AS
MATHEMATICS
7356/2
Paper 2

Mark scheme
June 2019

Version 1.0 Final

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	1.2	B1	–3
	Total		**1**	

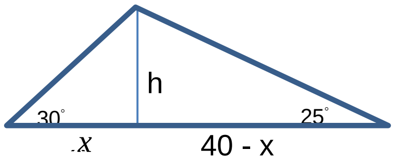
Q	Marking Instructions	AO	Marks	Typical Solution
2	Ticks correct answer	1.1b	B1	(–2, 3)
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	Substitutes $\sin\theta$ value into the equation $\sin^2\theta + \cos^2\theta = 1$ ACF or Uses $\sin\theta = -0.1$ and right-angled triangle to get magnitude of $\cos\theta$ or Obtains $\cos^2\theta = 0.99$ CAO	1.1a	M1	$\sin^2\theta + \cos^2\theta = 1$ $0.01 + \cos^2\theta = 1$ $\cos^2\theta = 0.99$ $\cos\theta = -\frac{3}{10}\sqrt{11}$
	Solves and selects correct sign Accept $\cos\theta = -\sqrt{0.99}$ or exact equivalent $-\frac{3}{10}\sqrt{11}$ ISW if exact answer seen and then evaluated NB Any full numerical approach scores M0A0	1.1b	A1	
	Total		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
4	Uses a law of logarithms correctly in their working from the list below: Multiplication / Division / Power NB **Any attempt to show the result with numerical values scores 0/4**	1.1a	M1	$\log_{10}\dfrac{x^4}{100}+\log_{10}9x-\log_{10}x^3$ $=4\log_{10}x-\log_{10}100$ $+\log_{10}9+\log_{10}x$ $-3\log_{10}x$ $=-2\log_{10}10+2\log_{10}3+2\log_{10}x$ $=2(-\log_{10}10+\log_{10}3+\log_{10}x)$ $=2(-1+\log_{10}3x)$
	Uses a different law of logarithms correctly from above list NB $\log_{10}\frac{9x^2}{100}$ OE **scores M1 M1**	1.1a	M1	
	Obtains at least two terms equivalent to $-2\log_{10}10+2\log_{10}3+2\log_{10}x$	1.1b	A1	
	Completes rigorous argument with no slips to obtain $2(-1+\log_{10}3x)$ correctly with Base 10 identified in the final answer AG	2.1	R1	
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
5	Uses sine rule with 125° (or 55°)	1.1a	M1	$\frac{AB}{\sin 30} = \frac{40}{\sin 125}$
	Finds one of the sides as an expression or value given to at least 1 decimal place *AB* = 24.4 or *AC* = 20.6	1.1b	A1	$AB = \frac{40 \sin 30}{\sin 125}$ $AB = 24.415$
	Uses ½ ab sin C to find area for 'their' a, b and c OE NB Must be a valid set	1.1a	M1	Area = $\frac{1}{2} \times 40 \times \frac{40 \sin 30}{\sin 125} \times \sin 25$
	Obtains the correct volume of 61900 CAO Condone missing units	1.1b	A1	= 206.4 Volume = 61900 mm^3
	Total		**4**	

Alternative solution to Q5:

