AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

A-level
MATHEMATICS
Paper 1

Wednesday 5 June 2019 Morning Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

Answer **all** questions in the spaces provided.

1 Given that $a > 0$, determine which of these expressions is **not** equivalent to the others.

Circle your answer.

[1 mark]

$-2\log_{10}\left(\frac{1}{a}\right)$ $\quad$ $2\log_{10}(a)$ $\quad$ $\log_{10}(a^2)$ $\quad$ $-4\log_{10}(\sqrt{a})$

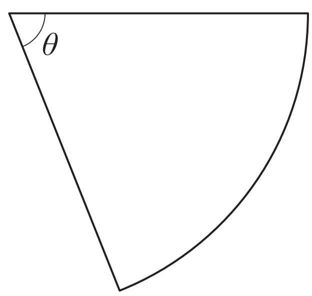
2 Given $y = e^{kx}$, where k is a constant, find $\frac{dy}{dx}$

Circle your answer.

[1 mark]

$\frac{dy}{dx} = e^{kx}$ $\quad$ $\frac{dy}{dx} = ke^{kx}$ $\quad$ $\frac{dy}{dx} = kxe^{kx-1}$ $\quad$ $\frac{dy}{dx} = \frac{e^{kx}}{k}$

3 The diagram below shows a sector of a circle.



The radius of the circle is 4 cm and $\theta = 0.8$ radians.

Find the area of the sector.

Circle your answer.

[1 mark]

1.28 cm^2 $\quad$ 3.2 cm^2 $\quad$ 6.4 cm^2 $\quad$ 12.8 cm^2

4 The point A has coordinates $(-1, a)$ and the point B has coordinates $(3, b)$

The line AB has equation $5x + 4y = 17$

Find the equation of the perpendicular bisector of the points A and B.

[4 marks]

Turn over for the next question

5 An arithmetic sequence has first term a and common difference d.

The sum of the first 16 terms of the sequence is 260

5 (a) Show that $4a + 30d = 65$

[2 marks]

5 (b) Given that the sum of the first 60 terms is 315, find the sum of the first 41 terms.

[3 marks]

5 (c) S_n is the sum of the first n terms of the sequence.

Explain why the value you found in part **(b)** is the maximum value of S_n

[2 marks]

Turn over for the next question

6 The function f is defined by

$$f(x) = \frac{1}{2}(x^2 + 1),\ x \geq 0$$

6 (a) Find the range of f.

[1 mark]

6 (b) (i) Find $f^{-1}(x)$

[3 marks]

6 (b) (ii) State the range of $f^{-1}(x)$

[1 mark]

6 (c) State the transformation which maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}^{-1}(x)$

[1 mark]

6 (d) Find the coordinates of the point of intersection of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$

[2 marks]

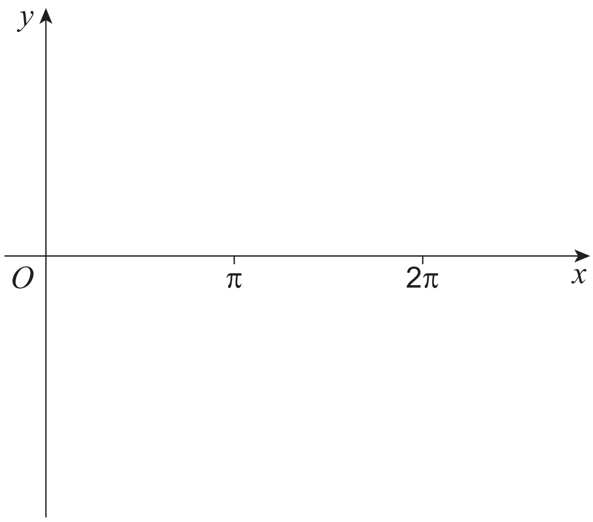
Turn over for the next question

7 (a) By sketching the graphs of $y = \frac{1}{x}$ and $y = \sec 2x$ on the axes below, show that the equation

$$\frac{1}{x} = \sec 2x$$

has exactly one solution for $x > 0$

[3 marks]



7 (b) By considering a suitable change of sign, show that the solution to the equation lies between 0.4 and 0.6

[2 marks]

7 (c) Show that the equation can be rearranged to give

$$x = \frac{1}{2}\cos^{-1} x$$

[2 marks]

