Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE
In Mathematics (9MA0_31) Statistics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.

- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.

- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is awarded.

- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 50.

2. These mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - **bod** – benefit of doubt

   - **ft** – follow through

   - the symbol $\sqrt{}$ will be used for correct ft

   - **cao** – correct answer only

   - **cso** - correct solution only. There must be no errors in this part of the question to obtain this mark

   - **isw** – ignore subsequent working

   - **awrt** – answers which round to

   - **SC**: special case

   - **o.e.** – or equivalent (and appropriate)

   - **d** or **dep** – dependent

   - **indep** – independent

   - **dp** decimal places

   - **sf** significant figures

   - **∗** The answer is printed on the paper or ag- answer given

4. All M marks are follow through.

   A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but the response is deemed to be valid, examiners must escalate the response for a senior examiner to review.

Question	Scheme	Marks	AOs
1(a)		B1	1.1b
		dB1	1.1b
		(2)	
(b)	$\dfrac{9}{10} \times \dfrac{4}{5} \times \dfrac{2}{3}$	M1	1.1b
	$= \dfrac{12}{25} \ (= 0.48)$	A1	1.1b
		(2)	
(c)	$\dfrac{9}{10} \times \dfrac{1}{5} + \dfrac{9}{10} \times \dfrac{4}{5} \times \dfrac{1}{3}$ or $1 - \left(\dfrac{1}{10} + \dfrac{9}{10} \times \dfrac{4}{5} \times \dfrac{2}{3} \right)$	M1	3.1b
	$= \dfrac{21}{50} \ (= 0.42)$	A1	1.1b
		(2)	
(d)	$[\text{P(Red from } B|\text{Red selected)}] = \dfrac{\frac{9}{10} \times \frac{1}{5}}{\frac{1}{10} + \frac{9}{10} \times \frac{1}{5} + \frac{9}{10} \times \frac{4}{5} \times \frac{1}{3}} \left[= \dfrac{\frac{9}{50}}{\frac{13}{25}} \right]$	M1	3.1b
	$= \dfrac{9}{26}$	A1	1.1b
		(2)	
		(8 marks)	

<div align="center">Notes</div>

<div align="center">Allow decimals or percentages throughout this question.</div>

(a)	**B1:** for correct shape (3 pairs) and at least **one** label on **at least two** pairs G(reen) and R(ed) allow G and G' **or** R and R' as labels, etc. condone 'extra' pairs if they are labelled with a probability of 0 **dB1:** (dep on previous B1) all correct i.e. for all 6 correct probabilities on the correct branches with at least one label on **each** pair
(b)	**M1:** Multiplication of 3 correct probabilities (allow ft from their tree diagram) **A1:** $\dfrac{12}{25}$ oe
(c)	**M1:** Either addition of only two correct products (product of two probs + product of three probs) which may ft from their tree diagram or for $1 - ('\frac{1}{10}' + '(b)')$ **A1:** $\dfrac{21}{50}$ oe
(d)	**M1:** Correct ratio of probabilities **or** correct ft ratio of probabilities e.g. $\dfrac{'\frac{9}{10}' \times '\frac{1}{5}'}{1 - '(b)'}$ or $\dfrac{'\frac{9}{10}' \times '\frac{1}{5}'}{'\frac{1}{10}' + '(c)'}$ with num < den **A1:** $\dfrac{9}{26}$ (allow awrt 0.346)

Question	Scheme	Marks	AOs
2(a)	IQR = 26.6 – 19.4 [= 7.2]	B1	2.1
	19.4 – 1.5 × '7.2' [= 8.6] or 26.6 + 1.5 × '7.2' [= 37.4]	M1	1.1b
	Plotting one upper whisker to 32.5 **and** one lower whisker to 8.6 or 9.1	A1	1.1b
	Plotting 7.6 and 8.1 as the only two outliers	A1	1.1b
		(4)	
(b)	<u>October</u> (since it is the month with the coldest temperatures between May and October in Beijing)	B1	2.4
		(1)	
(c)	$[\sigma =]\sqrt{\dfrac{4952.906}{184}}$ or e.g. $[\sigma =]\sqrt{\dfrac{S_{xx}}{n}} = 5.188...$ [=5.19*]	B1cso*	1.1b
		(1)	
(d)	$z = (\pm)\,1.28(16)$	$[P_{90} =]29.251...$ or $[P_{10} =]15.948...$	B1	3.1b
	$2 \times 1.2816 \times 5.19$	'29.251…' – '15.948…'	M1	1.1b
		= awrt **13.3**	A1	1.1b
			(3)	
(e)	Daily mean <u>wind speed</u>/Beaufort conversion since it is <u>qualitative</u>	B1	2.4
	<u>Rain</u>fall since it is not symmetric/lots of days with 0 rainfall	B1	2.4
		(2)	
			(11 marks)