Q	Marking Instructions	AO	Marks	Typical Solution
5	Uses tan30° and tan25° separately to obtain expressions for the vertical height	1.1a	M1	
	Obtains a correct expression for h PI by correct area	1.1b	A1	$\tan 30 = \frac{h}{x}$ $\quad$ h = x tan30 $\tan 25 = \frac{h}{40-x}$ $\quad$ h = (40 – x) tan25
	Uses ½ × base × 'their calculated height' Must see a calculated height)	1.1a	M1	$h = (40 - \frac{h}{\tan 30}) \tan 25$ $h + \frac{h}{\tan 30}\tan 25 = 40\tan 25$
	Obtains the correct volume of 61900 CAO Condone missing units	1.1b	A1	$h = \frac{40\tan 25}{\left(1+\frac{\tan 25}{\tan 30}\right)} = 10.3184\ldots$ Area = $\frac{1}{2} \times 40 \times 10.3184..$ = 206.4 Volume = 61900 mm^3
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
6	Expresses $\frac{1}{x\sqrt{x}}$ as $x^{-\frac{3}{2}}$ or $x^{-1.5}$ or $x^{-1\frac{1}{2}}$ PI completes correct integration Condone incorrect use of ‘2’ NB $a = 16$ **with no justification scores 0/5**	1.1a	M1	$\frac{2}{x\sqrt{x}} = 2x^{-\frac{3}{2}}$ $3 = \int_1^a 2x^{-\frac{3}{2}}\,dx$ $= [-4x^{-\frac{1}{2}}]_1^a$ $3 = -4a^{-\frac{1}{2}} + 4$ $a^{-\frac{1}{2}} = \frac{1}{4}$ $a = 16$
	Carries out correct integration to obtain $-4x^{-\frac{1}{2}}$ OE	1.1b	A1	
	Forms an equation by equating 3 PI by • correct integral $[-4x^{-\frac{1}{2}}]_1^a$ • original expression as integral with powers $\int 2x^{-\frac{3}{2}}\,dx$ • original expression as integral $\int \frac{2}{x\sqrt{x}}\,dx$ • ‘Their’ integration with limits 1 and a • ‘Their’ expression after integration and after using limits 1 and a Condone limits interchanged **If assuming $a = 16$ and then trying to verify scores M1A1M1 max**	3.1a	M1	
	Substitutes $x = 1$ as the lower limit and $x = a$ as the upper limit into ‘their’ integrated expression and subtracts	1.1a	M1	
	Obtains $a = 16$ CAO	1.1b	A1	
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
7	Uses gradient or equation of *AB* or vectors or proportionate division to find a PI by obtaining $a = -2$	3.1a	M1	Gradient (2, 4) to $B = \frac{6-4}{10-2} = \frac{1}{4}$
	Obtains $a = -2$	1.1b	A1	$\frac{6-3}{10-a} = \frac{1}{4}$
	Finds midpoint of *AB* PI by either coordinate being correct NB Knowledge of value of a is not required for this mark	1.1a	M1	So $a = -2$ Midpoint = $\left(\frac{a+10}{2}, \frac{3+6}{2}\right)$ = (4, 4.5) $c = 4$, $d = 4.5$
	Deduces $c = 4$ and $d = 4.5$	2.2a	A1	Radius$^2 = 6^2 + 1.5^2 = 38.25$
	Uses an appropriate distance formula to find length of radius or radius squared NB Must be fully numerical PI by use of 'their' $(10 - c)^2 + (6 - d)^2$ or 38.25 seen anywhere or $\frac{1}{2}\sqrt{(10-a)^2 + 3^2}$ for 'their' a	1.1a	M1	$e = 38.25$
	Deduces correct value of e Accept 38.25 or $\frac{153}{4}$ or $38\frac{1}{4}$ OE CAO Do not ISW if e is square rooted or squared	2.2a	A1	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Substitutes coordinates of *R* into $y = x^3 + px^2 + qx - 45$ to form a correct equation in terms of p and q ACF	1.1b	B1	$3 = 2^3 + 2^2p + 2q - 45$ $40 = 4p + 2q$
	Differentiates $y = x^3 + px^2 + qx - 45$ with at least two terms correct	1.1a	M1	$\frac{dy}{dx} = 3x^2 + 2px + q$
	Obtains a fully correct derivative	1.1b	A1	
	Substitutes $x = 2$ and $\frac{dy}{dx} = 8$ into differential equation to give a correct equation ACF	1.1b	A1	$8 = 3 \times 2^2 + 4p + q$ $-4 = 4p + q$
	Obtains $p = -12$ $q = 44$	1.1b	A1	$p = -12$ $q = 44$
8(b)	States that gradient of normal is $-\frac{1}{8}$ PI	1.2	B1	Gradient of normal is $-\frac{1}{8}$
	Writes down equation of line through (2, 3) with 'their' gradient of the normal ACF	1.1a	M1	$(y-3) = -\frac{1}{8}(x-2)$ $y = -\frac{1}{8}x + \frac{13}{4}$
