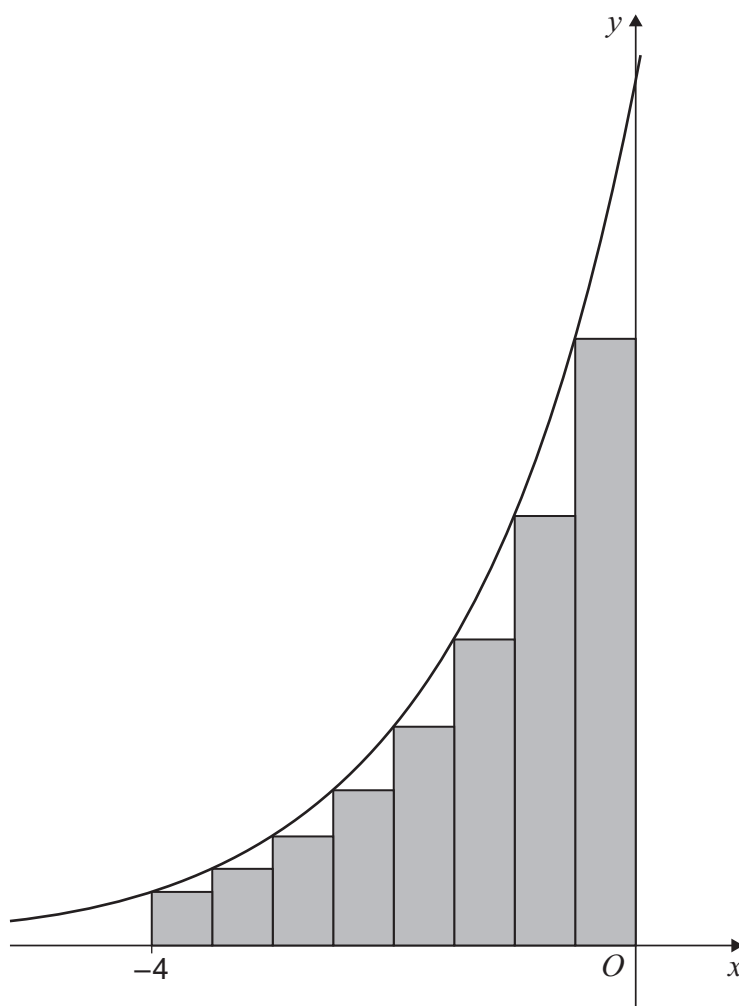
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- 14 (b)** The area,  $A$ , bounded by the curve with equation  $y = 2^x$ , the  $x$ -axis, the  $y$ -axis and the line  $x = -4$  is approximated using eight rectangles of equal width as shown in the diagram below.



- 14 (b) (i)** Show that the exact area of the largest rectangle is  $\frac{\sqrt{2}}{4}$

**[2 marks]**

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**14 (b) (ii)** The areas of these rectangles form a geometric sequence with common ratio  $\frac{\sqrt{2}}{2}$

Find the exact value of the total area of the eight rectangles.

Give your answer in the form  $k(1 + \sqrt{2})$  where  $k$  is a rational number.

**[3 marks]**

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**14 (b) (iii)** More accurate approximations for  $A$  can be found by increasing the number,  $n$ , of rectangles used.

Find the exact value of the limit of the approximations for  $A$  as  $n \rightarrow \infty$

**[3 marks]**

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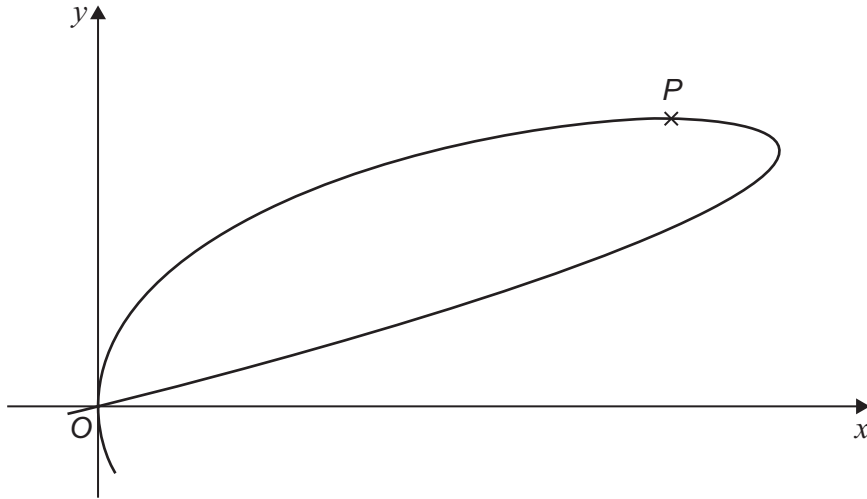
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15

The curve with equation

$$x^2 + 2y^3 - 4xy = 0$$

has a single stationary point at  $P$  as shown in the diagram below.

15 (a)

Show that the  $y$ -coordinate of  $P$  satisfies the equation

$$y^2(y - 2) = 0$$

**[7 marks]**


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**15 (b)**

Hence, find the coordinates of  $P$

**[2 marks]**

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16 (a) Given that

$$\frac{1}{16 - 9x^2} \equiv \frac{A}{4 - 3x} + \frac{B}{4 + 3x}$$

find the values of  $A$  and  $B$

[3 marks]

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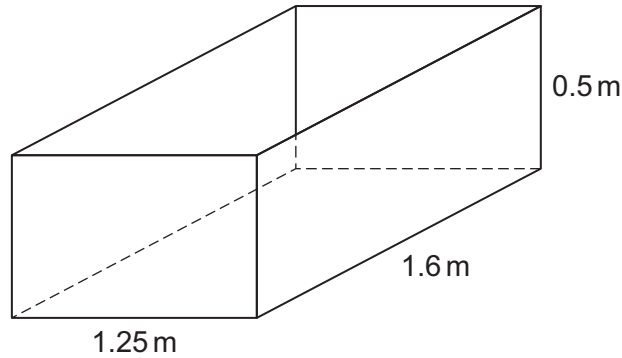
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- 16 (b)** An empty container, in the shape of a cuboid, has length 1.6 metres, width 1.25 metres and depth 0.5 metres, as shown in the diagram below.



The container has a small hole in the bottom.

Water is poured into the container at a rate of 0.16 cubic metres per minute.

At time  $t$  minutes after the container starts to be filled, the depth of water is  $d$  metres and water leaks out at a rate of  $0.36d^2$  cubic metres per minute.

At time  $t$  minutes after the container starts to be filled, the volume of water in the container is  $V$  cubic metres.

- 16 (b) (i)** Show that

$$\frac{dV}{dt} = \frac{16 - 9V^2}{100}$$

**[4 marks]**

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16 (b) (ii) Hence, find  $t$  in terms of  $V$

[5 marks]

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16 (b) (iii) Determine how long it takes to fill the container with water.

Give your answer to the nearest minute.

[2 marks]

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