GCE

Mathematics A

H240/03: Pure Mathematics and Mechanics

A Level

Mark Scheme for June 2022

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question includes the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
   - When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.
   - When a value is **not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
     NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

   Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
   Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g. Rules for replaced work and multiple attempts:
   - If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
   - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
   - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			$x = 6$	**B1**	1.1	**B0** for $\lvert x \rvert = 6$ unless replaced with $x = 6$	
			$\lvert 2x-3 \rvert = 9 \Rightarrow 2x-3 = -9$	**M1**	1.1a	oe e.g. $(2x-3)^2 = 81$ or $2x-3 = \pm 9$	**M0** for $(2x-3)^2 = 9$
			$x = -3$	**A1**	1.1	**A0** for $\lvert x \rvert = -3$ unless replaced with $x = -3$ The correct answer implies the previous **M** mark	Two marks maximum if more than two answers given
				[3]			

Question			Answer	Marks	AO	Guidance	
2	(a)		Translation	B1	2.5	Do not accept shift, move, transformation, etc. for first **B1**	
			-8 units parallel to the y-axis or $\begin{pmatrix}0\\-8\end{pmatrix}$	B1	1.1	Correct description e.g. correct vector (not as a coordinate), '8 units down'. **Do not allow second B1 after incorrect type of transformation e.g. stretch/rotation etc. but allow after shift/move etc.** Condone lack of 'units' but **do not accept** 'factor -8', '8 <u>places/spaces/steps</u> down' etc. If more than a single transformation, then no marks (unless two translations equivalent to the correct answer)	For 'parallel to the y axis' allow 'vertically', 'in the y direction'. **Do not accept** across/up/along/to/in/towards the y axis' **Mark vector before description/words**
				[2]			
2	(b)		$y = x^3 - 8 \Rightarrow y + 8 = x^3$ $x = \ldots$	M1	1.1	Attempt to make x the subject (allow sign errors only)	May use either f(x) or y
			$\mathrm{f}^{-1}(x) = (x+8)^{\frac{1}{3}}$	A1	1.1	Must be in terms of x. Allow the expression only e.g. $(x+8)^{\frac{1}{3}}$, $\sqrt[3]{x+8}$, oe	Ignore what this expression is equated to
				[2]			
2	(c)		One graph is the reflection of the other graph in the line $y = x$	B1	1.2	Must include both 'reflection' or 'mirror image' or 'mirrored' **and** '$y = x$'	**B0** for 'symmetrical' unless clearly describing reflective symmetry
				[1]			

Question			Answer	Marks	AO	Guidance	
3			$\left(\frac{2+3}{2}, \frac{-5+1}{2}\right)$	**M1***	2.1	Attempt at the midpoint (3 out of 4 values used correctly **or** one correct coordinate)	If correct $\left(\frac{5}{2}, -2\right)$
			$d^2 = (2-3)^2 + (-5-1)^2$	**M1***	1.1	Attempt to calculate diameter d or radius r (or the square of either of these two) e.g. $r^2 = \left(2-\frac{5}{2}\right)^2 + \left(-5-(-2)\right)^2$ **M0** if implying that $r^2 = (2-3)^2 + (-5-1)^2$. If the value is not labelled (as either r or d) then consider how this value is used in their circle equation	3 out of 4 values used correctly – if correct $d^2 = 37$ or $r^2 = \frac{37}{4}$
			$\left(x-\frac{5}{2}\right)^2 + (y+2)^2 = \frac{37}{4}$	**M1dep***	1.1	Correct form for the equation of a circle with their values for the centre and radius	Dependent on both previous **M** marks
			$x^2 - 5x + \frac{25}{4} + y^2 + 4y + 4 = \frac{37}{4}$				
			$x^2 + y^2 - 5x + 4y + 1 = 0$	**A1**	2.2a	Must have = 0 Order of terms on the lhs irrelevant	a, b and c need not be explicitly stated
				[4]			

Question			Answer	Marks	AO	Guidance	
4	(a)		GP: $x,\ \frac{15}{y},\ z \Rightarrow \frac{\frac{15}{y}}{x} = \frac{z}{\frac{15}{y}}$	M1*	3.1a	**M1** for un-simplified correct use of $r = \frac{u_n}{u_{n-1}}$ so allow this mark for stating that $r = \frac{15}{xy}$ or $r = \frac{yz}{15}$ or $r^2 x = z$ oe	Or for the terms of the GP as $y+4,\ \frac{15}{y},\ y-4$ in y only
			AP: $x, y, z \Rightarrow y - x = -4$ or $z - y = -4$	M1*	1.1	**M1** for $y - x = \pm 4$ or $z - y = \pm 4$ or $z - x = \pm 8$ oe For reference: $xy^2 z = 225$, $x = y + 4$ and $z = y - 4$	Or for the terms of the AP as $y+4,\ y,\ y-4$ in y only
			$(y+4)y^2(y-4) = 225$	M1dep*	1.1	Eliminate x and z to form an equation in y only (must be equivalent to a quartic in y)	Or for $\frac{y-4}{\frac{15}{y}} = \frac{\frac{15}{y}}{y+4}$ $\left(\Rightarrow y^2 - 16 = \frac{225}{y^2}\right)$
			$y^2(y^2 - 4y + 4y - 16) = 225 \Rightarrow y^4 - 16y^2 - 225 = 0$	A1	2.2a	**AG** so sufficient working must be shown www – note that $y = x + 4$ and $y = z - 4$ (from $d = +4$) can lead to the correct equation (which can score the **M** marks only)	
				[4]			