7 (d) (i) Use the iterative formula

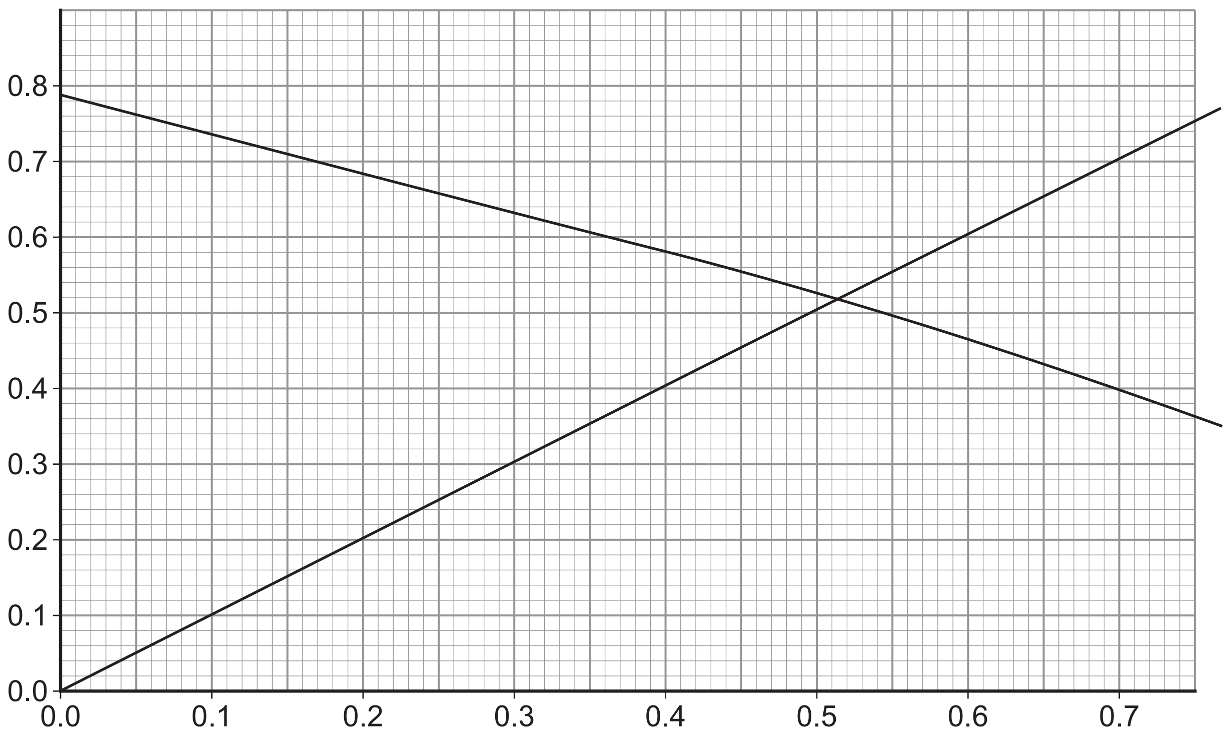
$$x_{n+1} = \frac{1}{2}\cos^{-1} x_n$$

with $x_1 = 0.4$, to find x_2, x_3 and x_4, giving your answers to four decimal places.

[2 marks]

7 (d) (ii) On the graph below, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2, x_3 and x_4.

[2 marks]



8 $\mathrm{P}(n) = \sum_{k=0}^{n} k^3 - \sum_{k=0}^{n-1} k^3$ where n is a positive integer.

8 (a) Find $\mathrm{P}(3)$ and $\mathrm{P}(10)$

[2 marks]

8 (b) Solve the equation $\mathrm{P}(n) = 1.25 \times 10^8$

[2 marks]

9 Prove that the sum of a rational number and an irrational number is always irrational.
[5 marks]

Turn over for the next question

10 The volume of a spherical bubble is increasing at a constant rate.

Show that the rate of increase of the radius, r, of the bubble is inversely proportional to r^2

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

[4 marks]

Turn over for the next question

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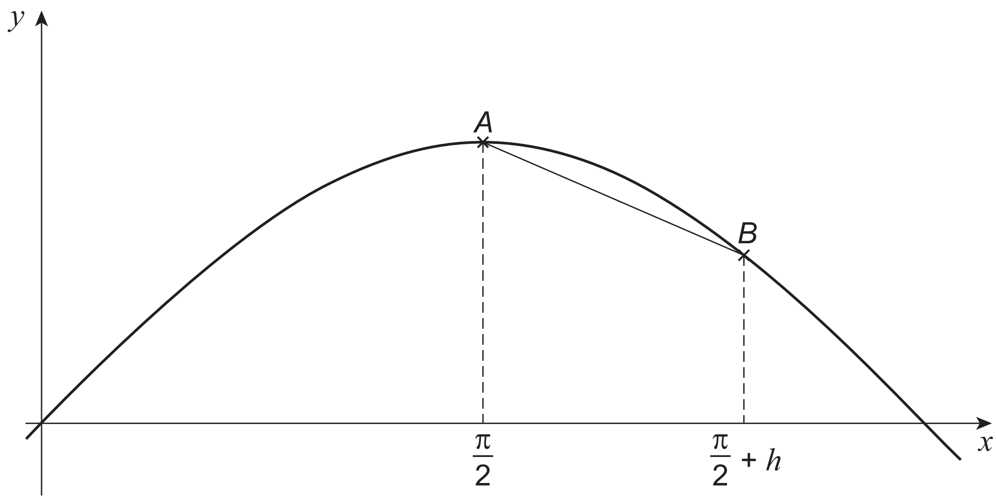
ANSWER IN THE SPACES PROVIDED

11 Jodie is attempting to use differentiation from first principles to prove that the gradient of $y = \sin x$ is zero when $x = \frac{\pi}{2}$

Jodie's teacher tells her that she has made mistakes starting in Step 4 of her working. Her working is shown below.



