

GCE

Mathematics A

H230/02: Pure Mathematics and Mechanics

Advanced Subsidiary GCE

Mark Scheme for June 2019

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	Meaning
√and ×	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in	Meaning
manula a a barra a	
mark scheme	
E1	Mark for explaining a result or establishing a given result
E1 dep*	Mark for explaining a result or establishing a given result Mark dependent on a previous mark, indicated by *
E1 dep* cao	Mark for explaining a result or establishing a given result Mark dependent on a previous mark, indicated by * Correct answer only
E1 dep* cao oe	Mark for explaining a result or establishing a given result Mark dependent on a previous mark, indicated by * Correct answer only Or equivalent
E1 dep* cao oe rot	Mark for explaining a result or establishing a given result Mark dependent on a previous mark, indicated by * Correct answer only Or equivalent Rounded or truncated
El dep* cao oe rot soi	Mark for explaining a result or establishing a given result Mark dependent on a previous mark, indicated by * Correct answer only Or equivalent Rounded or truncated Seen or implied
E1 dep* cao oe rot soi www	Mark for explaining a result or establishing a given result Mark dependent on a previous mark, indicated by * Correct answer only Or equivalent Rounded or truncated Seen or implied Without wrong working
mark schemeE1dep*caooerotsoiwwwAG	Mark for explaining a result or establishing a given result Mark dependent on a previous mark, indicated by * Correct answer only Or equivalent Rounded or truncated Seen or implied Without wrong working Answer given
mark schemeE1dep*caooerotsoiwwwAGawrt	Mark for explaining a result or establishing a given result Mark dependent on a previous mark, indicated by * Correct answer only Or equivalent Rounded or truncated Seen or implied Without wrong working Answer given Anything which rounds to
El dep* cao oe rot soi www AG awrt BC	Mark for explaining a result or establishing a given result Mark dependent on a previous mark, indicated by * Correct answer only Or equivalent Rounded or truncated Seen or implied Without wrong working Answer given Anything which rounds to By Calculator

Subject-specific Marking Instructions for AS Level Mathematics A

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
 If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

Μ

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

В

Mark for a correct result or statement independent of Method marks.

Е

Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

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Mark Scheme

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
 - When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
 - When a value is not given in the paper accept any answer that agrees with the correct value to **3 s.f**. unless the question specifically asks for another level of accuracy.

Follow through should be used so that only one mark is lost for each distinct accuracy error.

- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Qı	uestion	Answer		Marks	AO	Guidance	
1		DR $24(3+\sqrt{5})$		M1	1.1	Multiplying numerator and	Alternative: M1
		$x = \frac{24}{3} = \frac{24(3+\sqrt{3})}{(2\sqrt{5})(2+\sqrt{5})}$				denominator by $3 + \sqrt{5}$ or $-3 - \sqrt{5}$	Correct method to
		$(3-\sqrt{3})(3-\sqrt{3})(3+\sqrt{3})$					solve simultaneous
							equating expressions to
							$a+b\sqrt{5}$
		$24(3+\sqrt{5})$ $24(3+\sqrt{5})$		A1	1.1	Correct simplified denominator	A1 Either <i>a</i> or <i>b</i> correct
		$=\frac{7}{9-3\sqrt{5}+3\sqrt{5}-5}=\frac{7}{4}$					
		$=18+6\sqrt{5}$		A1	1.1	Final answer cao , therefore final	A1 Both correct
						answer of only $6(3+\sqrt{5})$ is A0	
				[3]			
2	(a)	$5\left[x^2-4x\right]+3$				No marks until attempt to complete the	
				D1	11	square	
		$=5\lfloor (x-2)^2 - 4 \rfloor + 3$	p = 5	БІ	1.1	Must be of the form $5(x \pm \alpha)^2 \pm \cdots$	
			$(x-2)^2$	B 1	1.1		
		$=5(x-2)^2-17$	r = -17	B1	1.1		
				[3]			
2	(b)	Minimum point $(2, -17)$		B1ft	1.1	Follow through their $-q$	Or by differentiation
				B1ft	1.1	Follow through their r	
				[2]			
2	(c)	x = 2		B1ft	1.1	Follow through their <i>x</i> coordinate in	
						part (b)	
				[1]			