Question			Answer	Marks	AO	Guidance	
4	(b)		$y^4 - 16y^2 - 225 = 0 \Rightarrow y = \pm 5$ but $y > 0 \Rightarrow y = 5$	B1	1.1	**BC** Allow implied e.g. $y = \pm 5$ then using $y = 5$ only (**B0** if the sum to infinity for other values of y are not rejected)	$x = 9, z = 1$
			$\Rightarrow r^2 = \frac{z}{x} = \frac{1}{9} \Rightarrow r = \frac{1}{3}$	M1	1.1	Calculate r (soi) corresponding to $y = 5$ allow unsimplified e.g. $r = \frac{(5)(1)}{15}$ and allow if more than one value of y stated	Possibly done implicitly in formula for S_∞
			$S_\infty = \frac{x}{1-r} = \frac{9}{\frac{2}{3}}$	M1	1.1	Using the correct formula for the sum to infinity of a GP with their value of x (= their $y \pm 4$) and a value of r where $-1 < r < 1$	
			$S_\infty = 13.5$	A1	1.1	cao oe eg $\frac{27}{2}$ - do not award this mark if more than one value for $S_¥$ stated	**A0** for a triple-decker fraction e.g. $\frac{9}{\frac{2}{3}}$
				[4]			

Question			Answer	Marks	AO	Guidance	
5	(a)		**DR**				
			$y=(2x-3)\left(4x^2+1\right)^{-1}$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=2\left(4x^2+1\right)^{-1}+(2x-3)(-1)\left(4x^2+1\right)^{-2}(8x)$	**M1***	1.1	Attempt use of quotient rule or equivalent (e.g. product rule). Condone one incorrect term only (of the five terms) but must be subtraction in the numerator (but allow subtraction the wrong way round); condone absence of brackets; no denominator (if using quotient rule) is **M0**	By the five terms we mean the four in the numerator and the fifth is the term in the denominator
			$y=\frac{2x-3}{4x^2+1}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left(4x^2+1\right)(2)-(2x-3)(8x)}{\left(4x^2+1\right)^2}$	**A1**	1.1	cao must include brackets as necessary	Any correct equivalent form
			$\frac{2\left(1+12x-4x^2\right)}{\left(4x^2+1\right)^2}=2$	**M1dep***	3.1a	Sets their derivative (in any form) equal to 2 (**M0** if equating to normal gradient)	May equate at any stage (even after incorrect manipulation of their derivative)
			$1+12x-4x^2=\left(4x^2+1\right)^2\Rightarrow 16x^4+12x^2-12x=0$	**M1**	1.1	Multiply both sides by $\left(4x^2+1\right)^2$ and simplify (so combining like terms) to obtain a quartic equation (must be expanded with at least three terms – condone lack of = 0 if all terms on the same side) – allow sign errors/minor slips but the expansion of $\left(4x^2+1\right)^2$ must be three terms of the form $16x^4+ax^2+1$ where $a=\pm4,\pm8$	**Dependent on both previous M marks**
			$x\left(4x^3+3x-3\right)=0\Rightarrow 4x^3+3x-3=0$ as $x\neq 0$	**A1**	2.3	**AG** with explicit rejection of $x = 0$ – as a minimum must indicate that x cannot equal 0	Just cancelling x is **A0**
				[5]			