Notes

(a)	**B1:** for a correct calculation for the IQR (implied by 10.8 **or** 8.6 **or** 37.4 seen) **M1:** for a complete method for either lower outlier limit or upper outlier limit (allow ft on their IQR) (may be implied by the 1st A1 or a lower whisker at 8.6) **A1:** both whiskers plotted correctly (allow ½ square tolerance) **A1:** only two outliers plotted, 7.6 and 8.1 (must be disconnected from whisker) **NOTE:** A fully correct box plot with no incorrect working scores 4/4
(c)	**B1cso*:** Correct expression with square root **or** correct formula and 5.188 or better Allow a complete correct method finding $\sum x^2 =$ awrt 98720 and $\sigma = \sqrt{\dfrac{98715.9...}{184} - \left(\dfrac{4153.6}{184}\right)^2}$
(d)	**B1:** Identifying z-value for 10th or 90th percentile (allow awrt $(\pm)\,1.28$) or for identifying $[P_{90} =]29.251...$ (awrt 29.3) or $[P_{10} =]15.948...$ (awrt 15.9) (This may be implied by a correct answer awrt 13.3) **M1:** for $2 \times z \times 5.19$ where $1 < z < 2$ or for their $P_{90} - P_{10}$ where $25< P_{90} <35$ and $10< P_{10} <20$ **A1:** awrt 13.3
(e)	**B1:** for one variable identified and a correct supporting reason **B1:** for two variables identified and a correct supporting reason for each Allow any two of the following: • <u>Wind speed</u>/Beaufort since the data is <u>non-numeric</u> (o.e.). They need not mention Beaufort provided there is a description of the data as non-numeric (Do not allow wind direction/wind gust) • <u>Rain</u>fall as not symmetric/is skewed/is not bell shaped/lots of 0s /many days with no rain/mean≠mode or median • <u>Date</u> since each data value appears once/it is uniformly distributed • Daily mean <u>pressure</u> since it is not symmetric/is skewed/not bell shaped • Daily mean <u>wind speed</u> since it is not symmetric/is skewed/not bell shaped Do not allow 'not continuous' or 'discrete' as a supporting reason. Ignore extraneous non-contradicting statements

Question	Scheme		Marks	AOs
3(a)	$H_0 : \rho = 0 \qquad H_1 : \rho > 0$		B1	2.5
	Critical value 0.3438		M1	1.1a
	(0.446 > 0.3438) so there is evidence that the product moment correlation coefficient (pmcc) is greater than 0/there is positive correlation		A1	2.2b
			(3)	
(b)	The value is close(r) to 1 **or** there is strong(er) (positive) correlation		B1	2.4
			(1)	
(c)	$\log_{10} y = -1.82 + 0.89(\log_{10} x)$	$y = ax^n \rightarrow$ $\log_{10} y = \log_{10}(ax^n)$	M1	1.1b
	$y = 10^{-1.82 + 0.89(\log_{10} x)}$	$\log_{10} y = \log_{10} a + \log_{10} x^n$	M1	2.1
	$y = 10^{-1.82} \times 10^{0.89(\log_{10} x)}$ $[= 10^{-1.82} \times 10^{(\log_{10} x)^{0.89}}]$	$\log_{10} y = \log_{10} a + n\log_{10} x$ $[\log_{10} a = -1.82, n = 0.89]$	M1	1.1b
	$y = 0.015x^{0.89}$	$y = 0.015x^{0.89}$	A1A1	1.1b 1.1b
			(5)	
				(9 marks)