Question		Answer	Marks	AO	Guidance		
3	(a)		B1	1.1	 Curve in both quadrants: Correct shape, symmetrical, not touching axis Asymptote the axes Not finite Allow slight movement away from asymptote at one end but not more 	N.B. Ignore 'feathering' now that answers are scanned.	
			[1]				
3	(b)		M1	1.1	$(y=) - \frac{1}{(x-2)^2}$ or $(y=) - \frac{1}{(x+2)^2}$		
		$y = -\frac{1}{(x-2)^2}$	A1	2.5	Fully correct, must include 'y ='		
			[2]				
3	(c)	$\begin{pmatrix} 1 & 2 \end{pmatrix}$	B2	1.1	B1 for each coordinate		
		$\left(\frac{1}{2}, -2\right)$		1.1			
			[2]				

Qı	iestion	An	swer	Marks	AO	O Guidance	
4	(a)	$(2-5x)^5 = 2^5 + {}^5C_1 2^4 (-5x)^5$	$(+)^{5}C_{2}2^{3}(-5x)^{2}+$	M1	1.1a	Attempt at least 2 terms – products of	Allow $\pm 5x$ – allow
) 2 ()			binomial coefficients and correct	expansion of $\left(1\pm\frac{5}{2}x\right)^5$
						powers of 2 and $-5x$	(1-2n)
		32 - 400x		A1	1.1		
		$+2000x^{2}$		A1	1.1		Do not allow from $+5x$
				[3]			
4	(b)	$(1+2ax+a^2x^2)(32-400x)$	$+2000x^{2}+)$	M1*	2.1	Expand first bracket, multiply by part	Ignore terms in x^2
)			(a) to obtain the two relevant terms in x	
		$64a - 400 = 48 \Longrightarrow a = \dots$		Dep*M1	1.1	Equate sum of the two relevant terms to	M1 only for 2 <i>a</i> – 400 =
						48 and attempt to solve for <i>a</i>	48 (oe e.g. with
							consistent <i>x</i>)
		<i>a</i> = 7		A1	2.2a	Obtain $a = 7$ only	
				[3]			
5	(a)	<i>k</i> = 3		B1	1.1		
				[1]		11	
5	(b)	$(1 4)^2 + (2 k)^2 - 13$		M1	1 10	oe e.g. allow consistent use of square	May be implied by one
5	(0)	(1-4) + (2-k) = 15		IVII	1.1a	brackets	correct value for k
		k = 0		A1	1.1		
		<i>k</i> = 4		A1	1.1		
				[3]			
			5 0 1 1			or $\frac{5-4}{2-7} = \frac{k-2}{4-1}$ oe – must be	Any one of these three
5	(c)	$\frac{4-2}{7-1} = \frac{k-5}{4-3}$ oe	or $\frac{5-2}{3-1} = \frac{4-k}{7-4}$ oe	M1	3.1a	3-7 $4-1$	solutions
			51 / 7			(allow one sign error)	5010010115
		$k = \frac{16}{2}$	$k = -\frac{1}{2}$	A1	1.1	$k = \frac{5}{4}$	
		5	2	[2]		4	

6 (a) DR

Question		Answer	Marks	AO	Guidance	
		$6(1-\sin^2\theta) = \frac{\sin\theta}{\cos\theta}(\cos\theta) + 4$	M1	3.1 a	Correct use of both $\cos^2 \theta = 1 - \sin^2 \theta$	
		$\cos(1-\sin^2\theta) \cos^2\theta$			and $\tan \theta = \frac{\sin \theta}{2}$	
					$\cos\theta$	
		$6 - 6\sin^2\theta = \sin\theta + 4 \implies 6\sin^2\theta + \sin\theta - 2 = 0$	A1	2.1	AG	Must show sufficient
			[2]			working to justify the
6	(b)	DR	[4]			given answer
U	(0)	$(2\sin\theta - 1)(3\sin\theta + 2)$	M1	1.1a	Attempt to solve 3-term quadratic	Ignore incorrect use of
		(25110 - 1)(55110 + 2)				inequalities for first
						three marks
		Critical values occur when $\sin \theta = \frac{1}{2}$ and $\sin \theta = -\frac{2}{2}$	B 1	1.1		
		$\frac{1}{2}$				
		Critical values are $\theta = 30,150,222,318$	B1	1.1	Any three correct critical values	221.8103
		0 < 0 < 20 or $150 < 0 < 222$ or $218 < 0 < 260$	D1	11	P1 for one correct interval 2 of or	318.1896
		$0 < \theta < 5001 130 < \theta < 222 01 518 < \theta < 500$	DI	1.1	better (condone use of x)	Condone ≤ oe
			A1	2.5	Cao (all three intervals) -3 sf or better	Allow $\theta < 30$,
						$150 < \theta < 222, 318 < \theta$
					For those that have $\sin \theta = -\frac{1}{2}$ and	
					$\sin\theta = \frac{2}{3}$ can score M1 (if DR seen)	
					then SC B1 for one 'correct' interval	
					(condone \leq oe) or SC B2 for all three	
					'correct' intervals which are	
					$\theta < 42,138 < \theta < 210, \theta > 330$ (so max.	
			[5]		3/3)	
			[5]			
7		32	B 1	1.1	Seen or implied by later working	
		$\frac{1}{3}$				