Question			Answer	Marks	AO	Guidance	
5	(b)		**DR** Consider both $\mathrm{f}(0.5)$ and $\mathrm{f}(1)$ Where $\mathrm{f}(x) = \pm(4x^3 + 3x - 3)$	M1	1.1	Working or correct answer for one value is sufficient evidence of correct method but both 0.5 and 1 must be seen	Just stating that $\mathrm{f}(0.5) < 0$ and $\mathrm{f}(1) > 0$ is **M0**
			$\mathrm{f}(0.5) = -1 < 0$ and $\mathrm{f}(1) = 4 > 0$ (or $\mathrm{f}(0.5) = 1 > 0$ and $\mathrm{f}(1) = -4 < 0$) Change of sign indicates that the *x*-coordinate lies between 0.5 and 1	A1 **[2]**	2.4	Correct values together with explanation (change of sign) and correct conclusion (as a minimum 'root' oe)	
			Alternative				
			Considers both $\mathrm{g}(0.5)$ and $\mathrm{g}(1)$ where $\mathrm{g}(x) = \dfrac{(4x^2+1)(2)-(2x-3)(8x)}{(4x^2+1)^2}$	M1		Must be using the correct derivative. Working or correct answer for one value is sufficient evidence of correct method but both 0.5 and 1 must be seen	Just stating that $\mathrm{g}(0.5) > 2$ and $\mathrm{g}(1) < 2$ is **M0**
			$\mathrm{g}(0.5) = 3 > 2$ and $\mathrm{g}(1) = 0.72 < 2$ Values either side of 2 indicates that the *x*-coordinate lies between 0.5 and 1	A1		Correct values together with explanation (values either side of 2) and correct conclusion (as a minimum 'root' oe)	
				[2]			

Question			Answer	Marks	AO	Guidance	
5	(c)		**DR**				
			Let $h(x)=\frac{3-4x^3}{3} \Rightarrow h'(x)=-4x^2$	**B1***	**2.1**	Calculates correct derivative of rhs of given iterative formula	
			As the root α lies in the interval (0.5, 1) $\Rightarrow h'(\alpha)<-1$ so iterative formula cannot converge to the *x*-coordinate of *P*	**B1dep***	**2.2a**	Correct explanation that any value in the given interval gives a gradient which is less than –1	**No marks** for just showing that the iteration doesn't converge using different starting values
				[2]			
5	(d)		**DR**				
			$f(x_n)=4x_n^3+3x_n-3 \Rightarrow f'(x_n)=12x_n^2+3$	**B1**	**1.1**	Correct derivative (possibly seen in N-R formula)	Condone x for x_n oe
			$x_{n+1}=x_n-\left\{\frac{4x_n^3+3x_n-3}{12x_n^2+3}\right\}$	**M1**	**2.1**	Correct N-R formula seen with correct $f(x_n)$ and their $f'(x_n)$ substituted	Condone x for x_n oe
			$x_0=0.5,\quad x_1=\frac{2}{3}$ or 0.666666..., $x_2=\frac{29}{45}$ or 0.644444..., ($x_3=0.64395510...$)	**A1**	**1.1**	First two iterations correctly stated to at least 5 decimal places (or exact) (truncated or rounded)	The correct first two iterations can imply **B1 M1**
			x coordinate of *P* is 0.64395	**A1**	**2.2a**	Independent of previous **A** mark (but must have scored **B1 M1**) – must be stated to exactly **5** decimal places	This **A** mark does not imply the previous **A** mark
			y coordinate of *P* is –0.64395	**B1**	**1.1**	Independent of all previous marks – must be stated to exactly **5** decimal places	The correct answers with no evidence of N-R (e.g. no iterations stated **and** no N-R formula) then **B0M0A0A0B1** max.
				[5]			

6			**DR**	M1	2.1	**M1** for $k(2x+9)^{\frac{3}{2}}$ with non-zero k	$k \neq 1$
			$\int (2x+9)^{\frac{1}{2}}\,dx = \frac{1}{3}(2x+9)^{\frac{3}{2}}$	A1	1.1	cao (allow unsimplified)	
			$\left[\frac{(2x+9)^{\frac{3}{2}}}{3}\right]_{-\frac{9}{2}}^{0} = 9$	A1	1.1	Uses correct limits (or implies correct limits) to get 9. Condone limits the wrong way round leading to -9 but must be changed to +9	
			$4e^{-2x} - 1 = 0 \Rightarrow e^{-2x} = \frac{1}{4}$ $-2x = \ln\left(\frac{1}{4}\right)$	M1*	3.1a	Attempt to solve $4e^{-2x} - 1 = 0$ by correctly taking logs of both sides leading to $\pm 2x = \pm \ln \alpha$ where $\alpha > 0$	Allow sign errors and other minor slips only
			$x = -\frac{1}{2}\ln\left(\frac{1}{4}\right)$	A1	1.1	Or equivalent exact value (soi possibly by correct exact value used later)	e.g. $\frac{1}{2}\ln 4$ or $\ln 2$
			$\int \left(4e^{-2x} - 1\right)dx = -2e^{-2x} - x$	M1*	1.1	Integrate $4e^{-2x} - 1$ to obtain $ce^{-2x} \pm x$	Where c is non-zero and $c \neq 4$
			$\int_0^{\frac{1}{2}\ln 4}\left(4e^{-2x} - 1\right)dx = \left(-2e^{-\ln 4} - \frac{1}{2}\ln 4\right) - (-2)$	M1dep*	1.1	Uses limits correctly $F(\frac{1}{2}\ln 4) - F(0)$ (with their $\frac{1}{2}\ln 4$) – **dependent on the previous two M marks** (allow non-exact top limit). Condone limits the wrong way round only if the sign of their answer is subsequently changed	If zero limit is assumed to give 0 (with no working) then **M0**
			Area $= 9 + \frac{3}{2} - \frac{1}{2}\ln 4 = \frac{21}{2} - \frac{1}{2}\ln 4 = \frac{21}{2} - \ln 2$	A1	2.2a	**If the values of the integral(s) are changed from negative to positive (e.g. from limits the wrong way round) with no justification given then A0**	p and q need not be explicitly stated. $p = \frac{21}{2}$ (oe) and $q = -1$
				[8]			

