Mark Scheme (Result)

October 2020

Pearson Edexcel GCE
In AS Level Mathematics
8MA0 Paper 1 Pure Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.
6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to $x=\ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to $x=\ldots$

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

<u>Use of a formula</u>

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

<u>Exact answers</u>

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1	Attempts to differentiate $x^n \to x^{n-1}$ seen once	M1	1.1b
	$y = 2x^3 - 4x + 5 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 - 4$	A1	1.1b
	For substituting $x = 2$ into their $\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 - 4$	dM1	1.1b
	For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x-2)$	ddM1	1.1b
	$y = 20x - 27$	A1	1.1b
		(5)	
			(5 marks)

Notes

M1: Attempts to differentiate $x^n \to x^{n-1}$ seen once. Score for $x^3 \to x^2$ or $\pm 4x \to 4$ or $+5 \to 0$

A1: $\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right) 6x^2 - 4$ which may be unsimplified $6x^2 - 4 + C$ is A0

dM1: Substitutes $x = 2$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$. The first M must have been awarded.

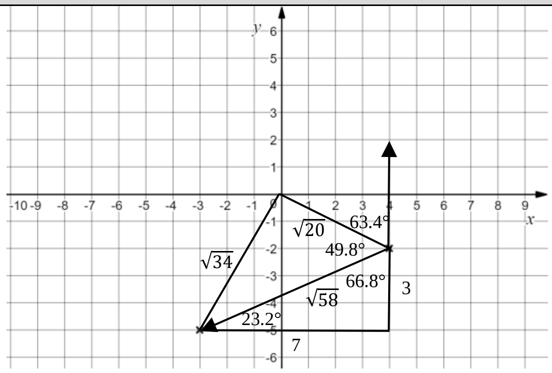
Score for sight of embedded values, or sight of "$\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$ is" or a correct follow through.

Note that 20 on its own is not enough as this can be done on a calculator.

ddM1: For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x-2)$
It is dependent upon both previous M's.

If the form $y = mx + c$ is used they must proceed as far as $c = ...$

A1: Completely correct $y = 20x - 27$ (and in this form)

Question	Scheme	Marks	AOs
2(a)			
	Attempts to find an "allowable" angle Eg $\tan\theta = \frac{7}{3}$	M1	1.1b
	A full attempt to find the bearing Eg $180° + "67°"$	dM1	3.1b
	Bearing = awrt $246.8°$	A1	1.1b
		(3)	
(b)	Attempts to find the distance travelled = $\sqrt{(4--3)^2 + (-2+5)^2} = (\sqrt{58})$	M1	1.1b
	Attempts to find the speed = $\frac{\sqrt{58}}{2.75}$	dM1	3.1b
	= awrt 2.77 km h^{-1}	A1	1.1b
		(3)	
			(6 marks)

Notes: Score these two parts together.

(a)

M1: Attempts an allowable angle. (Either the "66.8" , "23.2" or ("49.8" and "63.4"))

$\tan\theta = \pm\frac{7}{3}$, $\tan\theta = \pm\frac{3}{7}$, $\tan\theta = \pm\frac{-2--5}{4--3}$ etc

There must be an attempt to subtract the coordinates (seen or applied at least once)

If part (b) is attempted first, look for example for $\sin\theta = \pm\frac{7}{"\sqrt{58}"}$, $\cos\theta = \pm\frac{7}{"\sqrt{58}"}$, etc

They may use the cosine rule and trigonometry to find the two angles in the scheme. See above. Eg award for $\cos\theta = \frac{"58"+"20"-"34"}{2\times"\sqrt{58}"\times"\sqrt{20}"}$ **and** $\tan\theta = \pm\frac{4}{2}$ or equivalent.

dM1: A full attempt to find the bearing. $180° + \arctan\frac{7}{3}$, $270° - \arctan\frac{3}{7}$, $360° - "49.8°" - "63.4°"$. It is dependent on the previous method mark.

A1: Bearing = awrt $246.8°$ oe. Allow S $66.8°$ W

(b)

M1: Attempts to find the distance travelled. Allow for $d^2 = (4 - -3)^2 + (-2 + 5)^2$

You may see this on a diagram and allow if they attempt to find the magnitude from their "resultant vector" found in part (a).

dM1: Attempts to find the speed. There must have been an attempt to find the distance using the coordinates and then divide it by 2.75. Alternatively they could find the speed in km min^{-1} and then multiply by 60

A1: awrt 2.77 km h^{-1}

Question	Scheme	Marks	AOs
3 (i)	$x\sqrt{2}-\sqrt{18}=x\Rightarrow x\left(\sqrt{2}-1\right)=\sqrt{18}\Rightarrow x=\dfrac{\sqrt{18}}{\sqrt{2}-1}$	M1	1.1b
	$\Rightarrow x=\dfrac{\sqrt{18}}{\sqrt{2}-1}\times\dfrac{\sqrt{2}+1}{\sqrt{2}+1}$	dM1	3.1a
	$x=\dfrac{\sqrt{18}\left(\sqrt{2}+1\right)}{1}=6+3\sqrt{2}$	A1	1.1b
		(3)	
(ii)	$4^{3x-2}=\dfrac{1}{2\sqrt{2}}\Rightarrow 2^{6x-4}=2^{-\frac{3}{2}}$	M1	2.5
	$6x-4=-\dfrac{3}{2}\Rightarrow x=...$	dM1	1.1b
	$x=\dfrac{5}{12}$	A1	1.1b
		(3)	
			(6 marks)