Qu	iestion	Answer	Marks	AO	Guidance	
			M1*	3.1 a	Attempt integration on a 3 term	(increase in power by 1
					quadratic in x	for at least 1 term but
						not just multiplying
						each term by x)
		$\int (-x^2 + 6x - 5) \mathrm{d}x = -\frac{x^3}{3} + 3x^2 - 5x$	A1	1.1	Ignore lack of $+c$	
		$-\frac{a^3}{3} + 3a^2 - 5a - \left(-\frac{5^3}{3} + 75 - 25\right)$	Dep*M1	1.1	$\pm (F(a) - F(5))$	
		$\frac{32}{3} + \frac{a^3}{3} - 3a^2 + 5a + \frac{25}{3} = 19$	A1	1.1	oe	
		$a^{3}-9a^{2}+15a=0 \Rightarrow a^{2}-9a+15=0 \therefore a \neq 0$	M1	3.1a	solve their cubic (which comes from attempt at both areas and 19) leading to an exact value for <i>a</i>	Dependent on both previous M marks
		$a \neq \frac{9 - \sqrt{21}}{2} \because a > 5$	B1	3.2a	BC – must give a reason for rejection of this value of <i>a</i>	Allow rejection of 2.21
		$a = \frac{9 + \sqrt{21}}{2}$ only	A1	2.2a	BC	
			[8]			

8 (a)
$$\log_2 x^2 (= \log_2 (kx - 1) + 3)$$

B1 1.2 Using $a \log b = \log(b^a)$

Question		Answer	Marks	AO	Guidance	
		(x^2)	M1*	2.1	Re-arranging and correctly combining	Or re-write 3 as $\log_2 8$
		$\log_2\left(\frac{1}{kx-1}\right) = 3$			both log terms	and then combining
						e.g. $2\log_2 x$
						$= \log_2(8(kx-1))$
		$\frac{x^2}{2} = 2^3$	Dep*M1	1.1	Correctly remove logs	$x^2 = 8(kx - 1)$
		kx-1				
		$x^2 = 8(kx-1)$				
		$x^2 - 8kx + 8 = 0$	A1	1.1	AG	Must show sufficient
						working to justify the
						given answer (i.e. at
						least one more line of
						M mark)
			[4]			WI IIIIIK)
8	(b)	$b^2 - 4ac = 0 \Rightarrow (-8k)^2 - 4(1)(8) = 0$	M1	3.1a	Use of $b^2 - 4ac = 0$	Or state equation must
						be of the form
						$(x+p)^2 = 0$
		$k = (\pm) \frac{1}{\sqrt{2}}$	A1	1.1	oe exact	with $p^2 = 8$
		$\sqrt{\sqrt{2}}$				
		$k = \frac{1}{\sqrt{2}} \Longrightarrow x = 2\sqrt{2}$	A1	2.2a	BC oe exact	so $x = (\pm)2\sqrt{2}$
		$\sqrt{2}$	A 1	3.2h	BC as statement for rejection of	
		$k = -\frac{1}{\sqrt{2}} \Rightarrow x = -2\sqrt{2}$ and as $\log_2 x$ is only defined for	AI	5.20	negative value of x (allow decimal	reject $x = -2\sqrt{2}$ with
		$\sqrt{\frac{\sqrt{2}}{2}}$			argument)	vanu teason
		$x > 0$ so $x \neq -2\sqrt{2}$	[4]			
			[-+]			
L						