7	(a)		**DR** $m\sec\theta + 3\cos\theta = 4\sin\theta$ $\left(\Rightarrow m\sec\theta + \frac{3}{\sec\theta} = 4\sin\theta\right)$ $m\sec^2\theta + 3 = 4\sin\theta\sec\theta$	**M1**	2.1	Or for $m\sec\theta + 3\cos\theta = 4\sin\theta$ $\left(\Rightarrow \frac{m\sec\theta}{\cos\theta} + 3 = 4\frac{\sin\theta}{\cos\theta}\right)$ $m\sec^2\theta + 3 = 4\tan\theta$ The first **M** mark is for a valid method arriving at a three term equation containing $\sec^2\theta$	**Squaring each individual term of the original equation scores no marks**
			$m\left(1 + \tan^2\theta\right) + 3 = 4\tan\theta$	**M1**	1.1	Correctly uses the identity $1 + \tan^2\theta \equiv \sec^2\theta$ to obtain an equation in $\tan\theta$ only	
			$m + m\tan^2\theta + 3 = 4\tan\theta$ $\Rightarrow m\tan^2\theta - 4\tan\theta + (m+3) = 0$	**A1** **[3]**	2.2a	**AG** so sufficient working must be shown	**A0** if angle missing from any trigonometric terms

7	(b)		**DR**				
			$\Delta = (-4)^2 - 4m(m+3)$	**M1***	**3.1a**	Considers discriminant of given quadratic equation in tan (*c* must be two terms) to get an expression in *m* only. **M0** for embedded discriminant in quadratic formula unless explicitly considered	Allow $4^2 - 4m(m+3)$
			As the quadratic equation in tan has only one solution for θ in the given interval (and as the range of tan in the given interval is all non-zero real values) this implies that the given equation must only have one real root and therefore $(-4)^2 - 4m(m+3) = 0 \Rightarrow m^2 + 3m - 4 = 0$	**M1dep***	**1.1**	Sets their discriminant equal to zero and obtains an expanded three-term quadratic in *m*	**Reasoning for setting the discriminant equal to zero is not required for this mark**
			$(m+4)(m-1) = 0 \Rightarrow m = -4$ only as *m* is a negative integer	**A1**	**1.1**	State or imply $m = -4$ **only**	
			$m = -4 \Rightarrow (2\tan\theta + 1)^2 = 0$ so $\tan\theta = -0.5$	**M1**	**1.1**	Uses their negative integer value of *m*, and solves the equivalent of their three term quadratic equation in tan, to obtain (at least) $\tan\theta = k$ - **dependent on both previous M marks** Allow $-4\tan^2\theta - 4\tan\theta - 1 = 0$ $\Rightarrow \tan\theta = -0.5$ for this mark	where *k* is non-zero. **If no method shown for solving their quadratic, then award this mark if the solution is correct for their quadratic**
			$\theta = 2.68$ (3 sf)	**A1** **[5]**	**2.4**	For full marks must explain why the discriminant should be set equal to zero – must say that as there is only one value of θ or tan $\theta \Rightarrow \Delta = 0$ (as a minimum must see explicit mention of 'one' together with 'θ' or 'tanθ' for this mark). Allow awrt 2.68	2.677945045...

8			$T\cos 15 = 60$	**M1**	**1.1**	Resolve horizontally, correct number of terms, allow sin/cos confusion (e.g. $T\sin 15 = 60$ scores **M1** but $T = 60\cos 15$ scores **M0**)	Where T is the tension in the rope
			$T = 62.1$ (N)	**A1**	**1.1**	awrt 62.1 – allow the exact answer of $60\left(\sqrt{6}-\sqrt{2}\right)$	62.11657082…
				[2]			