	Substitutes $x = 0$ or $y = 0$ into 'their' straight line equation to find at least one intercept **M1M1 PI by $x = 26$ or $y = 3\frac{1}{4}$**	1.1a	M1	Meets x-axis at (26, 0) Meets y-axis at $(0, 3\frac{1}{4})$
	Calculates area of triangle using both 'their' intercepts or Calculates area of triangle by using integration of 'their' line between $x = 0$ and $x =$ 'their' x intercept	1.1a	M1	Area $= \frac{1}{2} \times 26 \times 3\frac{1}{4} = \frac{169}{4}$
	Obtains correct area as $\frac{169}{4}$ or $42\frac{1}{4}$ or 42.25 CAO	1.1b	A1	
	Total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
9(a)	Multiplies out $\mathrm{f}(x)$ correctly	1.1b	B1	$\mathrm{f}(x) = (x-2)(x^2-6x+9)$ $= x^3 - 8x^2 + 21x - 18$
	Differentiates, with at least one term of $3x^2 - 16x + 21$ correct	1.1a	M1	$\mathrm{f}'(x) = 3x^2 - 16x + 21$
	Explains that $\mathrm{f}'(x) = 0$ for a turning point	2.4	E1	$\mathrm{f}'(x) = 0$ for a turning point
	Sets 'their' differential equal to zero and solves to find 'their' two x values PI	1.1a	M1	$3x^2 - 16x + 21=0$
	Obtains correct x coordinates of turning points	1.1b	A1	$x = \frac{7}{3}$ and 3
	Substitutes 'their' x values into $\mathrm{f}(x)$ to obtain 'their' y values	1.1a	M1	$y = \frac{4}{27}$ and 0
	Differentiates a second time, using 'their' $\mathrm{f}'(x)$ and tests each of the x coordinates of 'their' turning points or Tests the gradient either side of each value or Justifies fully from shape of cubic with reference to a sketch or using the nature of a positive cubic graph	1.1a	M1	$\mathrm{f}''(x) = 6x - 16$ $\mathrm{f}''\left(\frac{7}{3}\right) = -2 < 0$ $\mathrm{f}''(3) = 2 > 0$ Maximum at $\left(\frac{7}{3}, \frac{4}{27}\right)$ Minimum at $(3, 0)$
	Determines correct nature of turning points at the correct coordinates, clearly identifying which is maximum and which is minimum It is not necessary to obtain E1 to obtain R1	2.1	R1	
9(b)	Deduces at least one fully correct coordinate FT 'their' coordinates	2.2a	B1F	$\left(\frac{4}{3}, -\frac{104}{27}\right)$ (2, -4)
	Deduces both coordinates correctly CSO	2.2a	B1	
	Total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(a)	Substitutes t = 0 to obtain $\theta = A$ Or States when t = 0, $10^{-kt} = 1$ **and** Infers correctly that A is the initial temperature of the water	2.2b	R1	t = 0 gives $\theta = A$ A is the starting temperature of the water
10(b)	Uses logarithms correctly to achieve given answer Must see clear evidence of use AG $\log_{10} A \times \log_{10} 10^{-kt}$ scores B0	1.1b	B1	$\log_{10}\theta = \log_{10} A + \log_{10} 10^{-kt}$ $= \log_{10} A - kt$
10(c)	Substitutes correct t and θ values to form at least one correct equation	3.3	M1	t = 10, θ = 30, t = 20, θ = 12 $\log_{10} 30 = \log_{10} A - 10k$ $\log_{10} 12 = \log_{10} A - 20k$ $k = \dfrac{1}{10}\log_{10} 2.5 = 0.0398$ A = 75
	Substitutes correct t and θ values to form two correct equations	3.1b	A1	
	Solves the equations to find exact k ACF or AWFW 0.039 to 0.04	1.1a	M1	
	Solves to find A AWRT 75	1.1b	A1	
10(d)	Substitutes 'their' calculated values of k , A and t = 45 into the given equation or Solves $75 \times 10^{-0.039 \times t} = 1$	3.4	M1	$75 \times 10^{-0.039 \times 45}$ = 1.2 1.2 > 1 Model does not support Zena's statement
	Obtains correct answer for θ AWFW 1.18 to 1.32 Or Obtains t = 47.1 AWFW 46.8 to 48.1	1.1b	A1	
	Compares AWFW 1.18 to 1.32 with 1 and states that the model does not support Zena's statement or Compares AWFW 46.8 to 48.1 with 45 and states that the model does not support Zena's statement	3.2b	R1	
10(e)	States a valid problem with the **model.** For example: Change in outside temperature Model implies water never cools	3.5b	E1	After 45 minutes the outside temperature may have changed