Notes

(i)

M1: Combines the terms in x, factorises and divides to find x. Condone sign slips and ignore any attempts to simplify $\sqrt{18}$

Alternatively squares both sides $x\sqrt{2}-\sqrt{18}=x\Rightarrow 2x^2-12x+18=x^2$

dM1: Scored for a complete method to find x. In the main scheme it is for making x the subject and then multiplying both numerator and denominator by $\sqrt{2}+1$

In the alternative it is for squaring both sides to produce a 3TQ and then factorising their quadratic equation to find x. (usual rules apply for solving quadratics)

A1: $x=6+3\sqrt{2}$ only following a correct intermediate line. Allow $\dfrac{6+3\sqrt{2}}{1}$ as an intermediate line.

In the alternative method the $6-3\sqrt{2}$ must be discarded.

(ii)

M1: Uses correct mathematical notation and attempts to set both sides as powers of 2 or 4.

Eg $2^{ax+b}=2^c$ or $4^{dx+e}=4^f$ is sufficient for this mark.

Alternatively uses logs (base 2 or 4) to get a linear equation in x.

$4^{3x-2}=\dfrac{1}{2\sqrt{2}}\Rightarrow \log_2 4^{3x-2}=\log_2\dfrac{1}{2\sqrt{2}}\Rightarrow 2\left(3x-2\right)=\log_2\dfrac{1}{2\sqrt{2}}$.

Or $4^{3x-2}=\dfrac{1}{2\sqrt{2}}\Rightarrow 3x-2=\log_4\dfrac{1}{2\sqrt{2}}$

Or $4^{3x-2}=\dfrac{1}{2\sqrt{2}}\Rightarrow 4^{3x}=4\sqrt{2}\Rightarrow 3x=\log_4 4\sqrt{2}$

dM1: Scored for a complete method to find x.
Scored for setting the indices of 2 or 4 equal to each other and then solving to find x.
There must be an attempt on both sides.

You can condone slips for this mark Eg bracketing errors $4^{3x-2} = 2^{2\times 3x-2}$ or $\frac{1}{2\sqrt{2}} = 2^{-1+\frac{1}{2}}$

In the alternative method candidates cannot just write down the answer to the rhs.

So expect some justification. E.g. $\log_2 \frac{1}{2\sqrt{2}} = \log_2 2^{-\frac{3}{2}} = -\frac{3}{2}$

or $\log_4 \frac{1}{2\sqrt{2}} = \log_4 2^{-\frac{3}{2}} = -\frac{3}{2}\times\frac{1}{2}$ condoning slips as per main scheme

or $3x = \log_4 4\sqrt{2} \Rightarrow 3x = 1+\frac{1}{4}$

A1: $x = \frac{5}{12}$ with correct intermediate work

Question	Scheme	Marks	AOs
4 (a)	Attempts $A = mn + c$ with either $(0,190)$ or $(8,169)$ Or attempts gradient eg $m = \pm\frac{190-169}{8}(= -2.625)$	M1	3.3
	Full method to find a linear equation linking A with n E.g. Solves $190 = 0n + c$ and $169 = 8n + c$ simultaneously	dM1	3.1b
	$A = -2.625n + 190$	A1	1.1b
		(3)	
(b)	Attempts $A = -2.625 \times 19 + 190 = ...$	M1	3.4
	$A = 140.125$ g km^{-1}	A1	1.1b
	It is predicting a much higher value and so is not suitable	B1ft	3.5a
		(3)	
			(6 marks)

Notes

(a)

M1: Attempts $A = mn + c$ with either $(0,190)$ or $(8,169)$ considered.
Eg Accept sight of $190 = 0n + c$ or $169 = 8m + c$ or $A - 169 = m(n-8)$ or $A = 190 + mn$ where m could be a value.
Also accept an attempt to find the gradient $\pm\frac{190-169}{8}$ or sight of ± 2.625 or $\pm\frac{21}{8}$ oe

dM1: A full method to find both constants of a linear equation
Method 1: Solves $190 = 0n + c$ and $169 = 8n + c$ simultaneously
Method 2: Uses gradient and a point Eg $m = \pm\frac{190-169}{8}(= -2.625)$ and $c = 190$
Condone different variables for this mark. Eg. y in terms of x.

A1: $A = -2.625n + 190$ or $A = -\frac{21}{8}n + 190$ oe

(b)

M1: Attempts to substitute "n" $= 19$ into their linear model to find A. They may call it $x = 19$
Alternatively substitutes $A = 120$ into their linear model to find n.