9	(a)		$8 = 20 + 30a$ $a = -0.4$ so deceleration is 0.4 (m s^{-2})	M1 A1 [2]	3.4 1.1	Use of $v = u + at$ with given values (allow $v = 20$ and $u = 8$) Allow 0.4 or –0.4	
9	(b)		Distance travelled by *B*: $12t$ Distance travelled by *A*: $\frac{1}{2}(8+20)(30)+8(t-30)$ or $\frac{1}{2}(30)(12)+8t$ $12t = '420' + 8t - '240'$ $t = 45$	B1 B1 M1 A1 [4]	1.1 1.1 3.1b 1.1	Or first 30 seconds: *B* travels 12(30) (= 360) Or first 30 seconds: *A* travels $0.5(8+20)(30)$ $(=420)$ M1 for a (possibly unsimplified) correct equation/inequality in *t* where '360' and '420' must have been correct unsimplified e.g. $'420' - '360' = (t-30)(12-8)$, $12(t-30) + '360' = 8(t-30) + '420'$, $'420' + 8(t-30) = 12t$ etc. Or for a correct equation in another time variable e.g. $12T + '360' = 8T + '420'$ **and** $T = t - 30$ seen or implied Allow $t > 45 \Rightarrow t = 45$	If an inequality used, then allow incorrect direction or strict inequality symbol for this mark

10	**(a)**		$3g-16.8=3a$	**M1**	**3.3**	N2L for Q – correct number of terms with the correct mass. Condone sign errors	**M0** if using $3g$ for the mass but allow g missing from the net force
			$a=\dfrac{3g-16.8}{3}=4.2\,(\mathrm{m\,s^{-2}})$	**A1** **[2]**	**1.1**		
10	**(b)**		$16.8-F_P=2.5(4.2)$	**M1***	**3.3**	N2L for P horizontally using T = 16.8 and their a (but not ± 9.8) from **(a)** – allow sign errors but must have correct number of terms and correct mass ($\neq 3$) – if correct $F_P=6.3$	F_P is the friction between P and B
			$16.8-2.5(4.2)=\mu(2.5g)$	**M1dep***	**3.4**	Use of $F=\mu R$ for P with $R=2.5g$	
			$\mu=0.257$	**A1** **[3]**	**1.1**	awrt 0.257 – condone working with $F\leqslant\mu R$ provided that the value of μ is explicitly stated (and not left in an inequality)	0.257142857… allow $\dfrac{9}{35}$
10	**(c)**		$R_B=2.5g+Mg$	**M1***	**3.1b**	Resolving vertically for B – correct number of terms, allow sign errors and condone g's missing	Where M is the mass of B
			$6.3\leqslant\dfrac{5}{49}(2.5g+Mg)$	**M1dep***	**3.4**	Use of $F\leqslant\mu R$ or $F=\mu R$ with correct R and $\mu=\dfrac{5}{49}$ with F being their F_P from **(b)** where $F_P\neq 16.8$	No g's missing for this mark
			$M\geqslant 3.8$ so least possible value for the mass of B is 3.8 (kg)	**A1** **[3]**	**2.2a**	3.8 - allow use of '=' throughout this part	No justification required

11	**(a)**			**M1**	**3.1b**	Taking moments about A – correct number of terms, allow sin/cos confusion and sign errors – may take moments about another point (and resolve) but must end up after elimination with an equation in T and x only	Dimensionally correct. Must be xT and not $xT\cos(...)$ or $xT\sin(...)$
			$4g\left(\frac{3}{2}\cos 60\right)+g\left(3\cos 60\right)=xT$	**A1**	**1.1**	Correct equation in g, x and T only – condone g replaced by 9.8	T is the tension in the string
			$T=\dfrac{9g}{2x}$ (N)	**A1**	**2.2a**	An answer of $\frac{44.1}{x}$ (or with trigonometric terms) is **A0** unless correct answer in terms of g seen	oe exact answers in terms of g and x (condone correct triple decker fractions)
				[3]			

For reference for parts **(a)** and **(b)**:

Moments about C: $R_A(x\cos 60)+g(3-x)\cos 60=4g(x-1.5)\cos 60+F_A(x\sin 60)$

Moments about B: $T(3-x)+R_A(3\cos 60)=4g(1.5\cos 60)+F_A(3\sin 60)$

Moments about midpoint of AB: $R_A(1.5\cos 60)+g(1.5\cos 60)=T(x-1.5)+F_A(1.5\sin 60)$