Step 1 Gradient of chord $AB = \dfrac{\sin\left(\frac{\pi}{2} + h\right) - \sin\left(\frac{\pi}{2}\right)}{h}$

Step 2 $= \dfrac{\sin\left(\frac{\pi}{2}\right)\cos(h) + \cos\left(\frac{\pi}{2}\right)\sin(h) - \sin\left(\frac{\pi}{2}\right)}{h}$

Step 3 $= \sin\left(\frac{\pi}{2}\right)\left(\dfrac{\cos(h) - 1}{h}\right) + \cos\left(\frac{\pi}{2}\right)\dfrac{\sin(h)}{h}$

Step 4 For gradient of curve at A,

let $h = 0$ then

$\dfrac{\cos(h) - 1}{h} = 0$ and $\dfrac{\sin(h)}{h} = 0$

Step 5 Hence the gradient of the curve at A is given by

$\sin\left(\frac{\pi}{2}\right) \times 0 + \cos\left(\frac{\pi}{2}\right) \times 0 = 0$

Complete Steps 4 and 5 of Jodie's working below, to correct her proof.

[4 marks]

Step 4 For gradient of curve at *A*,

____

____

____

Step 5 Hence the gradient of the curve at *A* is given by

____

____

____

Turn over for the next question

12 (a) Show that the equation

$$2\cot^2 x + 2\operatorname{cosec}^2 x = 1 + 4\operatorname{cosec} x$$

can be written in the form

$$a\operatorname{cosec}^2 x + b\operatorname{cosec} x + c = 0$$

[2 marks]

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12 (b) Hence, given x is obtuse and

$$2\cot^2 x + 2\,\text{cosec}^2 x = 1 + 4\,\text{cosec}\, x$$

find the exact value of $\tan x$

Fully justify your answer.

[5 marks]

Turn over for the next question

13 A curve, C, has equation

$$y = \frac{e^{3x-5}}{x^2}$$

Show that C has exactly one stationary point.

Fully justify your answer.

[7 marks]

Turn over for the next question

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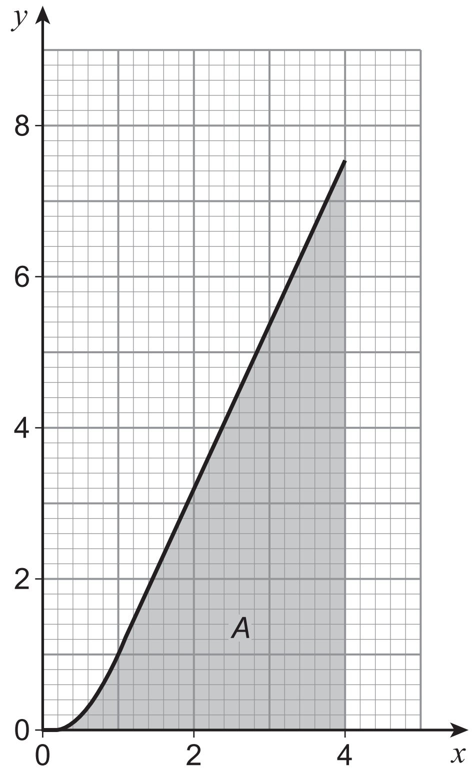
14 The graph of $y = \dfrac{2x^3}{x^2 + 1}$ is shown for $0 \le x \le 4$



Caroline is attempting to approximate the shaded area, *A*, under the curve using the trapezium rule by splitting the area into n trapezia.

14 (a) When $n = 4$

14 (a) (i) State the number of ordinates that Caroline uses.

[1 mark]

14 (a) (ii) Calculate the area that Caroline should obtain using this method.

Give your answer correct to two decimal places.

[3 marks]

14 (b) Show that the exact area of *A* is

$$16 - \ln 17$$

Fully justify your answer.