Qı	iestion	Answer	Marks	AO	Guidance	
9		\mathbf{r} (-9)	B2	1.1	B1 for one correct value	Allow $-9\mathbf{i} + \mathbf{j}$
		$\mathbf{F} = \begin{pmatrix} 1 \end{pmatrix}$		1.1		
			[2]			SC B1 for (-9,1) or
						(-9 1)
10	(a)	$v = pt^2 + qt + r$				
		$t = 0, v = 18 \Longrightarrow r = 18$	B1	3.4		
		$t = 5, v = 9 \Longrightarrow 25 p + 5q + 18 = 9$	M1	1.1	Substitutes $t = 5, v = 9$ into quadratic	Allow with <i>r</i>
		$\frac{\mathrm{d}v}{\mathrm{d}t} = 2pt + q$	B1	3.1b		
		$t = 5, \frac{\mathrm{d}v}{\mathrm{d}t} = 0 \Longrightarrow 10p + q = 0$	M1	1.1	Substitutes $t = 5$ and sets $\frac{dv}{dt} = 0$	Dependent on one term differentiated correctly
		$p = \frac{9}{25}, q = -\frac{18}{5}$	A1	1.1	BC (oe e.g. exact decimals)	
			[5]			
10	(b)	$\int_{2}^{5} \left(\frac{9}{25}t^2 - \frac{18}{5}t + 18\right) \mathrm{d}t$	M1	3.4	Using their values of p , q and r in an attempt to find the distance travelled from 2 to 5 by integration	
		+9×5	B1	1.1	For distance travelled from 5 to 10	
		= 75.24 m	A1	1.1	BC cao (oe)	
			[3]			
1						

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Q	uestion	Answer	Marks	AO	Guidance	
11	(a)		M1*	3.3	Attempt N2L for P or Q	Must include correct
						number of terms – use
						of weight for mass is
						M0
		For <i>P</i> : $40 - T - 8 = 3a$	A1	1.1		
		For <i>Q</i> : $T - 2g = 2a$	A1	1.1		
		$32-2g=5a \Rightarrow a=$	Dep*M1	1.1	Attempt to solve simultaneous	
					equations	
		$a = 2.48 \text{ m s}^{-2}$	A1	2.2a	AG	Must show sufficient
						working to justify the
						given answer
					M1 A2 for $40-8-2g = 5a$ for M1	
					must have correct number of terms and	
					mass must be 5 not $5g$	
			[5]			
11	(b)	T - 2g = 2(2.48)	M1	3.4	Substitute given value of <i>a</i> into either	Must include correct
					equation	number of terms – use
						of weight for mass is
						M0
		$T = 24.56 \mathrm{N}$	A1	1.1	cao	Allow 24.6
11		2.49/0.5	[2]	2.4		1.04
11	(c)	v = 2.48(0.5)	BI	3.4	Speed after 0.5 seconds	1.24
		$s = 0.5(2.48)(0.5)^2$	B1	3.4	Distance travelled in this time	0.31
			M1	3.1b	Applying $s = ut + 0.5at^2$ correctly —	M0 if not using
					allow sign errors	relevant displacement
		$(2 + 0.21) = 1.244 + 0.5(0.8)4^{2}$	A1	1.1		
		-(2+0.51)=1.24t-0.5(9.8)t				
		$t = 0.825 \mathrm{s}$	A1	2.2a	BC	0.8246986

Qu	iestion	Answer	Marks	AO	Guidance	
11	(c)	ALT				
		v = 2.48(0.5)	B1		Speed after 0.5 seconds	1.24
		$s = 0.5(2.48)(0.5)^2$	B 1		Distance travelled in this time	0.31
		$0 = 1.24^2 - 2(9.8)s \ (\Rightarrow s = 0.0784)$	M1		Complete method to calculate the time	
		$2 + 0.31 + 0.0784 = 0.5(9.8)t_1^2$			down	
		$t_1 = 0.698$	A1		Correct value for time down	
		t = 0.825	A1			
			[5]			
11	(d)	$v^2 = 1.24^2 + 2(-9.8)(-2.31)$	M1	3.3	Applying $v^2 = u^2 + 2as$ correctly with	M0 if not using total
					their 1.24 and 2.31 or any other	time or relevant
					complete method	displacement
		$v = 6.84 \text{ m s}^{-1}$	A1	1.1	Allow 6.85	6.8420464
			[2]			
11	(e)	19.6 N	B1	3.4	Accept 2g	
			[1]			
11	(f)	e.g. include a more accurate value for g	B1	3.5c		
		e.g. include a variable resistance in the model rather than				
		a constant				
		e.g. include the dimension of the pulley in the model so				
		that the string is not parallel to the table				
		e.g. include a frictional force at the pulley				
			[1]			

OCR (Oxford Cambridge and RSA Examinations) The Triangle Building Shaftesbury Road Cambridge CB2 8EA

OCR Customer Contact Centre

Education and Learning

Telephone: 01223 553998 Facsimile: 01223 552627 Email: <u>general.gualifications@ocr.org.uk</u>

www.ocr.org.uk

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