Resolving perpendicular to AB: $T+R_A\cos 60=4g\cos 60+g\cos 60+F_A\sin 60$

Resolving parallel to AB: $R_A\sin 60+F_A\cos 60=4g\sin 60+g\sin 60$

11	(b)			M1	3.3	Resolve vertically or horizontally – correct number of terms with the tension at C in terms of cos/sin, condone sign errors, allow sin/cos confusion but forces that require resolving must be (and correspondingly those that don't require resolving e.g. the weights if resolving vertically, should not be resolved)	Or obtain an equation in F_A and/or R_A in terms of T (or their T) only (see list of equations below)
			$T\cos 60 + R_A = 4g + g \quad \left(\Rightarrow R_A = 5g - \frac{9g}{4x}\right)$ $F_A = T\sin 60 \left(\Rightarrow F_A = \frac{9\sqrt{3}g}{4x}\right)$	A1	1.1	Both correct (unsimplified) – allow with T or their (possibly incorrect) T (oe eg two valid equations in R_A and F_A)	R_A is the normal contact force at A F_A is the frictional contact force at A
			$\frac{9\sqrt{3}g}{4x} = \frac{9\sqrt{3}}{35}\left(5g - \frac{9g}{4x}\right)$	M1dep*	3.4	Use of $F = \mu R$ with correct μ to form an equation in x only – no forces missing from their R_A and F_A and all required forces resolved accordingly or not e.g. if resolving vertically the two weights should not contain sin/cos	
			$x = 2.2$	A1 [4]	2.2a	awrt 2.2	www

For reference for parts **(a)** and **(b)**:

Moments about C: $R_A(x\cos 60) + g(3 - x)\cos 60 = 4g(x - 1.5)\cos 60 + F_A(x\sin 60)$

Moments about B: $T(3 - x) + R_A(3\cos 60) = 4g(1.5\cos 60) + F_A(3\sin 60)$

Moments about midpoint of AB: $R_A(1.5\cos 60) + g(1.5\cos 60) = T(x - 1.5) + F_A(1.5\sin 60)$

Resolving perpendicular to AB: $T + R_A\cos 60 = 4g\cos 60 + g\cos 60 + F_A\sin 60$

Resolving parallel to AB: $R_A\sin 60 + F_A\cos 60 = 4g\sin 60 + g\sin 60$

12	(a)		$\mathbf{v} = (1-2t)\mathbf{i} + (2t^2 + t - 13)\mathbf{j}$ If P is stationary, then $1-2t=0$ and $2t^2+t-13=0$ $\mathbf{i}: 1-2t=0 \Rightarrow t=\frac{1}{2}$ $\mathbf{j}: 2t^2+t-13=0 \Rightarrow t=2.3117..., -2.8117...$ No value of t is common to both components, so P is never stationary	M1 A1 [2]	3.1b 2.2a	Considers **either** the **i or j** component equal to zero **or** forms a five-term quartic equation for $\|\mathbf{v}\|^2 = 0$ (oe) $(4t^4 + 4t^3 - 47t^2 - 30t + 170 = 0)$ **BC** – need not see the negative value of t **or** for substituting $t = 0.5$ into quadratic expression for **j** and showing this gives a non-zero answer (oe) with correct working **and** conclusion	**A1** for the **correct** quartic equation with roots stated as $2.3 \pm 0.35i$, $-2.8 \pm 0.65i$ + correct conclusion
12	(b)		$\Rightarrow \mathbf{a} = -2\mathbf{i} + (4t+1)\mathbf{j}$ (m s^{-2})	B1 [1]	1.1	Correct derivative – or as a column vector	Brackets must be around the $4t + 1$

12	(c)		$-2\left(2t^2+t-13\right)=1\left(1-2t\right)$	M1*	3.1b	Setting up a quadratic equation in t only – allow sign errors (including on the 1 and 2) and the 1 and –2 on the wrong side	Or multiples of 1 and –2
			$4t^2-25=0 \quad \Rightarrow t=2.5$	M1dep*	1.1	Solves their (two or three term) quadratic and selects their positive value of t	**Check unsupported solutions if incorrect quadratic equation**
			$\mathbf{F}=m\mathbf{a} \Rightarrow \mathbf{F}=0.5\{-2\mathbf{i}+\left(4t+1\right)\mathbf{j}\}$	M1*	3.4	Substitute their **a** into **F** = 0.5**a** or their $\lvert\mathbf{a}\rvert$ into $\lvert\mathbf{F}\rvert=0.5\lvert\mathbf{a}\rvert$. If **F** not stated in terms of t then one component must be correct following through from their **a** (and possibly t)	Must use correct value of 0.5 for m but can be in terms of t
			$\lvert\mathbf{F}\rvert=\sqrt{(-1)^2+5.5^2}$	M1dep*	1.1	Dependent on previous **M** mark only	From a value of $t>0$
			$\lvert\mathbf{F}\rvert=5.59$ (N)	A1 [5]	1.1	awrt 5.59 (exact: $\frac{5\sqrt{5}}{2}$)	5.590169…

