



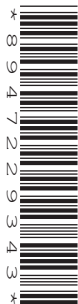
Oxford Cambridge and RSA

Wednesday 13 October 2021 – Afternoon

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Kinematics

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u+v)t$$

$$v^2 = u^2 + 2as$$

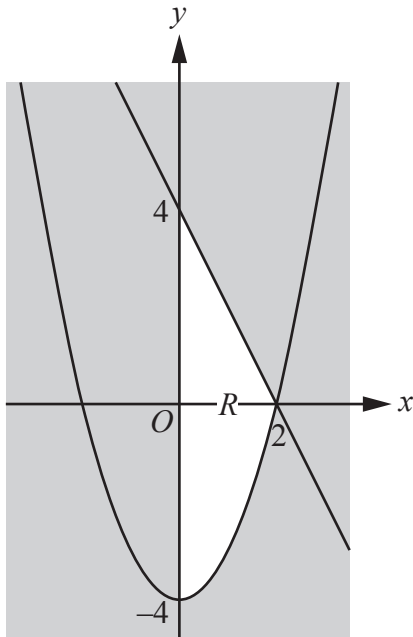
$$s = vt - \frac{1}{2}at^2$$

Section A: Pure Mathematics

Answer **all** the questions.

- 1 Given that $(x-2)$ is a factor of $2x^3 + kx - 4$, find the value of the constant k . [2]

2



The diagram shows the line $y = -2x + 4$ and the curve $y = x^2 - 4$. The region R is the unshaded region together with its boundaries.

Write down the inequalities that define R . [3]

- 3 Sam invested in a shares scheme. The value, £ V , of Sam's shares was reported t months after investment.

- Exactly 6 months after investment, the value of Sam's shares was £2375.
- Exactly 1 year after investment, the value of Sam's shares was £2825.

(a) Using a straight-line model, determine an equation for V in terms of t . [3]

Sam's original investment in the scheme was £1900.

(b) Explain whether or not this fact supports the use of the straight-line model in part (a). [2]

- 4 The quadratic polynomial $2x^2 - 3$ is denoted by $f(x)$.

Use differentiation from first principles to determine the value of $f'(2)$. [5]

- 5 (a) Show that the equation $2 \cos x \tan^2 x = 3(1 + \cos x)$ can be expressed in the form

$$5 \cos^2 x + 3 \cos x - 2 = 0. \quad [3]$$

- (b) In this question you must show detailed reasoning.

Hence solve the equation

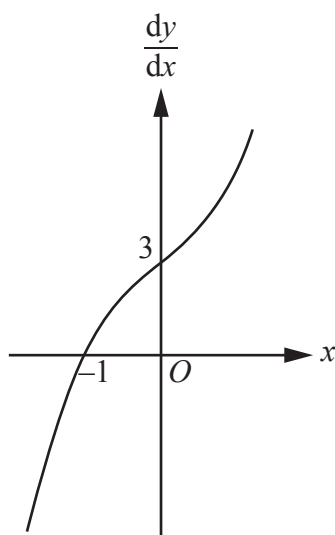
$$2 \cos 3\theta \tan^2 3\theta = 3(1 + \cos 3\theta),$$

giving all values of θ between 0° and 120° , correct to 1 decimal place where appropriate. [6]

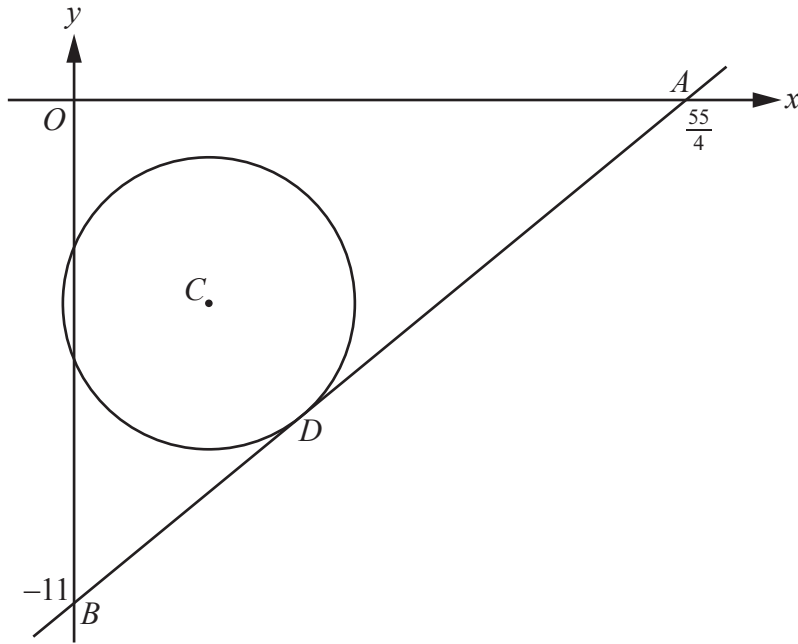
- 6 A curve C has an equation which satisfies $\frac{d^2y}{dx^2} = 3x^2 + 2$, for all values of x .