Notes

(a)	**B1:** for both hypotheses correct in terms of ρ
	M1: for the critical value: sight of 0.3438 or any cv such that $0.25 < |cv| < 0.45$
	A1: a comment suggesting a significant result/ H_0 is rejected on the basis of <u>seeing</u> +0.3438
	and which mentions "pmcc/correlation/relationship" and "greater than 0/positive" (not just $\rho > 0$)
	  or an answer in context e.g. 'as "income"(o.e.) increases, "CO$_2$/emissions"(o.e.) increases'
	  A **contradictory** statement scores A0 e.g. 'Accept H_0, therefore positive correlation'

(b)	**B1:** for suitable reason e.g. r is close(r) to 1 **or** "strong(er)"/"near perfect" "correlation" Do not allow 'association'
(c)	**For both methods, once an M0 is scored, no further marks can be awarded** **and condone missing base 10 throughout** **Method 1: (working to the model)** **M1:** Correct substitution for both c and m (may be implied by 2nd M1 mark) **M1:** Making y the subject to give an equation in the form $y = 10^{a+b(\log_{10} x)}$ (may be implied by 3rd M1 mark) **M1:** Correct multiplication to give an equation in the form $y = 10^a \times 10^{b(\log_{10} x)}$ (this line implies M1M1M1 provided no previous incorrect working seen) **Method 2: (working from the model)** **M1:** Taking the log of both sides (may be implied by 2nd M1 mark) **M1:** Correct use of addition rule (may be implied by 3rd M1 mark) **M1:** Correct multiplication of power (this line implies M1M1M1 provided no previous incorrect working seen) **A1:** $n = 0.89$ **or** $a =$ awrt 0.015 **or** $y = ax^{0.89}$ **or** $y =$ awrt$0.015x^n$ (**dep on M3**) **A1:** $n = 0.89$ **and** $a =$ awrt 0.015 / $y =$ awrt$0.015x^{0.89}$ (**dep on M3**) do not award the final A1 if answer is given in an incorrect form e.g $y = 0.015 + x^{0.89}$

Question	Scheme	Marks	AOs
4 (a)	$\dfrac{132}{184} = 0.71739...$ awrt **0.717**	B1	1.1b
		(1)	
(b)(i)	$P(X \geq 6) = 1 - P(X \leq 5)$ or $P([X=]6) + P([X=]7) + P([X=]8)$	M1	3.4
	$= 1 - 0.296722...$ awrt **0.703**	A1	1.1b
		(2)	
(b)(ii)	$184 \times P(X = 7)$ $[= 184 \times 0.2811…]$	M1	1.1b
	$= 51.7385...$ awrt **51.7**	A1	1.1b
		(2)	
(c)	Part (a) and part (b)(i) are similar **and** the expected number of 7s (51.7 or 0.281) matches with the number of 7s found in the data set (52 or 0.283) so Magali's model is supported.	B1ft	3.5a
		(1)	
(d)	$\dfrac{23}{28} = 0.82142...$ awrt **0.821**	B1	1.1b
		(1)	
(e)	Any one of… • Part (d)/'0.821' differs from part (a)/(b)(i)/(0.7…) • there is a **greater/different probability of high cloud cover/more likely to have high cloud cover** if the previous day had high cloud cover • **independence(o.e.)** does not hold	B1	2.4
	…therefore Magali's (binomial) model may not be suitable.	dB1	3.5a
		(2)	
			(9 marks)
	Notes		
	Allow fractions, decimals or percentages throughout this question.		
(a)	Allow equivalent fraction, e.g. $\frac{33}{46}$		
(b)(i)	**M1:** for writing or using $1 - P(X \leq 5)$ or $P(X = 6) + P(X = 7) + P(X = 8)$ **A1:** awrt 0.703 (correct answer scores 2 out of 2)		
(b)(ii)	**M1:** for $184 \times P(X = 7)$ o.e. e.g., $184 \times [P(X \leq 7) - P(X \leq 6)]$ **A1:** awrt 51.7		
(c)	**B1ft:** comparing '0.717' with '0.703' <u>and</u> '51.7 or '0.281' with 52 or 0.283 **and** concluding that Magali's model **is supported** (must be comparing prob. with prob. <u>and</u> days with days). Allow not supported or mixed conclusions if consistent with their f.t. answers in (a) and (b)		
(e)	**B1:** Any bullet point **dB1:** (dep on previous B1) for Magali's model may not be suitable (o.e.) Condone not accurate for not suitable **SC:** part (d) is similar to part (a)/(b)(i) **and** a compatible conclusion (i.e. Magali's model is supported) to score B1B1.		

