OCR
Oxford Cambridge and RSA

Wednesday 6 October 2021 – Afternoon

AS Level Mathematics A

H230/01 Pure Mathematics and Statistics

Time allowed: 1 hour 30 minutes

8324534812

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Section A: Pure Mathematics

Answer **all** the questions.

1 In the triangle ABC, $AB = 3$, $BC = 4$ and angle $ABC = 30°$. Find the following.

(a) The area of the triangle. **[2]**

(b) The length AC. **[2]**

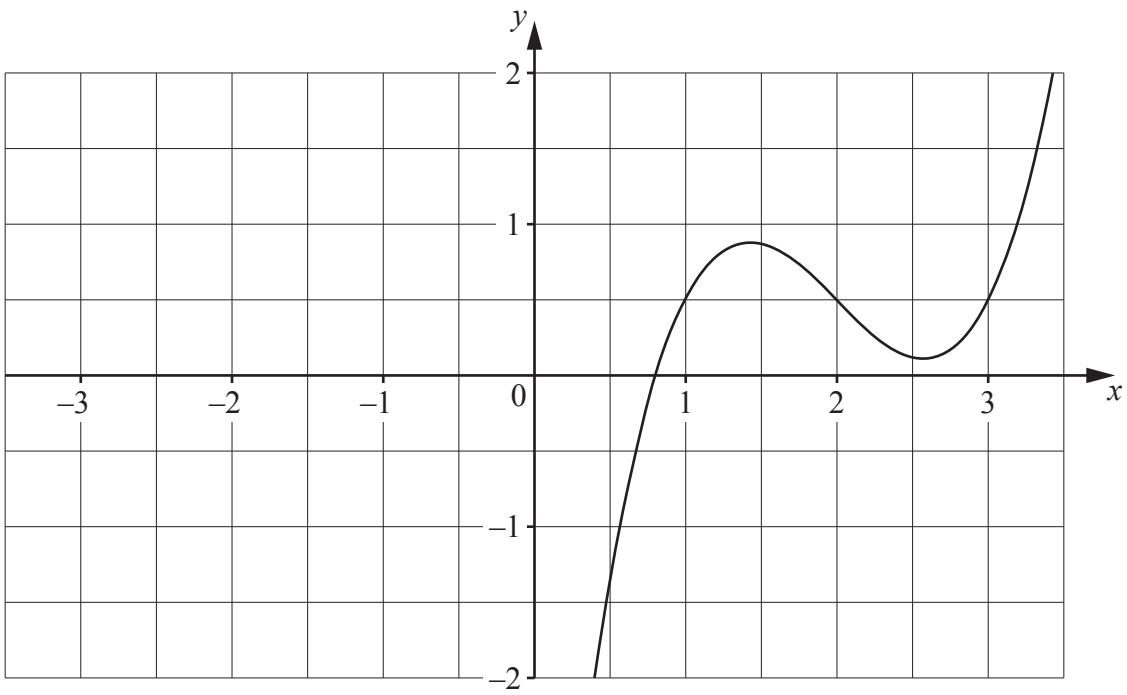
(c) The angle ACB. **[3]**

2 The number of people, n, living in a small town is changing over time. In an attempt to predict the future growth of the town, a researcher uses the following model for n in terms of t, where t is the time in years from the start of the research.

$n = 12\,500 + \frac{5000}{t}$, for $t \geqslant 1$

Find the rate of change of n when $t = 5$. **[4]**

3 The diagram shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is a cubic polynomial in x.
This diagram is repeated in the Printed Answer Booklet.



(a) State the values of x for which $\mathrm{f}(x) < \frac{1}{2}$, giving your answer in set notation. **[3]**

(b) On the diagram in the Printed Answer Booklet, draw the graph of $y = \mathrm{f}(-x)$. **[2]**

(c) Explain how you can tell that $\mathrm{f}(x)$ cannot be expressed as the product of three real linear factors. **[1]**

4 **(a)** Simplify $2\begin{pmatrix}6\\-3\end{pmatrix}-3\begin{pmatrix}-1\\2\end{pmatrix}$. **[2]**

(b) The vector **a** is defined by $\mathbf{a}=r\begin{pmatrix}6\\-3\end{pmatrix}+s\begin{pmatrix}-1\\2\end{pmatrix}$, where r and s are constants.

Determine two pairs of values of r and s such that **a** is parallel to the y-axis and $|\mathbf{a}|=3$. **[5]**

5 The fuel consumption of a car, C miles per gallon, varies with the speed, v miles per hour. Jamal models the fuel consumption of his car by the formula

$C=\frac{12}{5}v-\frac{3}{125}v^2$, for $0\leqslant v\leqslant 80$.