- (a) It is given that C has a single stationary point. Determine the nature of this stationary point. [1]

The diagram shows the graph of the **gradient function** for C .



- (b) Given that C passes through the point $(-1, \frac{1}{4})$, find the equation of C in the form $y = f(x)$. [5]



The diagram shows the circle with equation $x^2 + y^2 - 6x + 9y + 19 = 0$ and centre C .

(a) Find the following.

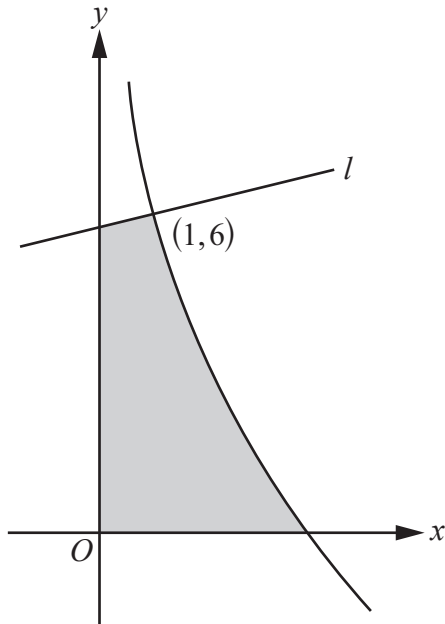
- The coordinates of C .
- The exact radius of the circle.

[3]

The tangent to the circle at D meets the x -axis at the point $A (\frac{55}{4}, 0)$ and the y -axis at the point $B (0, -11)$.

(b) Determine the area of triangle OBD .

[6]



The diagram shows the curve $y = 1 - x + \frac{6}{\sqrt{x}}$ and the line l , which is the normal to the curve at the point $(1, 6)$.

- (a) Determine the equation of l in the form

$$ax + by = c$$

where a , b and c are integers whose values are to be stated.

[5]

- (b) Verify that the curve intersects the x -axis at the point where $x = 4$.

[1]

- (c) **In this question you must show detailed reasoning.**

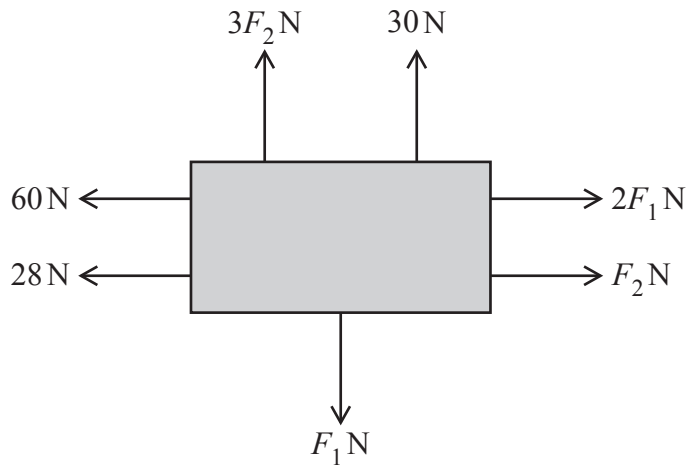
Determine the exact area of the shaded region enclosed between l , the curve, the x -axis and the y -axis.

[5]

Section B: Mechanics

Answer **all** the questions.

9



A body remains at rest when subjected to the horizontal and vertical forces shown in the diagram.