A1: $A = 140.125$ from $n = 19$ Allow $A = 140$
or $n = 26/27$ following $A = 120$

B1ft: Requires a correct calculation for their model, a correct statement and a conclusion
E.g For correct (a) $A = 140$ is (much) higher than 120 so the model is not suitable/appropriate.
Follow through on a correct statement for their equation. As a guide allow anything within [114,126] to be regarded as suitable. Anything less than 108 or more than 132 should be justified as unsuitable.

Note B0 Recorded value is not the same as/does not equal/does not match the value predicted

Question	Scheme	Marks	AOs
5 (a)	States $\frac{\sin\theta}{12}=\frac{\sin 27}{7}$	M1	1.1b
	Finds $\theta=$ awrt $51°$ or awrt $129°$	A1	1.1b
	$=$ awrt $128.9°$	A1	1.1b
		(3)	
(b)	Attempts to find part or all of *AD* Eg $AD^2=7^2+12^2-2\times12\times7\cos101.9=(AD=15.09)$ Eg $(AC)^2=7^2+12^2-2\times12\times7\cos(180-\text{"}128.9\text{"}-27)$ Eg $12\cos27$ or $7\cos\text{"}51\text{"}$	M1	1.1b
	Full method for the total length = $12+7+7+\text{"}15.09\text{"}=$	dM1	3.1a
	= 42 m	A1	3.2a
		(3)	
			(6 marks)

Notes

(a)

M1: States $\frac{\sin\theta}{12}=\frac{\sin 27}{7}$ oe with the sides and angles in the correct positions

Alternatively they may use the cosine rule on $\angle ACB$ and then solve the subsequent quadratic to find *AC* and then use the cosine rule again

A1: awrt $51°$ or awrt $129°$

A1: Awrt $128.9°$ only (must be seen in part a))

(b)

M1: Attempts a "correct" method of finding either *AD* or a part of *AD* eg (*AC* or *CD* or forming a perpendicular to split the triangle into two right angled triangles to find *AX* or *XD*) which may be seen in (a).
You should condone incorrect labelling of the side.
Look for attempted application of the cosine rule

$$(AD)^2=7^2+12^2-2\times12\times7\cos(\text{"}128.9\text{"}-27)$$

$$\text{or } (AC)^2=7^2+12^2-2\times12\times7\cos(180-\text{"}128.9\text{"}-27)$$

Or an attempted application of the sine rule $\frac{(AD)}{\sin(\text{"}128.9\text{"}-27)}=\frac{7}{\sin27}$

Or $\frac{(AC)}{\sin(180-\text{"}128.9\text{"}-27)}=\frac{7}{\sin27}$

Or an attempt using trigonometry on a right-angled triangle to find part of AD

$12\cos 27$ or $7\cos"51"$

This method can be implied by sight of awrt 15.1 or awrt 6.3 or awrt 8.8 or awrt 10.7 or awrt 4.4

dM1: A complete method of finding the TOTAL length.
There must have been an attempt to use the correct combination of angles and sides.
Expect to see $7+7+12+"AD"$ found using a correct method.
This is scored by either $7+7+12+"AD"$ if $\angle ACB = 128.9°$ in a) or
$7+7+12+\text{awrt}15.1$ by candidates who may have assumed $\angle ACB = 51.1°$ in a)

A1: Rounds correct 41.09 m (or correct expression) up to 42 m to find steel **bought**

Candidates who assumed $\angle ACB = 51.1°$ (acute) in (a):
Full marks can still be achieved as candidates may have restarted in (b) or not used the acute angle in their calculation which is often unclear. We are condoning any reference to $AC = 15.1$ so ignore any labelling of the lengths they are finding.

Diagram of the correct triangle with lengths and angles:

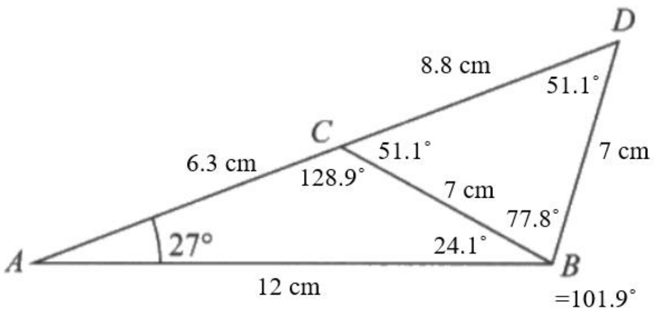
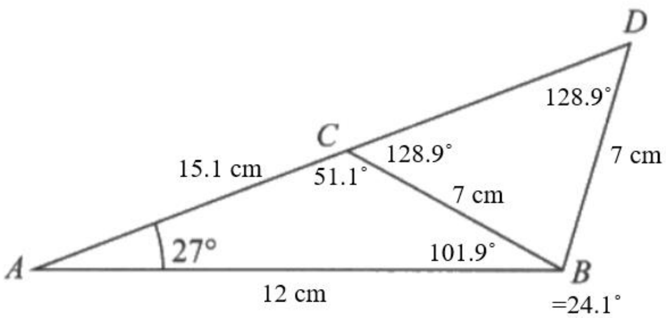