(a) Suggest a reason why Jamal has included an upper limit in his model. **[1]**

(b) Determine the speed that gives the maximum fuel consumption. **[4]**

Amaya's car does more miles per gallon than Jamal's car. She proposes to model the fuel consumption of her car using a formula of the form

$C=\frac{12}{5}v-\frac{3}{125}v^2+k$, for $0\leqslant v\leqslant 80$, where k is a positive constant.

(c) Give a reason why this model is **not** suitable. **[1]**

(d) Suggest a different change to Jamal's formula which would give a more suitable model. **[2]**

6 The power output, P watts, of a certain wind turbine is proportional to the cube of the wind speed $v\,\text{ms}^{-1}$.

When $v=3.6$, $P=50$.

Determine the wind speed that will give a power output of 225 watts. **[3]**

7 The relationship between the variables P and Q is modelled by the formula

$Q=aP^n$

where a and n are constants.

Some values of P and Q are obtained from an experiment.

(a) State appropriate quantities to plot so that the resulting points lie approximately in a straight line. **[1]**

(b) Explain how to use such a graph to estimate the value of n. **[2]**

8 **(a)** Prove that the following statement is **not** true.

p is a positive integer $\Rightarrow 2^p \geqslant p^2$ **[1]**

(b) Prove that the following statement **is** true.

m and n are consecutive positive odd numbers $\Rightarrow mn + 1$ is the square of an even number **[4]**

9 **In this question you must show detailed reasoning.**

Find the equation of the straight line with positive gradient that passes through (0, 2) and is a tangent to the curve $y = x^2 - x + 6$. **[6]**

Section B: Statistics

Answer **all** the questions.

10 Jane conducted a survey. She chose a sample of people from three towns, A, B and C. She noted the following information.

400 people were chosen.
230 people were adults.
55 adults were from town A.
65 children were from town A.
35 children were from town B.
150 people were from town B.

(a) In the Printed Answer Booklet, complete the two-way frequency table. **[2]**

	Town			
	A	B	C	Total
adult				
child				
Total				

(b) One of the people is chosen at random.

(i) Find the probability that this person is an adult from town A. **[1]**

(ii) Given that the person is from town A, find the probability that the person is an adult. **[1]**

For another survey, Jane wanted to choose a random sample from the 820 students living in a particular hostel. She numbered the students from 1 to 820 and then generated some random numbers on her calculator.

The random numbers were 0.114 287 562 and 0.081 859 817.

Jane's friend Kareem used these figures to write down the following sample of five student numbers.

114, 142, 428, 287 and 756

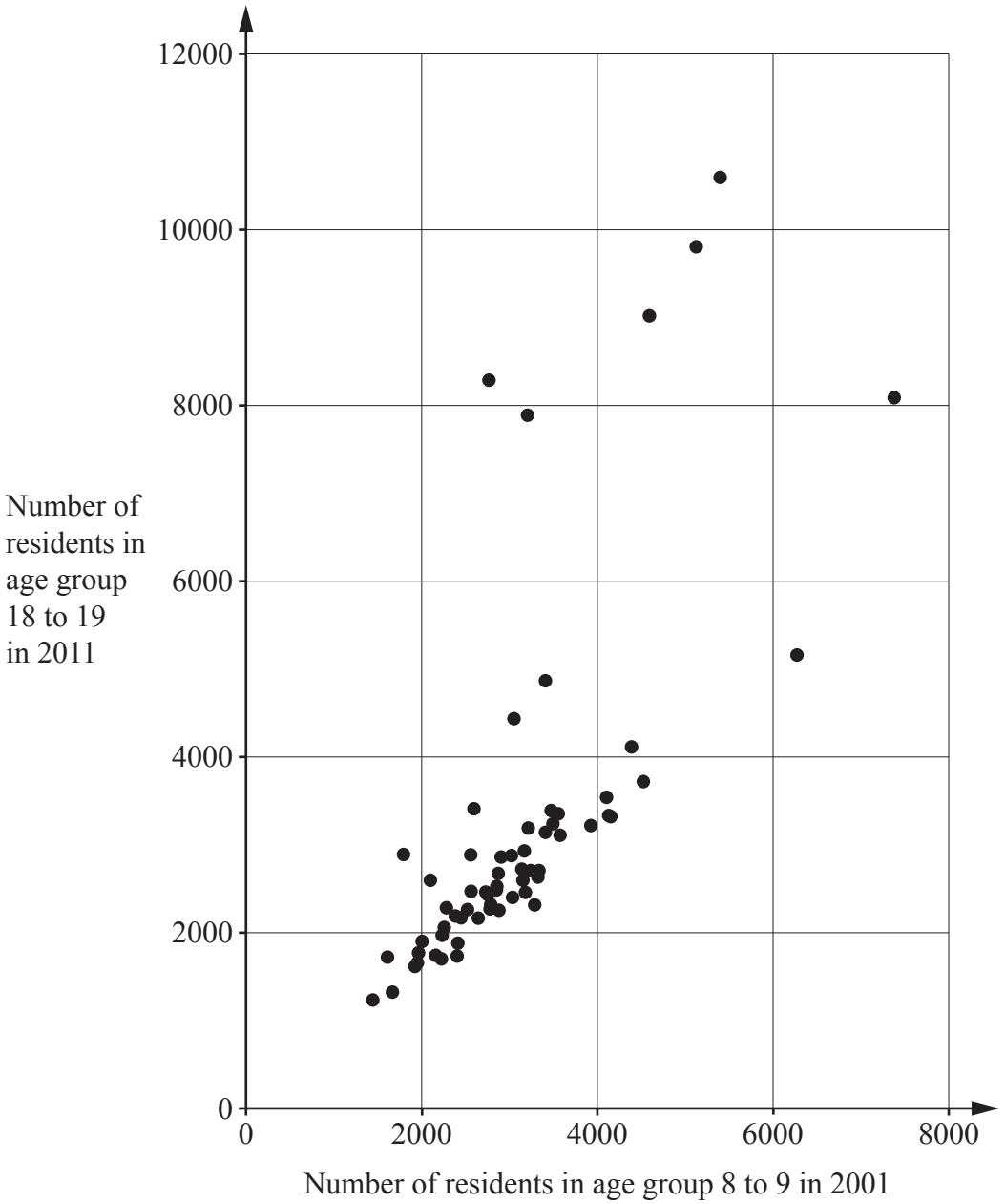
Jane used the same figures to write down the following sample of five student numbers.