12	(d)		$\mathbf{s}=\left(t-t^2\right)\mathbf{i}+\left(\frac{2}{3}t^3+\frac{1}{2}t^2-13t\right)\mathbf{j}(+\mathbf{c})$	M1*	1.1	Integrates **v** wrt t – at least three terms correct	Allow without +**c**
			$t=1, \mathbf{s}=\frac{1}{6}\mathbf{j} \Rightarrow \mathbf{c}=(0\mathbf{i}+)12\mathbf{j}$	A1	3.4	Uses given conditions to find correct **c** – dependent on a completely correct integrated expression for **s**	www
			When t = 1.5, $\mathbf{s}=-\frac{3}{4}\mathbf{i}-\frac{33}{8}\mathbf{j}$	M1dep*	1.1	Substitute t = 1.5 into their **s**	
			$tan^{-1}\left(\frac{\pm 3/4}{\pm 33/8}\right)$ or $tan^{-1}\left(\frac{\pm 33/8}{\pm 3/4}\right)$	M1	3.1b	Attempt to find a relevant angle using the components of their **s** (allow use of sin/cos with the magnitude of **s**)	**Dependent on both previous M marks.** Written in terms of arctan is sufficient
			Bearing = $180+\tan^{-1}\left(\frac{3/4}{33/8}\right)$ = 190°	A1	3.2a	awrt 190 (or from $270-tan^{-1}\left(\frac{33/8}{3/4}\right)$)	190.3048465…
				[5]			

13	(a)		$x = Ut$ $y = Vt - \frac{1}{2}gt^2$	M1*	3.3	Setting up expressions for x and y using $s = ut + \frac{1}{2}at^2$ with $a = 0$ in x and $\pm g$ in y oe. **M0** if using U instead of V vertically	Allow sign errors. May use $t = x / U$ in $y = Vt + \frac{1}{2}at^2$
			$y = V\left(\frac{x}{U}\right) - \frac{1}{2}g\left(\frac{x}{U}\right)^2$	M1dep*	3.4	Eliminating both t terms in y to get an equation in y, x, U, V (and possibly g)	
			$y = \frac{Vx}{U} - \frac{gx^2}{2U^2} \Rightarrow 2U^2y = 2UVx - gx^2$	A1	2.2a	**AG** so sufficient working must be shown	www
				[3]			
13	(b)		B passes through $\left(a, \frac{1}{2}a\right) \Rightarrow 2U^2\left(\frac{a}{2}\right) = 2UVa - ga^2$ $\left(\Rightarrow U^2 = 2UV - ga\right)$	B1	3.4	Substituting $\left(a, \frac{1}{2}a\right)$ into given result from **(a)**	
			B passes through $(4a, 0) \Rightarrow 2UV(4a) - g(4a)^2 = 0$ $(\Rightarrow UV - 2ga = 0)$	B1	3.1b	Substituting $(4a, 0)$ into given result from **(a) –** note that using $(3a, 0)$ is not a **MR**	
			$U = \sqrt{3ga},\ V = \frac{2\sqrt{ga}}{\sqrt{3}}$ or $2U = 3V$	M1*	2.1	Solve simultaneously (oe) to find either U or V (or their squares) in terms of a and g only, or for a linear equation (oe) in V and U only, if correct $2U = 3V$	oe e.g., if correct $V^2 = \frac{4}{3}ga$ and $U^2 = 3ga$
			$\tan\theta = \frac{V}{U} \Rightarrow \tan\theta = \frac{\frac{2}{\sqrt{3}}\left(\sqrt{ga}\right)}{\sqrt{3ga}}$	M1dep*	3.1b	Using $\tan\theta = \frac{V}{U}$ with their U and their V	
			$\tan\theta = \frac{2}{3} \Rightarrow \theta = 33.7°$ (3 sf)	A1	2.2a	awrt 33.7 (an answer of 36.9 from using $(3a, 0)$ scores (if from correct working) **B1 B0 M1 M1 A0**)	
				[5]			

13	(c)		$\sqrt{3ga + \frac{4}{3}ga} = 54.6$	M1	3.4	Using $\sqrt{U^2 + V^2} = 54.6$ to set up an equation in a (and possibly g)	
			$a = 70.2$	A1	1.1	www awrt 70.2	an answer of 97.344 (awrt 97.3) from using $(3a, 0)$ in **(b)** scores **M1 A0**
				[2]			
			Alternative 1				
			$54.6\cos(33.7...) = \sqrt{3ga}$ **or** $54.6\sin(33.7...) = \sqrt{\frac{4}{3}ga}$	M1		Setting either the horizontal component equal to their U (from **(b)**) or the vertical component equal to their V (from **(b)**). Allow sin/cos confusion	**M0** for an unsupported value of θ (if used)
			$a = 70.2$	A1		www awrt 70.2	
				[2]			
			Alternative 2				
			$a = \frac{UV}{2g} = \frac{54.6\cos(33.7...)' \; 54.6\sin(33.7...)}{2g}$	M1		Using their expression for a (possibly seen in **(b)**) in terms of U and V with 54.6 and their θ. **M0** for an unsupported value of θ	Allow sin/cos confusion
			$a = 70.2$	A1		www awrt 70.2	
				[2]			