Diagram using the incorrect acute angle:



Question	Scheme	Marks	AOs
6 (a)	$(1+kx)^{10}=1+\binom{10}{1}(kx)^1+\binom{10}{2}(kx)^2+\binom{10}{3}(kx)^3...$	M1 A1	1.1b 1.1b
	$=1+10kx+45k^2x^2+120k^3x^3...$	A1	1.1b
		(3)	
(b)	Sets $120k^3=3\times10k$	B1	1.2
	$4k^2=1\Rightarrow k=...$	M1	1.1b
	$k=\pm\frac{1}{2}$	A1	1.1b
		(3)	
			(6 marks)

(a)

M1: An attempt at the binomial expansion. This may be awarded for either the second or third term or fourth term. The coefficients may be of the form ${}^{10}C_1$, $\binom{10}{2}$ etc or eg $\frac{10\times9\times8}{3!}$

A1: A correct unsimplified binomial expansion. The coefficients must be numerical so cannot be of the form ${}^{10}C_1$, $\binom{10}{2}$. Coefficients of the form $\frac{10\times9\times8}{3!}$ are acceptable for this mark.

The bracketing must be correct on $(kx)^2$ but allow recovery

A1: $1+10kx+45k^2x^2+120k^3x^3...$ or $1+10(kx)+45(kx)^2+120(kx)^3...$
Allow if written as a list.

(b)

B1: Sets their $120k^3=3\times$ their $10k$ (Seen or implied)
For candidates who haven't cubed allow $120k=3\times$ their $10k$
If they write $120k^3x^3=3\times$ their $10kx$ only allow recovery of this mark if x disappears afterwards.

M1: Solves a cubic of the form $Ak^3=Bk$ by factorising out/cancelling the k and proceeding correctly to at least one value for k. Usually $k=\sqrt{\frac{B}{A}}$

A1: $k=\pm\frac{1}{2}$ o.e ignoring any reference to 0

Question	Scheme	Marks	AOs
7 (a)	$x^n \to x^{n+1}$	M1	1.1b
	$\int \left(\frac{5}{2\sqrt{x}}+3\right)\mathrm{d}x = 5\sqrt{x}+3x$	A1	1.1b
	$\left[5\sqrt{x}+3x\right]_1^k = 4 \Rightarrow 5\sqrt{k}+3k-8=4$	dM1	1.1b
	$3k+5\sqrt{k}-12=0$ *	A1*	2.1
		(4)	
(b)	$3k+5\sqrt{k}-12=0 \Rightarrow \left(3\sqrt{k}-4\right)\left(\sqrt{k}+3\right)=0$	M1	3.1a
	$\sqrt{k}=\frac{4}{3},(-3)$	A1	1.1b
	$\sqrt{k}=\ldots \Rightarrow k=\ldots$ oe	dM1	1.1b
	$k=\frac{16}{9}, \cancel{9}$	A1	2.3
		(4)	
			(8 marks)

Notes

(a)

M1: For $x^n \to x^{n+1}$ on correct indices. This can be implied by the sight of either $x^{\frac{1}{2}}$ or x

A1: $5\sqrt{x}+3x$ or $5x^{\frac{1}{2}}+3x$ but may be unsimplified. Also allow with + *c* and condone any spurious notation.

dM1: Uses both limits, subtracts, and sets equal to 4. They cannot proceed to the given answer without a line of working showing this.

A1*: Fully correct proof with no errors (bracketing or otherwise) leading to given answer.

(b)

M1: For a correct method of solving. This could be as the scheme, treating as a quadratic in $\sqrt{k}$ and using allowable method to solve including factorisation, formula etc.

Allow values for $\sqrt{k}$ to be just written down, e.g. allow $\sqrt{k}=\pm\frac{4}{3},(\pm 3)$

Alternatively score for rearranging to $5\sqrt{k}=12-3k$ and then squaring to get $\ldots k=(12-3k)^2$

A1: $\sqrt{k}=\frac{4}{3},(-3)$

Or in the alt method it is for reaching a correct 3TQ equation $9k^2-97k+144=0$

dM1: For solving to find at least one value for *k*. It is dependent upon the first M mark.
In the main method it is scored for squaring their value(s) of $\sqrt{k}$
In the alternative scored for solving their 3TQ by an appropriate method

A1: Full and rigorous method leading to $k=\frac{16}{9}$ only. The 9 must be rejected.

