A-level

MATHEMATICS

7357/2

Paper 2

Mark scheme

June 2022

Version: 1.0 Final Mark Scheme

226A7357/2/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

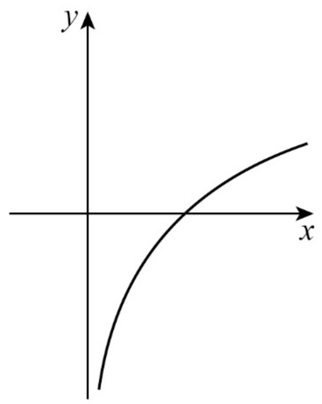
Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking instructions	AO	Marks	Typical solution
1	Ticks correct box	1.1b	B1	$(x-4)^2+(y+5)^2=36$
	Question 1 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	2.2a	R1	–1
	Question 2 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Ticks correct box	1.2	B1	
	Question 3 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4	Uses the sine rule Or Substitutes correctly into the cosine rule	1.1a	M1	$\frac{\sin\theta}{8.7}=\frac{\sin 38}{6.1}$ $\theta=61.4$ $A=180-61.4$ $=118.58...$ $=119^{\circ}$
	Obtains a value of 61 or 61.410964… rounded or truncated Condone answer (in radians) of 1.0718… or 0.4364… PI by correct obtuse angle or 81 Or Obtains correct length AB = 3.9367… Or $AB^2=15.4998...$	1.1b	A1	
	Deduces the largest angle is 119 AWRT CAO	2.2a	A1	
	Question 4 Total		**3**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Obtains 16 Not incorrectly labelled	1.1b	B1	$(2+5x)^4 = 16+160x+600x^2$ $+1000x^3+625x^4$ A = 16 B = 600
	Obtains 600 Not incorrectly labelled	1.1b	B1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Obtains the expansion of $(2-5x)^4$ = $A-160x+Bx^2-1000x^3+625x^4$ Accept A and B unsubstituted or their A and B Or Uses a valid method and obtains one of $C=320$ or $D=2000$	1.1a	M1	$(2+5x)^4-(2-5x)^4$ $=16+160x+600x^2+1000x^3+625x^4$ $-(16-160x+600x^2-1000x^3+625x^4)$ $=320x+2000x^3$
	Completes reasoned argument to show $(2+5x)^4-(2-5x)^4=320x+2000x^3$ Accept A and B unsubstituted or their A and B Must finish with $320x+2000x^3$ don't accept just C=320 and D = 2000	2.1	R1F	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(c)	Integrates one term correctly Accept C and D unsubstituted or their C and D Or Uses reverse of chain rule to obtain at least one term of the form $P(2\pm5x)^5$, $P=\pm\frac{1}{5}$ or $\pm\frac{1}{25}$	1.1a	M1	$\int\left((2+5x)^4-(2-5x)^4\right)\mathrm{d}x$ $=\int\left(320x+2000x^3\right)\mathrm{d}x$ $=160x^2+500x^4+c$
	Obtains $\frac{320}{2}x^2+\frac{2000}{4}x^4+c$ FT C and D unsubstituted or their C and D Or $\frac{(2+5x)^5}{5\times5}+\frac{(2-5x)^5}{5\times5}+c$ Condone missing +c	1.1b	A1F	
	Subtotal		**2**	

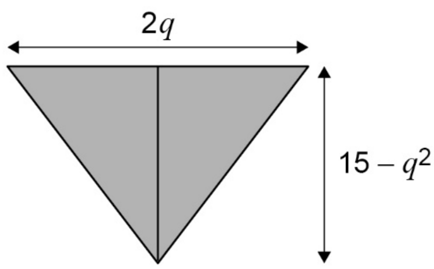
	Question 5 Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Squares a number with two or more digits and adds its digits. Must be explicit	1.1a	M1	$12^2 = 144$ $1+2=3$ $3 \neq 4$
	Completes argument to show that Asif's method is incorrect Must compare sum of digits with last digit of square number	2.3	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
6(b)	Obtains 1	1.1b	B1	1
	Subtotal		**1**	