13	(d)		$0 = V^2 - 2gH\left(\Rightarrow H = \dfrac{V^2}{2g}\right)$	M1*	3.3	Setting up the model using $v^2 = u^2 + 2as$ with $v = 0$ and $a = -g$	H is the maximum height of B
			$H = \dfrac{1}{2g}\left(\dfrac{4ga}{3}\right) = \dfrac{2}{3}a$	M1dep*	3.4	Using their expression for V from **(b)** to get an expression for H in terms of a (oe) e.g., $H = \dfrac{V^2}{2g}$ where $V = 54.6\sin\theta$ (allow $\cos\theta$) with their value of θ	**M0** for an unsupported value of θ
			$H = \dfrac{2}{3}(70.2) = 46.8$ (m)	A1 **[3]**	2.2a	awrt 46.8	an answer of 54.756 (awrt 54.8) from using ($3a$, 0) in **(b)** scores **M1M1A0**
			Alternative 1				
			$0 = V - gt \Rightarrow t = \dfrac{54.6\sin(33.7...)}{g}$	M1*		Find t at maximum height with $v = 0$, $a = -g$ and u = their $V \neq 54.6$ (allow sin/cos confusion). **M0** for an unsupported value of θ	$t = 3.090472522...$
			$H = (54.6\sin(33.7...))t - \frac{1}{2}gt^2$ $= (54.6\sin(33.7...))(3.09...) - \frac{1}{2}g(3.09...)^2$	M1dep*		Substituting their t into $s = Vt - \frac{1}{2}gt^2$	$V \neq 54.6$ (allow sin/cos confusion)
			$H = 46.8$ (m)	A1		awrt 46.8	
				[3]			
			Alternative 2				
			$x = 2a \Rightarrow 2U^2y = 4UVa - 4ga^2$	M1*		Setting $x = 2a$ or $1.5a$ and substituting into path equation from **(a)**	
			$2(54.6\cos(33.7))^2 H =$ $4(54.6\cos(33.7))(54.6\sin(33.7))(70.2) - 4g(70.2)^2$	M1dep*		Substituting their U, V and a to form an equation in H (and possibly g) only. **M0** for an unsupported value of θ	U and $V \neq 54.6$ (allow sin/cos confusion)
			$H = 46.8$ (m)	A1		awrt 46.8	
				[3]			

13	(e)		examples of possible refinements include • taking into account the size of *B* • taking into account that *B* is not a particle • taking into account the spin of *B* • taking into account the dimensions of *B* • taking into account the wind/weather • taking into account the thickness of the wall (which is assumed to be 'thin') • taking into account the clearance of the ball at the top of the wall	**B1** **[1]**	**3.5c**	Allow any correct refinement, including use of a more accurate value of g rather than the assumed 9.8 **B0** if referring to • the mass or weight or shape of *B* • the ground is unlikely to be horizontal • modelling the problem as three dimensional rather than two dimensional (unless specific detail given) • air resistance (only) If multiple refinements given, then all must be valid to score **B1**	

Method marks for solving/factorising quadratics of the form $ax^2 + bx + c$ - however note that candidates may solve quadratics on their calculators without showing any working so (unless the scheme says otherwise) unsupported answers from an incorrect quadratic equation need to be checked by examiners to award the corresponding M mark(s)

1. **Factorisation**: The modulus of the product of the first two terms in the brackets must equal $|a|x^2$. The modulus of the product of the second two terms in the bracket must equal $|c|$, e.g. $2x^2 + x - 6$ any of the following would score **M1**: $\pm 2x(x \pm 2) \pm 3(x \pm 2)$ $(\pm 2x \pm 3)(\pm x \pm 2)$
2. **Formula**: If the formula is written down correctly, then allow a maximum of one sign error in the substituted values for **M1**
3. **Completing the square**: Must get as far as halving the coefficient of x and dealing with the square of that term as well as moving the constant terms to one side ready to take the square root. Condone 1 sign error.

   e.g. $x^2 + 5x - 3 = 0 \Rightarrow \left(x + \frac{5}{2}\right)^2 = 3 \pm \left(\frac{5}{2}\right)^2$ would get **M1**
   e.g. $x^2 + 5x - 3 = 0 \Rightarrow \left(x + \frac{5}{2}\right)^2 = -3 + \left(\frac{5}{2}\right)^2$ would get **M1**

Need to get in touch?

If you ever have any questions about OCR qualifications or services (including administration, logistics and teaching) please feel free to get in touch with our customer support centre.

Call us on

01223 553998

Alternatively, you can email us on

support@ocr.org.uk

For more information visit

ocr.org.uk/qualifications/resource-finder

ocr.org.uk

Twitter/ocrexams

/ocrexams

/company/ocr

/ocrexams

CAMBRIDGE
UNIVERSITY PRESS & ASSESSMENT