Question	Scheme		Marks	AOs
8 (a)	Temperature = $83°C$		B1	3.4
			(1)	
(b)	$18+65e^{-\frac{t}{8}}=35 \Rightarrow 65e^{-\frac{t}{8}}=17$		M1	1.1b
	$t=-8\ln\left(\frac{17}{65}\right)$	$\ln 65-\frac{t}{8}=\ln 17 \Rightarrow t=...$	dM1	1.1b
	$t=10.7$		A1	1.1b
			(3)	
(c)	States a suitable reason • As $t \to \infty, \theta \to 18$ from above. • The minimum temperature is $18°C$		B1	2.4
			(1)	
(d)	$A+B=94$ or $A+Be^{-1}=50$		M1	3.4
	$A+B=94$ and $A+Be^{-1}=50$		A1	1.1b
	Full method to find at least a value for A		dM1	2.1
	Deduces $\mu=\frac{50e-94}{e-1}$ or accept $\mu=$ awrt 24.4		A1	2.2a
			(4)	
			(9 marks)	

Notes

(a)

B1: Uses the model to state that the temperature $=83°C$ Requires units as well

(b)

M1: Uses the information and proceeds to $Pe^{\pm\frac{t}{8}}=Q$ condoning slips

dM1: A full method using correct log laws and a knowledge that e^x and $\ln x$ are inverse functions. This cannot be scored from unsolvable equations, e.g $P>0, Q<0$. Condone one error in their solution.

A1: $t=$ awrt 10.7

(c)

B1: States a suitable reason with minimal conclusion

- As $t \to \infty, \theta \to 18$ from above.
- The minimum temperature is $18°C$ (so it cannot drop to $15°C$)
- Substitutes $\theta=15$ (or a value between 15 and 18) into $18+65e^{-\frac{t}{8}}=15$ (may be impied by $15-18=-3$ or similar) and makes a statement that $e^{-\frac{t}{8}}$ cannot be less than zero or else that $\ln(-ve)$ is undefined and hence $\theta \neq 15$. All calculations must be correct
- (According to the model) the room temperature is $18°C$ (so cannot fall below this)

(d)

M1: Attempts to use $(0,94)$ or $(8,50)$ in order to form at least one equation in A and B
Allow this to be scored with an equation containing e^0

A1: Correct equations $A+B=94$ and $A+Be^{-1}=50$ or equivalent. $e^0=1$ must have been processed. Condone $A+B=94$ and $A+0.37B=50$ where $e^{-1}=$ awrt 0.37

dM1: Dependent upon having two equations in A and B formed from the information given. It is a full and correct method leading to a value of A. Allow this to be solved from a calculator.
Note $B=69.6..$ or $\frac{44}{1-e^{-1}} \Rightarrow A=94-"B"$

A1: Deduces that $\mu=\frac{50e-94}{e-1}$ or accept $\mu=$ awrt 24.4. Condone $y=\ldots$

Question	Scheme	Marks	AOs
9 (a)	$(-180°, -3)$	B1	1.1b
		(1)	
(b)	(i) $(-720°, -3)$	B1ft	2.2a
	(ii) $(-144°, -3)$	B1 ft	2.2a
		(2)	
(c)	Attempts to use both $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\sin^2\theta + \cos^2\theta = 1$ and solves a quadratic equation in $\sin\theta$ to find at least one value of θ	M1	3.1a
	$3\cos\theta = 8\tan\theta \Rightarrow 3\cos^2\theta = 8\sin\theta$	B1	1.1b
	$3\sin^2\theta + 8\sin\theta - 3 = 0$ $(3\sin\theta - 1)(\sin\theta + 3) = 0$	M1	1.1b
	$\sin\theta = \frac{1}{3}$	A1	2.2a
	awrt $520.5°$ only	A1	2.1
		(5)	
			(8 marks)

(a)

B1: Deduces that $P(-180°, -3)$ or $c = -180^{(o)}, d = -3$

(b)(i)

B1ft: Deduces that $P'(-720°, -3)$ Follow through on their $(c, d) \to (4c, d)$ where d is negative

(b)(ii)

B1ft: Deduces that $P'(-144°, -3)$ Follow through on their $(c, d) \to (c + 36°, d)$ where d is negative

(c)

M1: An overall problem solving mark, condoning slips, for an attempt to

- use $\tan\theta = \frac{\sin\theta}{\cos\theta}$,
- use $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$
- find at least one value of θ from a quadratic equation in $\sin\theta$

B1: Uses the correct identity and multiplies across to give $3\cos\theta = 8\tan\theta \Rightarrow 3\cos^2\theta = 8\sin\theta$ oe

M1: Uses the correct identity $\sin^2\theta + \cos^2\theta = 1$ to form a 3TQ in $\sin\theta$ which they attempt to solve using an appropriate method. It is OK to use a calculator to solve this

A1: $\sin\theta = \frac{1}{3}$ Accept sight of $\frac{1}{3}$. Ignore any reference to the other root even if it is "used"

A1: Full method with all identities correct leading to the answer of awrt $520.5°$ and no other values.