Determine the value of F_1 and the value of F_2 . [3]

- 10** A cyclist starts from rest and moves with constant acceleration along a straight horizontal road. The cyclist reaches a speed of 6 m s^{-1} in 25 seconds. The cyclist then moves with constant acceleration 0.05 m s^{-2} until the speed is 10 m s^{-1} . The cyclist then moves with constant deceleration until coming to rest. The total time for the cyclist's journey is 150 seconds.

(a) Sketch a velocity-time graph to represent the cyclist's motion. [2]

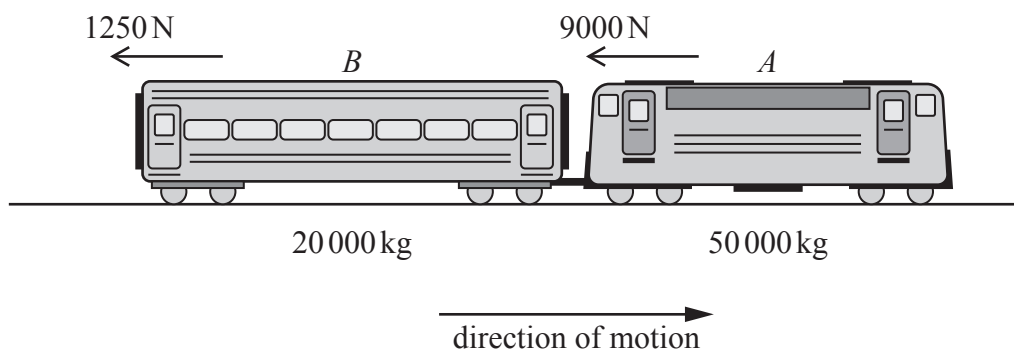
(b) Find the acceleration during the first 25 seconds of the cyclist's motion. [1]

The cyclist takes T seconds to decelerate from 10 m s^{-1} until coming to rest.

(c) Determine the value of T . [2]

(d) Determine the average speed for the cyclist's journey. [3]

11



A train consists of an engine *A* of mass 50 000 kg and a carriage *B* of mass 20 000 kg. The engine and carriage are connected by a rigid coupling. The coupling is modelled as light and horizontal.

The resistances to motion acting on *A* and *B* are 9000 N and 1250 N respectively (see diagram).

The train passes through station *P* with speed 15 m s^{-1} and moves along a straight horizontal track with constant acceleration 0.01 m s^{-2} towards station *Q*. The distance between *P* and *Q* is 12.95 km.

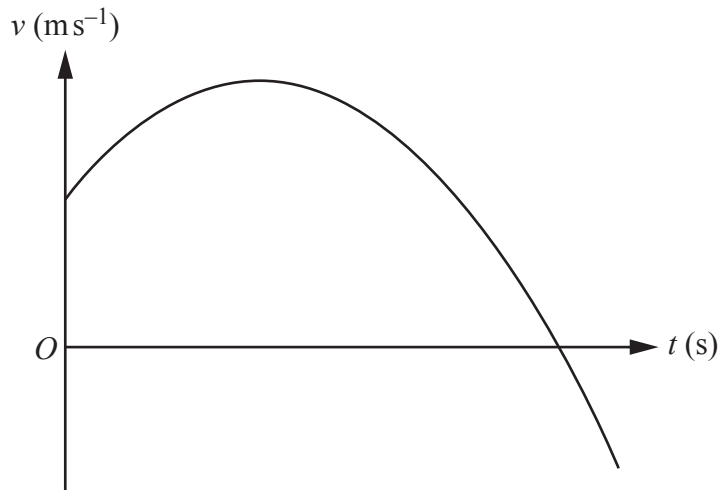
(a) Determine the time, in minutes, to travel between *P* and *Q*. [3]

For the train's motion between *P* and *Q*, determine the following.

(b) The driving force of the engine. [2]

(c) The tension in the coupling between *A* and *B*. [2]

12



A particle P moves in a straight line. At time t seconds, where $t \geq 0$, the velocity of P is $v \text{ m s}^{-1}$. It is given that $v = -3t^2 + 24t + k$, where k is a positive constant.

The diagram shows the velocity-time graph for the motion of P .

P attains its maximum velocity at time T seconds. Given that the distance travelled by P between times $t = 1$ and $t = T$ is 297 m, determine the time when P is instantaneously at rest. [7]

END OF QUESTION PAPER

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