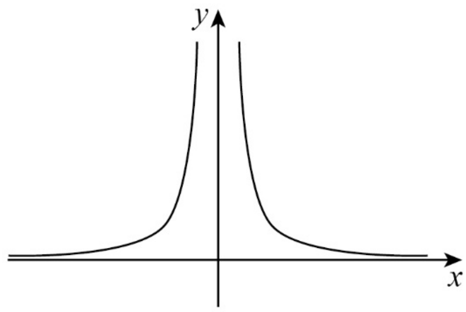
Q	Marking instructions	AO	Marks	Typical solution
6(c)	Lists at least four single digits and their squares Or Explains why odd digits do not need to be considered	1.1a	M1	$0^2 = 0$, $1^2 = 1$, $2^2 = 4$, $3^2 = 9$ $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, $7^2 = 49$ $8^2 = 64$, $9^2 = 81$
	Completes rigorous argument to prove that no square number has a last digit of 8 OE CSO	2.1	R1	Therefore, there can be no square number which has a last digit of 8
	Subtotal		**2**	

	Question 6 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Identifies the height of the triangle or rectangle as $15-q^2$ PI by $(q, 15-q^2)$, $h=15-q^2$ or $y=15-q^2$ may be indicated on diagram	3.1a	M1	 $A=\frac{1}{2}\times 2q(15-q^2)$ $=15q-q^3$ Since $A=q(15-q^2)>0$ then q's upper limit $c=\sqrt{15}$
	Completes rigorous argument to show the given result. It must be clear how they have defined the base and height with use of $\frac{1}{2}\times 2q(15-q^2)$ for whole triangle Or $\left[\frac{1}{2}q(15-q^2)\right]\times 2$ for two half triangles Or Explains why the area of the triangle is given by $q(15-q^2)$ with reference to the rectangle on either side of y-axis	2.1	R1	
	Deduces $c=\sqrt{15}$ ACF	2.2a	B1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)	Explains that **maximum** occurs when derivative **equals** 0 Condone incorrect variables in their derivative	2.4	E1	$\frac{dA}{dq} = 15 - 3q^2$ max occurs at $\frac{dA}{dq} = 0$ $15 - 3q^2 = 0$ $q = \sqrt{5}$ $\frac{d^2A}{dq^2} = -6\sqrt{5} < 0$ so local maximum $\therefore$ Max area = $15\sqrt{5} - 5\sqrt{5} = 10\sqrt{5}$
	Differentiates w.r.t. q At least one term correct	3.1a	M1	
	Obtains $15 - 3q^2$	1.1b	A1	
	Solves 'their $\frac{dA}{dq}$' $= 0$ to find q **and** substitutes to find maximum area	1.1a	M1	
	Obtains correct maximum area ACF	1.1b	A1	
	Gives justification for maximum Could be evaluation of second derivative as -13.42…< 0 Or Test of gradient either side, Or Explanation, for example: This must be a max value as only turning point in the interval $0 < q < \sqrt{15}$ **and** the area is 0 at the endpoints	2.4	E1	
	Subtotal		**6**	

	Question 7 Total		**9**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Sketches a branch of the curve with correct shape in 1st or 2nd quadrant with correct asymptotes and not touching the axes	1.2	M1	
	Sketches a fully correct curve with correct asymptotes and not touching the axes	1.1b	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	Describes a stretch of scale factor 3 or 9 in either direction	2.4	M1	The curve can be stretched in the y-direction by a scale factor of 9 The curve can be stretched in the x-direction by a scale factor of 3 Beth and Paul are both correct.
	Explains that the curve can be stretched in the y-direction by a scale factor of 9 Scale factor PI by $9\mathrm{f}\left(x\right)$ OR Explains that the curve can be stretched in the x-direction by a scale factor of 3 Scale factor PI by $\mathrm{f}\left(\frac{x}{3}\right)$	2.4	A1	
	Explains that the curve can be stretched in the y-direction by a scale factor of 9 PI by $9\mathrm{f}\left(x\right)$ AND Explains that the curve can be stretched in the x-direction by a scale factor of 3 PI by $\mathrm{f}\left(\frac{x}{3}\right)$ AND Concludes both are correct	2.2a	R1	
	Subtotal		**3**	