Question	Scheme	Marks	AOs
10 (a)	$g(5)=2\times5^3+5^2-41\times5-70=...$	M1	1.1a
	$g(5)=0\Rightarrow(x-5)$ is a factor, hence $g(x)$ is divisible by $(x-5)$.	A1	2.4
		(2)	
(b)	$2x^3+x^2-41x-70=(x-5)(2x^2...x\pm14)$	M1	1.1b
	$=(x-5)(2x^2+11x+14)$	A1	1.1b
	Attempts to factorise quadratic factor	dM1	1.1b
	$(g(x))=(x-5)(2x+7)(x+2)$	A1	1.1b
		(4)	
(c)	$\int 2x^3+x^2-41x-70\,dx=\frac{1}{2}x^4+\frac{1}{3}x^3-\frac{41}{2}x^2-70x$	M1 A1	1.1b 1.1b
	Deduces the need to use $\int_{-2}^{5} g(x)dx$ $-\frac{1525}{3}-\frac{190}{3}$	M1	2.2a
	Area $=571\frac{2}{3}$	A1	2.1
		(4)	
			(10 marks)

Notes

(a)

M1: Attempts to calculate $g(5)$ Attempted division by $(x-5)$ is M0

Look for evidence of embedded values or two correct terms of $g(5)=250+25-205-70=...$

A1: Correct calculation, reason and conclusion. It must follow M1. Accept, for example,

$g(5)=0\Rightarrow(x-5)$ is a factor, hence divisible by $(x-5)$

$g(5)=0\Rightarrow(x-5)$ is a factor ✓

Do not allow if candidate states

$f(5)=0\Rightarrow(x-5)$ is a factor, hence divisible by $(x-5)$ **(It is not f)**

$g(x)=0\Rightarrow(x-5)$ is a factor **(It is not g(x) and there is no conclusion)**

This may be seen in a preamble before finding $g(5)=0$ but in these cases there must be a minimal statement ie QED, "proved", tick etc.

(b)

M1: Attempts to find the quadratic factor by inspection (correct coefficients of first term and ± last term) or by division (correct coefficients of first term and ± second term). Allow this to be scored from division in part (a)

A1: $(2x^2+11x+14)$ You may not see the $(x-5)$ which can be condoned

dM1: Correct attempt to factorise their $(2x^2+11x+14)$

A1: $(\mathrm{g}(x)=)\ (x-5)(2x+7)(x+2)$ or $(\mathrm{g}(x)=)\ (x-5)(x+3.5)(2x+4)$
It is for the product of factors and not just a statement of the three factors
Attempts with calculators via the three roots are likely to score 0 marks. The question was "Hence" so the two M's must be awarded.

(c)

M1: For $x^n \to x^{n+1}$ for any of the terms in x for $\mathrm{g}(x)$ so
$2x^3 \to ...x^4,\ x^2 \to ...x^3,\ -41x \to ...x^2,\ -70 \to ...x$

A1: $\int 2x^3+x^2-41x-70\,\mathrm{d}x = \frac{1}{2}x^4+\frac{1}{3}x^3-\frac{41}{2}x^2-70x$ which may be left unsimplified (ignore any reference to $+C$)

M1: Deduces the need to use $\int_{-2}^{5}\mathrm{g}(x)\mathrm{d}x$.

This may be awarded from the limits on their integral (either way round) or from embedded values which can be subtracted either way round.

A1: For clear work showing all algebraic steps leading to area = $571\frac{2}{3}$ oe

So allow $\int_{-2}^{5} 2x^3+x^2-41x-70\,\mathrm{d}x = \left[\frac{1}{2}x^4+\frac{1}{3}x^3-\frac{41}{2}x^2-70x\right]_{-2}^{5} = -\frac{1715}{3} \Rightarrow$ area $=\frac{1715}{3}$ for 4 marks

Condone spurious notation, as long as the algebraic steps are correct. If they find $\int_{-2}^{5}\mathrm{g}(x)\mathrm{d}x$ then withhold the final mark if they just write a positive value to this integral since $\int_{-2}^{5}\mathrm{g}(x)\mathrm{d}x = -\frac{1715}{3}$

Note $\int_{-2}^{5} 2x^3+x^2-41x-70\,\mathrm{d}x \Rightarrow \frac{1715}{3}$ with no algebraic integration seen scores M0A0M1A0

Question	Scheme	Marks	AOs
11. (i)	$x^2+y^2+18x-2y+30=0 \Rightarrow (x+9)^2+(y-1)^2=...$	M1	1.1b
	Centre $(-9,1)$	A1	1.1b
	Gradient of line from $P(-5,7)$ to "$(-9,1)$" $= \frac{7-1}{-5+9} = \left(\frac{3}{2}\right)$	M1	1.1b
	Equation of tangent is $y-7=-\frac{2}{3}(x+5)$	dM1	3.1a
	$3y-21=-2x-10 \Rightarrow 2x+3y-11=0$	A1	1.1b
		(5)	
(ii)	$x^2+y^2-8x+12y+k=0 \Rightarrow (x-4)^2+(y+6)^2=52-k$	M1	1.1b
	Lies in Quadrant 4 if radius $<4 \Rightarrow$ "$52-k$" $<4^2$	M1	3.1a
	$\Rightarrow k>36$	A1	1.1b
	Deduces $52-k>0 \Rightarrow$ Full solution $36<k<52$	A1	3.2a
		(4)	
			(9 marks)

Notes

(i)

M1: Attempts $(x\pm9)^2....(y\pm1)^2=...$ It is implied by a centre of $(\pm9,\pm1)$

A1: States or uses the centre of C is $(-9,1)$

M1: A correct attempt to find the gradient of the radius using their $(-9,1)$ and P. E.g. $\frac{7-\text{"}1\text{"}}{-5-\text{"}-9\text{"}}$

dM1: For the complete strategy of using perpendicular gradients and finding the equation of the tangent to the circle. It is dependent upon both previous M's. $y-7=-\frac{1}{\text{gradient } CP}(x+5)$

Condone a sign slip on one of the -7 or the 5.