[5 marks]

Question 14 continues on the next page

Turn over ▶

14 (c) Explain what would happen to Caroline's answer to part **(a)(ii)** as $n \to \infty$

[1 mark]

Turn over for the next question

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15 (a) At time t hours **after a high tide**, the height, h metres, of the tide and the velocity, v knots, of the tidal flow can be modelled using the parametric equations

$$v = 4 - \left(\frac{2t}{3} - 2\right)^2$$

$$h = 3 - 2\sqrt[3]{t - 3}$$

High tides and low tides occur alternately when the velocity of the tidal flow is zero.

A high tide occurs at 2 am.

15 (a) (i) Use the model to find the height of this high tide.

[1 mark]

15 (a) (ii) Find the time of the first **low** tide after 2 am.

[3 marks]

15 (a) (iii) Find the height of this low tide.

[1 mark]

15 (b) Use the model to find the height of the tide when it is flowing with maximum velocity.

[3 marks]

15 (c) Comment on the validity of the model.

[2 marks]

Turn over for the next question

16 (a) $y = e^{-x}(\sin x + \cos x)$

Find $\frac{dy}{dx}$

Simplify your answer.

[3 marks]

16 (b) Hence, show that

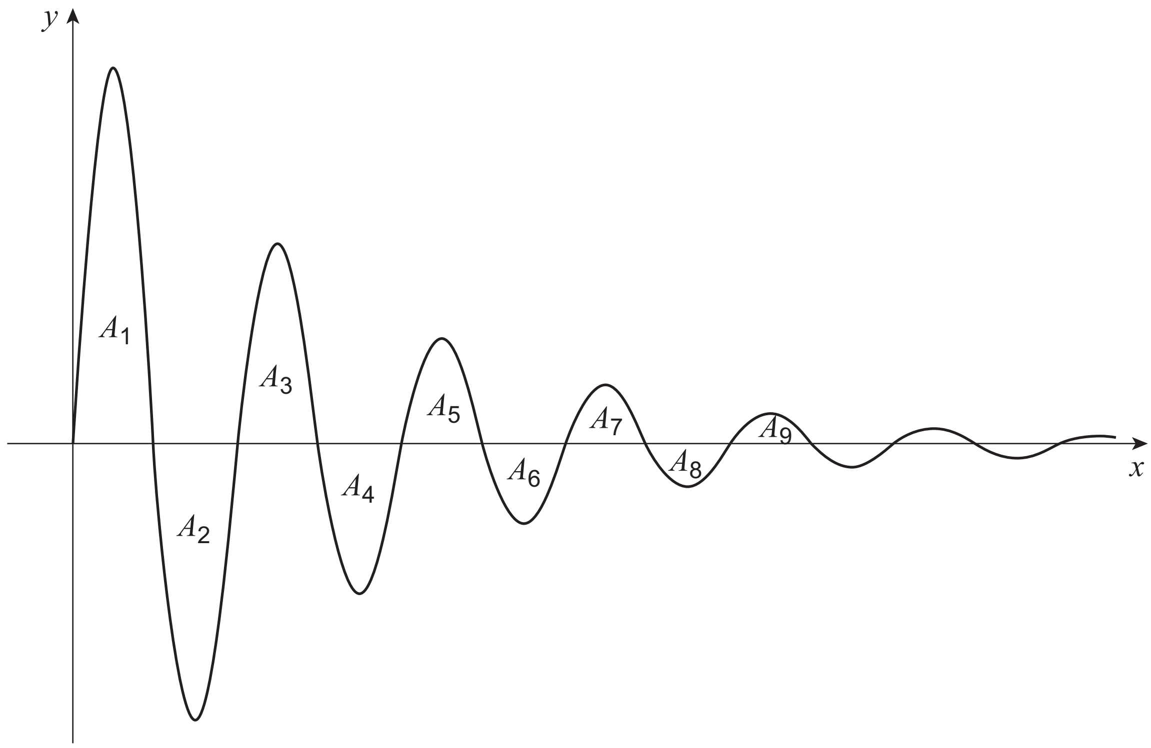
$$\int e^{-x} \sin x \, dx = a e^{-x}(\sin x + \cos x) + c$$

where a is a rational number.

[2 marks]

16 (c) A sketch of the graph of $y = e^{-x}\sin x$ for $x \geq 0$ is shown below.

The areas of the finite regions bounded by the curve and the x-axis are denoted by $A_1, A_2, ..., A_n, ...$



16 (c) (i) Find the exact value of the area A_1

[3 marks]

Question 16 continues on the next page

Turn over ▶

16 (c) (ii) Show that

$$\frac{A_2}{A_1} = \mathrm{e}^{-\pi}$$

[4 marks]

16 (c) (iii) Given that

$$\frac{A_{n+1}}{A_n} = e^{-\pi}$$

show that the exact value of the total area enclosed between the curve and the x-axis is

$$\frac{1 + e^{\pi}}{2(e^{\pi} - 1)}$$

[4 marks]

END OF QUESTIONS

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