	Question 8 Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
9	Uses a log (or index) law correctly on an algebraic term $\log A \pm \log B$ $n \log A$	1.1b	B1	$\log_2 x^3 - \log_2 y^2 = 9$ $\log_2 \frac{x^3}{y^2} = 9$ $\frac{x^3}{y^2} = 2^9$ $x^3 = 2^9 y^2$ $x = 8y^{\frac{2}{3}}$
	Raises 2 to the power of both sides (removal of $\log_2$) Or writes 9 as $9\log_2 2$ or $\log_2 512$ OE	1.1a	M1	
	Obtains correct equation without logs Or obtains $\log_2(x) = \log_2(8y^{2/3})$	1.1b	A1	
	Completes a reasoned argument to obtain $x = 8y^{\frac{2}{3}}$	2.1	R1	
	Question 9 Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)(i)	Forms correct model Or applies repeated percentage increase 4 times PI by AWRT 75.9	3.3	B1	$x = 25 \times 1.32^t$ $t = 5 \Rightarrow x = 25 \times 1.32^5 = 100.18...$
	Substitutes t = 5 into their model Or Applies repeated percentage increase 5 times	3.4	M1	
	Obtains 101 Condone 100 CAO	3.2a	A1	x = 101
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)(ii)	Explains that the **model** grows **exponentially** Must refer to model and exponential	3.5b	E1	The value predicted by the exponential model will grow without limit.
	Refers to 900 plants. eg this can't be true as there are only 900 tomato plants Condone reference to "tomato(es)" or "plants" in place of "tomato plants"	3.5a	E1	This can't be true as there are only 900 tomato plants in the greenhouse.
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(i)	Rearranges to obtain one of the following: $\frac{P}{x(900-x)}\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{Q}$ $\frac{P}{x(900-x)}\mathrm{d}x=\frac{1}{Q}\mathrm{d}t$ $\frac{P}{x(900-x)}=\frac{1}{Q}\frac{\mathrm{d}t}{\mathrm{d}x}$ where $P\times Q=2700$ If their P=2700 no need to see explicit $\frac{1}{Q}$ with dt, or $\frac{\mathrm{d}t}{\mathrm{d}x}$ May include integral signs	3.1a	B1	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{x(900-x)}{2700}$ $\frac{2700}{x(900-x)}\frac{\mathrm{d}x}{\mathrm{d}t}=1$ $\int\left(\frac{A}{x}+\frac{B}{900-x}\right)dx=\int dt$ $\frac{2700}{x(900-x)}\equiv\frac{A}{x}+\frac{B}{900-x}$ $2700\equiv A(900-x)+Bx$ $x=0\Rightarrow A=\frac{2700}{900}=3$ $x=900\Rightarrow B=3$ $\therefore$ $\int\left(\frac{3}{x}+\frac{3}{900-x}\right)dx=\int dt$
	Forms partial fraction equation with correct denominators and uses an appropriate method to find their numerators PI by correct A and B without incorrect working	3.1a	M1	
	Obtains correct A and B and concludes with $\int\left(\frac{3}{x}+\frac{3}{900-x}\right)dx=\int dt$ Accept $\int 1dt$ Condone missing brackets	2.1	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(ii)	Integrates to obtain $\ln x$ or $\pm\ln(900-x)$ Condone missing brackets for this mark	3.1a	M1	
	Integrates to obtain $\ln x$ and $\pm\ln(900-x)$ Condone missing brackets for this mark	1.1a	M1	
	Integrates to obtain $3(\ln x-\ln(900-x))+c=t$ OE Condone missing $+c$	1.1b	A1	$3(\ln x-\ln(900-x))+c=t$ $3(\ln 25-\ln(900-25))+c=0$ $c=10.67$
	Uses t = 0, x= 25, to obtain a value for c	3.4	M1	$t=3(\ln x-\ln(900-x))+10.67$
	Obtains correct equation for t in terms of x ACF If c given as a decimal accept AWRT 11 eg $t=3(\ln x-\ln(900-x))+3\ln 35$ $t=3\ln\left(\frac{35x}{900-x}\right)$	1.1b	A1	
	Subtotal		**5**	

Q	Marking instructions	AO	Marks	Typical solution
10(b)(iii)	Substitutes x = 450 into their model from part (b)(ii)	3.4	M1	$3(ln\,450 - ln(450)) + 10.67$ $= 10.67\ldots$
	Obtains 11 CAO	3.2a	A1	It takes 11 days from when the damage is first noticed until half of the plants are damaged by insects
	Subtotal		**2**	