	down to 0°C. Other factors may affect rate of cooling for example air currents She has not taken enough measurements to accurately determine the model parameters Water behaves differently as its temperature approaches 0°C We do not know what happens after t = 20 **Do not** accept any reference to rates of change unless fully qualified			
	Total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
11	Circles correct answer	1.2	B1	opportunity
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
12	Ticks correct box	3.2b	B1	definitely incorrect
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(a)	States correct propulsion type Accept hybrid or Category 8	2.2a	B1	Electric/petrol Only category with this many values
	Gives correct reason Accept only other category with more than one value	2.4	E1	
13(b)	Calculates correct value of mean AWRT 72.4	1.1b	B1	72.375
13(c)	Calculates correct value of standard deviation Accept 26.8 AWRT for either value	1.1b	B1	28.7
13(d)(i)	Calculates AWRT $72.4 - 2 \times s.d$ **and** shows clearly that a value greater than 13 is obtained Using 26.8 gives 18.8	2.3	R1	$72.4 - 2 \times 28.7 \approx 15 > 13$
13 (d)(ii)	Infers that standard deviation/it will decrease Accept one word answers Ignore any calculations unless contradictory to a decrease in standard deviation	2.2b	R1	Standard deviation will decrease
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
14(a)	Substitutes x values into formula at least 3 terms correct in terms of c ACF NB No need for addition of terms to be seen or Uses $c = \frac{1}{10}$ and shows the addition of the correct four probabilities summing to 1 **Max mark M1R0**	3.1a	M1	$4c + 3c + 2c + c = 1$ $10c = 1$ $c = \frac{1}{10}$
	Equates sum to 1 and shows convincingly that $c = \frac{1}{10}$	2.1	R1	
14(b)	Adds probabilities for x = 1, 2 and 3 NB Can be in terms of c or States $P(X = 0) = 4c$ OE **and** subtracts this from 1	1.1a	M1	$3c + 2c + c = 6c$ P(X ≥ 1)= 0.6
	Obtains correct value for probability CAO ACF	1.1b	A1	
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
15 (a)(i)	Uses the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ PI by $0.8 = 0.2 + P(B) -$ stated term/value or States $P(A' \cap B') = 0.2$ **and** $P(A') = 0.8$ OE	3.1a	M1	$0.8 = 0.2 + P(B) - P(A \cap B)$ $0.8 = 0.2 + P(B) - 0.2P(B)$ $0.6 = 0.8P(B)$ $P(B) = 0.75$
	Uses the formula $P(A \cap B) = P(A) \times P(B)$ or $P(A' \cap B') = P(A') \times P(B')$ OE	3.1a	M1	
	Obtains correct equation in $P(B)$ or $P(B')$ only PI	1.1a	A1	
	Finds correct value of $P(B)$	1.1b	A1	
15 (a)(ii)	Finds 'their' correct value of $P(A \cap B)$ **provided** P(B) lies between 0 and 0.8	1.1b	B1F	$P(A \cap B) = 0.15$
15(b)	Deduces not mutually exclusive and states a correct reason Other reasons: Not mutually exclusive, since $P(A \cap B) \neq 0$ or Shows $P(A \cup B) \neq P(A) + P(B)$ or Not mutually exclusive as they can both occur at the same time	2.2a	R1	A and B are not mutually exclusive since independent events cannot be mutually exclusive
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
16(a)	States both hypotheses correctly for one-tailed test Accept: • Population proportion = 0.12 • p = 12% • $H_1: p \leq 0.12$ or 12% Do not accept: x = 0.12, μ = 0.12 or $\bar{x}$ = 0.12	2.5	B1	$H_0: p = 0.12$ $H_1: p < 0.12$ Under null hypothesis, $X \sim B(60\,, 0.12)$ $P(X \leq 4) = 0.139$ $0.139 > 0.10$ Accept H_0 There is insufficient evidence to suggest that the proportion of faulty chargers has reduced
	States model used PI Can be implied by AWRT 0.06, 0.14, 0.040, 0.079	3.3	M1	
	Calculates $P(X \leq 4)$ or $P(X \leq 3)$ $P(X \leq 3) = 0.060$ Do **not** accept $P(X = 4)$ or $P(X = 3)$	1.1a	M1	
	Obtains correct value for $P(X \leq 4)$ Accept 0.14 AWRT	1.1b	A1	
	Evaluates Binomial model by comparing 0.139 (accept 0.14) or 0.060 with 0.10 Do **not** accept use of $P(X = 4)$ or $P(X = 3)$ Must be a clear comparison in words or inequality or diagram	3.5a	M1	
	Infers H_0 accepted or H_1 rejected Condone 'do not reject' If no hypothesis after comparison assume H_0	2.2b	A1	
	Concludes correctly in context. 'Insufficient evidence' required OE Only award for full complete solution	3.2a	R1	