A1: $2x+3y-11=0$ oe such as $k(2x+3y-11)=0, k\in\mathbb{Z}$

....

Attempt via implicit differentiation. The first three marks are awarded

M1: Differentiates $x^2+y^2+18x-2y+30=0 \Rightarrow ...x+...y\frac{dy}{dx}+18-2\frac{dy}{dx}....=0$

A1: Differentiates $x^2+y^2+18x-2y+30=0 \Rightarrow 2x+2y\frac{dy}{dx}+18-2\frac{dy}{dx}=0$

M1: Substitutes $P(-5,7)$ into their equation involving $\frac{dy}{dx}$

....

(ii)

M1: For reaching $(x \pm 4)^2 + (y \pm 6)^2 = P - k$ where P is a positive constant. Seen or implied by centre coordinates $(\mp 4, \mp 6)$ and a radius of $\sqrt{P-k}$

M1: Applying the strategy that it lies entirely within quadrant if "their radius" < 4 and proceeding to obtain an inequality in k only (See scheme). Condone $\ldots \leqslant 4$ for this mark.

A1: Deduces that $k > 36$

A1: A rigorous argument leading to a full solution. In the context of the question the circle exists so that as well as $k > 36$ $52 - k > 0 \Rightarrow 36 < k < 52$ Allow $36 < k \leqslant 52$

Question	Scheme		Marks	AOs
12 (a)	$\log_{10} V = 0.072t + 2.379$ $\Rightarrow V = 10^{0.072t+2.379}$ $\Rightarrow V = 10^{0.072t} \times 10^{2.379}$	$V = ab^t$ $\Rightarrow \log_{10} V = \log_{10} a + \log_{10} b^t$ $\Rightarrow \log_{10} V = \log_{10} a + t\log_{10} b$	B1	2.1
	States either $a = 10^{2.379}$ or $b = 10^{0.072}$	States either $\log_{10} a = 2.379$ or $\log_{10} b = 0.072$	M1	1.1b
	$a = 239$ or $b = 1.18$	$a = 239$ or $b = 1.18$	A1	1.1b
	Either $V = 239 \times 1.18^t$ or imply by $a = 239, b = 1.18$		A1	1.1b
			(4)	
(b)	The value of *ab* is the (total) number of views of the advert **1 day after** it went live.		B1	3.4
			(1)	
(c)	Substitutes $t = 20$ in either equation and finds V Eg $V = 239 \times 1.18^{20}$		M1	3.4
	Awrt 6500 or 6600		A1	1.1b
			(2)	
				(7 marks)

(a) **Condone** $\log_{10}$ **written** log **or** lg **written throughout the question**

B1: Scored for showing that $\log_{10} V = 0.072t + 2.379$ can be written in the form $V = ab^t$ or vice versa

Either starts with $\log_{10} V = 0.072t + 2.379$ (may be implied) and **shows lines**

$$V = 10^{0.072t+2.379} \text{ and } V = 10^{0.072t} \times 10^{2.379}$$

Or starts with $V = ab^t$ (implied) and **shows the lines**

$$\log_{10} V = \log_{10} a + \log_{10} b^t \text{ and } \log_{10} V = \log_{10} a + t\log_{10} b$$

M1: For a correct equation in *a* or a correct equation in *b*

A1: Finds either constant. Allow $a =$ awrt 240 or $b =$ awrt 1.2 following a correct method

A1: Correct solution: Look for $V = 239 \times 1.18^t$ or $a = 239, b = 1.18$

Note that this is NOT awrt

(b)

B1: See scheme. Condone not seeing total. Do not allow number of views at the start or similar.

(c)

M1: Substitutes $t = 20$ in either their $V = 239 \times 1.18^t$ or $\log_{10} V = 0.072t + 2.379$ and uses a correct method to find V

A1: Awrt 6500 or 6600

Question	Scheme	Marks	AOs
13 (a)	States $(2a-b)^2 \geqslant 0$	M1	2.1
	$4a^2 + b^2 \geqslant 4ab$	A1	1.1b
	(As $a > 0, b > 0$) $\dfrac{4a^2}{ab} + \dfrac{b^2}{ab} \geqslant \dfrac{4ab}{ab}$	M1	2.2a
	Hence $\dfrac{4a}{b} + \dfrac{b}{a} \geqslant 4$ * CSO	A1*	1.1b
		(4)	
(b)	$a = 5, b = -1 \Rightarrow \dfrac{4a}{b} + \dfrac{b}{a} = -20 - \dfrac{1}{5}$ which is less than 4	B1	2.4
		(1)	
			(5 marks)

Notes

(a) (condone the use of $>$ for the first three marks)

M1: For the key step in stating that $(2a-b)^2 \geqslant 0$

A1: Reaches $4a^2 + b^2 \geqslant 4ab$

M1: Divides each term by $ab \Rightarrow \dfrac{4a^2}{ab} + \dfrac{b^2}{ab} \geqslant \dfrac{4ab}{ab}$

A1*: Fully correct proof with steps in the correct order and gives the reasons why this is true:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a > 0$ and $b > 0$

(b)

B1: Provides a counter example and shows it is not true.
This requires values, a calculation or embedded values(see scheme) and a conclusion. The conclusion must be in words eg the result does not hold or not true
Allow 0 to be used as long as they explain or show that it is undefined so the statement is not true.