	Question 10 Total		**15**	

Q	Marking instructions	AO	Marks	Typical solution
11	Circles correct answer	1.1b	B1	1.63 m s^{-2}
	Question 11 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
12	Circles correct answer	1.1b	B1	$a = 0.6u$
	Question 12 Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)	Obtains $7\sin\theta$ for vertical component of initial velocity	1.1b	B1	$u = 7\sin\theta$
	Uses $v^2 = u^2 + 2as$ with $v = 0$ Or uses appropriate constant acceleration equations that form a complete method to obtain h eg finds t then subs into $s = ut + \frac{1}{2}at^2$	3.3	M1	$v^2 = u^2 + 2as$ $0 = 49\sin^2\theta - 19.6h$
	Completes argument substituting $s = h, v = 0$, $u = 7\sin\theta$ and $a = -9.8$ to show the given result. Accept negative values used consistently **AG**	2.1	R1	so $h = 2.5\ \sin^2\theta$
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
13(b)	Substitutes any value of θ in the range $0 < \theta \leq 60$ to obtain a height greater than zero	1.1a	M1	$\theta = 60°$ $h = 2.5\sin^2 60$ h =1.9
	Deduces h = 1.9 AWRT 1.9 or $\frac{15}{8}$	2.2a	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
13(c)	States that Nisha is incorrect **and** refers to air resistance or ball modelled as a particle	3.5a	E1	Nisha is incorrect The model ignores air resistance which gets greater as ball gets larger
	Subtotal		**1**	

	Question 13 Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
14(a)	Forms a dimensionally correct non-zero moment. For example $0.012g \times 0.066$ $0.012g \times 66$ $12g \times 66$ $160mg$	3.3	B1	$66 \times 12g = 80mg$
	Obtains a correct single equation where the only unknown is the mass. This might come from correct moments about a point other than the centre with forces correctly resolved in the vertical direction, but these equations must be combined to eliminate the force at the centre. PI by 9.9	1.1a	M1	$m = 9.9$
	Obtains 9.9 Condone $m = 0.0099kg$ OE provided units are included if m is not in grams	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
14(b)	States a valid assumption about the £2 coin with no contradictions or incorrect statements For example Models the coin as uniform, or weight acts through the centre or centre of mass is at the centre Do not accept ‘mass acts’ or ‘centre of mass acts’ at the centre or ‘centre of weight’ or ‘weight is at’	3.5a	E1	The coin is uniform
	Subtotal		**1**	

	Question 14 Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
15	Obtains a correct expression for the area of a triangle above the time axis in terms of a variable for time	1.1b	B1	Let t be the next time when v = 0 Area above $= 2t$
	Obtains a correct expression for the area of the triangle or trapezium below the time axis in terms of a variable for time accept negative area for displacement	1.1b	B1	Area below $= 2(10 - t) + 20$
	Forms equation with a single variable using their expressions for area consistent with area above – area below = $\pm k$ Or Forms equation with a single variable using their expressions for displacement consistent with disp above + disp below = k Where k is one of 3, 7, 13 or 17	3.1b	M1	$2t + 7 = 2(10 - t) + 20$
	Obtains 8.25 seconds OE Condone missing or incorrect units	1.1b	A1	$t = 8.25$ seconds
	Question 15 Total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
16(a)	States or uses the direction of motion is $\begin{bmatrix}3\\-4\end{bmatrix}$ or $\begin{bmatrix}9\\c+1\end{bmatrix}$ Or States or uses the gradient of the direction of motion is $-\frac{4}{3}$ or $\frac{c+1}{9}$	3.1a	M1	$\begin{bmatrix}10\\c\end{bmatrix}-\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}9\\c+1\end{bmatrix}$ $\begin{bmatrix}9\\c+1\end{bmatrix}=k\begin{bmatrix}3\\-4\end{bmatrix}$ $k=3$ $c+1=-12$ $\Rightarrow c=-13$
	Obtains a correct vector equation eg $\begin{bmatrix}10\\c\end{bmatrix}=\begin{bmatrix}1\\-1\end{bmatrix}+k\begin{bmatrix}3\\-4\end{bmatrix}$ OE Or Obtains both gradients or both direction vectors $\begin{bmatrix}3\\-4\end{bmatrix}$, $\begin{bmatrix}9\\c+1\end{bmatrix}$ or Obtains a correct cartesian equation for Q. eg $y+1=-\frac{4}{3}(x-1)$	1.1b	A1	
	Obtains or eliminates parameter in their vector equation Or Equates gradients or the reciprocals $\frac{c+1}{9}=-\frac{4}{3}$ Or substitutes $x=10$ into their cartesian equation	1.1a	M1	
	Shows that $c=-13$ **AG** A correct verification method using the given $c=-13$ scores a maximum of M1A1M0A0	1.1b	A1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
16(b)(i)	Obtains $\begin{pmatrix} -4 \\ 5 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 3 \\ -4 \end{pmatrix} t^2$ metres OE Condone missing units	2.2a	B1	$\mathbf{r} = \begin{bmatrix} -4 \\ 5 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 3 \\ -4 \end{bmatrix} t^2$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
16(b)(ii)	Equates their position vector from **(b)(i)** to one of the two known position vectors given for *Q*. Their position vector **must** be **quadratic** in t for **both** components *Or* Substitutes a known point for *P* into their Cartesian equation for the path of *Q* from part (a) OE Or Substitutes a known point for *Q* into their Cartesian equation for the path of *P* from part (a) OE Or forms Cartesian equations for the path of *P* and the path of *Q* Or Calculates the difference between $(-4\mathbf{i}+5\mathbf{j})$ and $(\mathbf{i}-\mathbf{j})$ or between $(-4\mathbf{i}+5\mathbf{j})$ and $(10\mathbf{i}-13\mathbf{j})$	3.1b	M1	$\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}-4\\5\end{bmatrix}+\frac{1}{2}t^2\begin{bmatrix}3\\-4\end{bmatrix}$ $1 = 1.5t^2 - 4$ $t^2 = \frac{10}{3}$ $5 - 2t^2 = -1$ $t^2 = 3$ Since the t^2 values are not the same no single value of t exists which satisfies both components. Therefore, the paths are not collinear.
	Obtains $t^2 = \frac{10}{3} or 3$ or $t=\sqrt{\frac{10}{3}}=1.82... or \sqrt{3}=1.73...$ Or Shows that $y \neq 5$ for $x=-4$ OE Or Writes the two correct cartesian equations in a comparable form eg $y=-\frac{4}{3}x+\frac{1}{3}$ and $y=-\frac{4}{3}x-\frac{1}{3}$ Or Compares two appropriate direction vectors	1.1b	A1	
	Completes reasoned argument by explaining that there is an inconsistency and deduces that paths are not collinear CSO	2.1	R1	
	Subtotal		**3**	