Question	Scheme	Marks	AOs
5(a)	$\dfrac{24.63-25}{'\sigma'}=-1.0364$	M1	3.1b
	$[\sigma =]0.357$ (must come from compatible signs)	A1	1.1b
	$P(D > k) = 0.4$ or $P(D < k) = 0.6$	B1	1.1b
	$\dfrac{k-25}{'0.357'}=0.2533$	M1	3.4
	$k = $ awrt **25.09**	A1	1.1b
		(5)	
(b)	$[Y \sim B(200, 0.45) \rightarrow]\ W \sim N(90, 49.5)$	B1	3.3
	$P(Y < 100) \approx P(W < 99.5)\left[=P\left(Z < \dfrac{99.5-90}{\sqrt{49.5}}\right)\right]$	M1	3.4
	$= 0.9115\ldots$ awrt **0.912**	A1	1.1b
		(3)	
(c)	$H_0 : \mu = 25 \qquad H_1 : \mu < 25$	B1	2.5
	$[\bar{D} \sim]N\left(25, \dfrac{0.16^2}{20}\right)$	M1	3.3
	$P(\bar{D} < 24.94)[= P(Z < -1.677...)] = 0.046766...$	A1	3.4
	$p = 0.047 < 0.05$ <u>or</u> $z = -1.677\ldots < -1.6449$ <u>or</u> $24.94 < 24.94115\ldots$ <u>or</u> reject H_0/in the critical region/significant	M1	1.1b
	There is sufficient evidence to support <u>Hannah's</u> <u>belief</u>.	A1	2.2b
		(5)	
		(13 marks)	

Notes

(a)	**M1:** for standardising 24.63, 25 and 'σ' (ignore label) and setting $=$ to z where $1 < \|z\| < 2$ **A1:** $[\sigma =]$ awrt 0.36. Do **not** award this mark if signs are not compatible. **B1:** for either correct probability statement (may be implied by correct answer) this mark may be scored for a correct region shown on a diagram **M1:** for a correct expression with $z = $ awrt 0.253 (may be implied by correct answer) **A1:** awrt 25.09 (Correct answer with no incorrect working scores 5 out of 5)
(b)	**B1:** setting up normal distribution approximation of binomial N(90, 49.5) (may be implied by a correct answer) Look out for e.g. $\sigma = \frac{3\sqrt{22}}{2}$ or $\sigma = $ awrt 7.04 **M1:** attempting a probability using a continuity correction i.e. $P(W < 100.5)$, $P(W < 99.5)$ or $P(W < 98.5)$ condone $\leq$ (The continuity correction may be seen in a standardisation). **A1:** awrt 0.912 [Note: 0.911299… from binomial scores 0 out of 3]

(c)	**B1:** for both hypotheses in terms of μ
	 M1: selecting suitable model must see N(ormal), mean 25, sd $= \frac{0.16}{\sqrt{20}}$ (o.e.) or var $= \frac{4}{3125}$ (o.e.)
	      Condone N(25, $\frac{0.16}{\sqrt{20}}$) if $\frac{0.16}{\sqrt{20}}$ then used as s.d.
	 A1: p value $= $ awrt 0.047 <u>or</u> **test statistic** awrt -1.68 <u>or</u> **CV** awrt 24.941
	 (any of these values imply the M1 provided they do not come from Normal mean $= 24.94$)
	 M1: a correct comparison (including compatible signs) **or** correct non-contextual
	      conclusion (f.t. their p value, test statistic or critical value in the comparison)
	      M1 may be implied by a correct contextual statement
	 NB Any contradictory non contextual statements/comparisons score M0A0 e.g. '$p < 0.05$, not significant'
	 A1: correct conclusion in context mentioning <u>Hannah's</u> <u>belief</u>
	      <u>or</u> the mean <u>amount</u>/<u>liquid</u> in each bottle is now <u>less than 25ml</u> (**dep on M1A1M1**)