Alternative Solution

Q	Marking Instructions	AO	Marks	Typical Solution
16(a)	States both hypotheses correctly for one-tailed test Accept: • Population proportion = 0.12 • p = 12% • $H_1: p \leq 0.12$ or 12% Do not accept: x = 0.12, μ = 0.12 or $\bar{x}$ = 0.12	2.5	B1	$H_0: p = 0.12$ $H_1: p < 0.12$ Under null hypothesis, $X \sim B(60\,, 0.12)$
	States model used PI Can be implied by AWRT 0.06, 0.14, 0.040, 0.079	3.3	M1	$P(X \leq 4) = 0.139$ > 0.1 $P(X \leq 3) = 0.060$ < 0.1
	Calculates $P(X \leq 4)$ **and** $P(X \leq 3)$ but **not** $P(X = 4)$ and $P(X = 3)$	1.1a	M1	Critical region is $X \leq 3$
	Identifies correct critical region. Must have considered both $P(X \leq 4)$ **and** $P(X \leq 3)$	1.1b	A1	As 4 does not lie in the critical region we accept H_0
	Evaluates Binomial model by comparing $X = 4$ or $X = 3$ with critical region Must be a clear comparison in words or inequality or diagram	3.5a	M1	There is insufficient evidence to suggest that the proportion of faulty chargers has reduced.
	Infers H_0 accepted or H_1 rejected Condone 'do not reject' If no hypothesis after comparison assume H_0	2.2b	A1	
	Concludes correctly in context. 'Insufficient evidence' required OE Only award for full complete solution	3.2a	R1	

Q	Marking Instructions	AO	Marks	Typical Solution
16(b)	States a first assumption in context Must include 'faulty' if assumption refers to probability or independence	3.5b	E1	The probability of a faulty charger is fixed A charger being faulty is independent of any other charger being faulty
	States a second assumption in context Must include 'faulty' if assumption refers to probability or independence Also accept: The sample of chargers was randomly selected Do **not** accept the number of trials is fixed at 60 Do **not** accept the charger is either faulty or not faulty	3.5b	E1	
	Total		**9**	