	Question 16 Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
17	Differentiates with evidence of correct use of product rule. Condone sign errors	3.4	M1	$\mathbf{v} = \dfrac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} =$ $\left(\mathrm{e}^t\cos t - \mathrm{e}^t\sin t\right)\mathbf{i} + \left(\mathrm{e}^t\sin t + \mathrm{e}^t\cos t\right)\mathbf{j}$ $\mathbf{a} = \dfrac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} =$ $\left(\mathrm{e}^t\cos t - \mathrm{e}^t\sin t - \mathrm{e}^t\sin t - \mathrm{e}^t\cos t\right)\mathbf{i} +$ $\left(\mathrm{e}^t\sin t + \mathrm{e}^t\cos t + \mathrm{e}^t\cos t - \mathrm{e}^t\sin t\right)\mathbf{j}$ $= -2\mathrm{e}^t\sin t\mathbf{i} + 2\mathrm{e}^t\cos t\mathbf{j}$ $\lvert\mathbf{a}\rvert = \sqrt{\left(-2\mathrm{e}^t\sin t\right)^2 + \left(2\mathrm{e}^t\cos t\right)^2}$ $= \sqrt{4\mathrm{e}^{2t}\left(\sin^2 t + \cos^2 t\right)}$ $\therefore \lvert\mathbf{a}\rvert = 2\mathrm{e}^t$
	Finds expression for $\mathbf{v}$ or $\dfrac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$ with either **i** or **j** component fully correct	1.1a	M1	
	Finds fully correct expression for $\mathbf{v}$ or $\dfrac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$ $\left(\mathrm{e}^t\cos t - \mathrm{e}^t\sin t\right)\mathbf{i} +$ $\left(\mathrm{e}^t\sin t + \mathrm{e}^t\cos t\right)\mathbf{j}$ Condone poor use of brackets provided fully correct acceleration seen	1.1b	A1	
	Differentiates their $\mathbf{v}$ or $\dfrac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$ with evidence of correct use of product rule to find an expression for $\mathbf{a}$ with at least one component correct. Condone sign errors	3.4	M1	
	Finds correct expression for $\mathbf{a}$ May be unsimplified	1.1b	A1	
	Obtains an expression for the magnitude of their $\mathbf{a}$ provided their $\mathbf{a}$ has non-zero **i** and **j** components	1.1a	M1	
	Completes reasoned argument from a correct $\mathbf{a}$ to show given result. Must see a factor of $\left(\sin^2 t + \cos^2 t\right)$ eg $\sqrt{4\mathrm{e}^{2t}(\sin^2 t + \cos^2 t)}$ AG	2.1	R1	
	Question 17 Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
18(a)	Obtains both $\sin^{-1}\left(\frac{0.6}{0.8}\right)$ **and** $\sin^{-1}\left(\frac{0.6}{1.2}\right)$ OE Accept complementary angles or exact values $\frac{\sqrt{3}}{2}$ **and** $\frac{\sqrt{7}}{4}$	1.1b	B1	Angle for $OA = \sin^{-1}\left(\frac{0.6}{0.8}\right) = 48.59°$ Angle for $OB = \sin^{-1}\left(\frac{0.6}{1.2}\right) = 30°$
	Resolves forces horizontally to form equilibrium equation, one component correct Or Uses a triangle of forces and applies the sine rule	3.3	M1	$T_{OA}\cos A = T_{OB}\cos B$ $T_{OA} = T_{OB}\frac{\cos B}{\cos A}$
	Obtains correct equation with angles substituted	1.1b	A1	$T_{OA} = T_{OB}\frac{\cos 30}{\cos 48.59}$
	Rearranges the correct equation to show that $T_{OA} = k\,T_{OB}$ OE where $1.305 \le k \le 1.325$ Completes argument to conclude that the **tension** in the shorter string is **over** 30% more than the tension in the longer string	2.1	R1	$\therefore T_{OA} = 1.309T_{OB}$ $\therefore T_{OA} > 1.3T_{OB}$ So, the tension in the shorter string is more than 30% greater than the tension in the longer string
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
18(b)	Obtains $T_{OA} = 2g \times$ their ratio	1.1b	B1	
	Resolves forces vertically to form a three-term equilibrium equation, with at least two terms correct Or Uses a triangle of forces and applies the sine rule	3.3	M1	$mg = T_{OA}\sin A + T_{OB}\sin B$ $mg = 2.6g\sin 48.59 + 2g\sin 30$ $m = 3.0$
	Forms fully correct equation of forces in equilibrium This mark can be awarded for $mg = T_{OA}\sin A + T_{OB}\sin B$	1.1b	A1	
	Substitutes their T_{OA} and T_{OB} and correct values for angles into the correct equation **and** obtains AWRT m = 3 Might come from 2.96.. FT their ratio from part (a) provided their m = AWRT 3	3.4	A1F	
	Subtotal		**4**	