Proof by contradiction: Scores all marks

M1: Assume that there exists an $a, b > 0$ such that $\frac{4a}{b} + \frac{b}{a} < 4$

A1: $4a^2 + b^2 < 4ab \Rightarrow 4a^2 + b^2 - 4ab < 0$

M1: $(2a - b)^2 < 0$

A1*: States that this is not true, hence we have a contradiction so $\frac{4a}{b} + \frac{b}{a} \ldots 4$ with the following reasons given:

- when you square any (real) number it is always greater than or equal to zero
- dividing by ab does not change the inequality as $a > 0$ and $b > 0$

Attempt starting with the left-hand side

M1: $(\text{lhs} =) \frac{4a}{b} + \frac{b}{a} - 4 = \frac{4a^2 + b^2 - 4ab}{ab}$

A1: $= \frac{(2a - b)^2}{ab}$

M1: $= \frac{(2a - b)^2}{ab} \ldots 0$

A1*: Hence $\frac{4a}{b} + \frac{b}{a} - 4 \ldots 0 \Rightarrow \frac{4a}{b} + \frac{b}{a} \ldots 4$ with the following reasons given:

- when you square any (real) number it is always greater than or equal to zero
- ab is positive as $a > 0$ and $b > 0$

Attempt using given result: For 3 out of 4

$\frac{4a}{b} + \frac{b}{a} \ldots 4$ M1 $\Rightarrow 4a^2 + b^2 \ldots 4ab \Rightarrow 4a^2 + b^2 - 4ab \ldots 0$

A1 $\Rightarrow (2a - b)^2 \ldots 0$ oe

M1 gives both reasons why this is true

- "square numbers are greater than or equal to 0"
- "multiplying by *ab* does not change the sign of the inequality because *a* and *b* are positive"

Question	Scheme	Marks	AOs
14 (a)	Deduces $g(x) = ax^3 + bx^2 + ax$	B1	2.2a
	Uses $(2,9) \Rightarrow 9 = 8a + 4b + 2a$ $\Rightarrow 10a + 4b = 9$	M1 A1	2.1 1.1b
	Uses $g'(2) = 0 \Rightarrow 0 = 12a + 4b + a$ $\Rightarrow 13a + 4b = 0$	M1 A1	2.1 1.1b
	Solves simultaneously $\Rightarrow a, b$	dM1	1.1b
	$g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$	A1	1.1b
		(7)	
(b)	Attempts $g''(x) = -18x + \frac{39}{2}$ and substitutes $x = 2$	M1	1.1b
	$g''(2) = -\frac{33}{2} < 0$ hence maximum	A1	2.4
		(2)	
			(9 marks)

Notes

(a)

B1: Uses the information given to deduce that $g(x) = ax^3 + bx^2 + ax$. (Seen or implied)

M1: Uses the fact that $(2,9)$ lies on the curve so uses $x = 2, y = 9$ within a cubic function

A1: For a simplified equation in just two variables. E.g. $10a + 4b = 9$

M1: Differentiates their cubic to a quadratic and uses the fact that $g'(2) = 0$ to obtain an equation in *a* and *b*.

A1: For a different simplified equation in two variables E.g. $13a + 4b = 0$

dM1: Solves simultaneously $\Rightarrow a = ..., b = ...$ It is dependent upon the B and both M's

A1: $g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$

(b)

M1: Attempts $g''(x) = -18x + \frac{39}{2}$ and substitutes $x = 2$. Award for second derivatives of the form $g''(x) = Ax + B$ with $x = 2$ substituted in.
Alternatively attempts to find the value of their $g'(x)$ or $g(x)$ either side of $x = 2$ (by substituting a value for *x* within 0.5 either side of 2)

A1: $g''(2) = -\frac{33}{2} < 0$ hence maximum. (allow embedded values but they must refer to the sign or that it is less than zero)
If $g'(x) = -9x^2 + \frac{39}{2}x - 3$ or $g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$ is calculated either side of $x = 2$ then the values must be correct or embedded correctly (you will need to check these) they need to compare $g'(x) > 0$ to the left of $x = 2$ and $g'(x) < 0$ to the right of $x = 2$ or $g(x) < 9$ to the left and $g(x) > 9$ to the right of $x = 2$ hence maximum.

Note If they only sketch the cubic function $g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$ then award M1A0