114, 287, 562, 81 and 817

(c) (i) State, with a reason, which one of these samples is not random. **[1]**

(ii) Explain why Jane omitted the number 859 from her sample. **[1]**

11 A student is investigating changes in the number of residents in Local Authorities in the South-East Region between 2001 and 2011. The scatter diagram shows the number x of residents in these Local Authorities in the age group 8 to 9 in 2001 and the number y of residents in the same Local Authorities in the age group 18 to 19 in 2011.



(a) Suggest a reason why the student is comparing these two age groups in 2001 and 2011. **[1]**

The student notices that most of the data points are close to the line $y = x$.

(b) (i) Explain what this suggests about the residents in these Local Authorities. **[1]**

(ii) The student says that correlation does not imply causation, so there is no causal link between the values of x and the values of y.

Explain whether or not they are correct. **[1]**

(c) Some of these Local Authorities contain universities.

(i) On the diagram in the Printed Answer Booklet, circle three points that are likely to represent Local Authorities containing universities. **[1]**

(ii) Give a reason for your choice of points in part **(c)(i)**. **[1]**

Assume that the proportion of residents in age group 8 to 9 in 2001 was roughly the same in each Local Authority in the South-East. The Local Authority in this region with the largest population is Medway.

(d) On the diagram in the Printed Answer Booklet, label clearly with the letter *M* the point that corresponds to Medway. **[1]**

12 The variable X has the distribution B(50, $\frac{1}{6}$). The probabilities $\mathrm{P}(X = r)$ for $r = 0$ to 50 are given by the terms of the expansion of $(a+b)^n$ for specific values of a, b and n.

(a) State the values of a, b and n. **[1]**

A student has an ordinary 6-sided dice. They suspect that it is biased so that it shows a 2 on fewer throws than it would if it were fair. In order to test the suspicion the dice is thrown 50 times and the number of 2s is noted. The student then carries out a hypothesis test at the 5% significance level.

(b) Write down suitable hypotheses for the test. **[2]**

(c) Determine the rejection region for the test, showing the values of any relevant probabilities. **[4]**

13 **(a)** The probability distribution of a random variable X is shown in the table, where p is a constant.

x	0	1	2	3
$P(X = x)$	$\frac{1}{12}$	$\frac{1}{4}$	p	$3p$

Two values of X are chosen at random. Determine the probability that their product is greater than their sum. **[5]**

(b) A random variable Y takes n values, each of which is equally likely. Two values, Y_1 and Y_2, of Y are chosen at random.

It is given that $\mathrm{P}(Y_1 = Y_2) = 0.02$.

Find $\mathrm{P}(Y_1 > Y_2)$. **[2]**

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