	Question 18 Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
19(a)	States $F = \mu R$ seen anywhere PI by use of μR in their 4-term equation of motion or on diagram	3.3	B1	$T - weight - Friction = ma$ $T - 20g\sin 25 - F = m \times 1.2$ $T - 20g\sin 25 - 20g\cos 25\mu = m \times 1.2$ $230 - 196\sin 25 - 196\cos 25\mu = 24$ $\mu = 0.69$
	Resolves the weight parallel to the slope to obtain $m\,g$ sin25 or better	1.1b	B1	
	Resolves perpendicular to the slope to obtain $R = m\,g$ cos25 or better	1.1b	B1	
	Uses F = ma to form a four-term equation with consistent signs. eg $T - weight - Friction = ma$ Condone omission of g in weight and friction component	3.3	M1	
	Substitutes T = 230 and $F = \mu mg\cos 25$ into their four term $F = ma$ equation with consistent signs. Condone 'mga' in $F = ma$ for this mark	1.1a	M1	
	Obtains single correct equation with all numerical values substituted. eg $230 - 196\sin 25 - 196\cos 25\mu = 24$ Scores B1B1B1M1M1A1	1.1b	A1	
	Obtains $\mu = 0.69$ CAO	3.2a	A1	
	Subtotal		**7**	

Q	Marking instructions	AO	Marks	Typical solution
19(b)(i)	Substitutes u = 0, a = 1.2 and t = 3.8 into $s = ut + \frac{1}{2}at^2$ Or uses appropriate constant acceleration equations that forms a complete method to obtain s	1.1a	M1	$s = ut + \frac{1}{2}at^2$
	Obtains AWRT 8.7	1.1b	A1	$s = \frac{1}{2} \times 1.2 \times 3.8^2 = 8.664$
	Obtains 10 – their 8.7 AWRT 1.3 or FT their 8.7 provided it is less than 10 Condone missing units	1.1b	A1F	*OA* = 10 – 8.664 = 1.3 m
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
19(b)(ii)	States the crate has been modelled as a particle OE	3.5b	E1	The crate is a particle
	Subtotal		**1**	

	Question 19 Total		**11**	

	Question Paper Total		**100**	