AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

A-level MATHEMATICS

Paper 1

Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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2	
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TOTAL	

Answer **all** questions in the spaces provided.

1 A curve is defined by the parametric equations

$$x = \cos\theta \text{ and } y = \sin\theta \qquad \text{where } 0 \leq \theta \leq 2\pi$$

Which of the options shown below is a Cartesian equation for this curve?

Circle your answer.

[1 mark]

$\frac{y}{x} = \tan\theta$ $\quad$ $x^2 + y^2 = 1$ $\quad$ $x^2 - y^2 = 1$ $\quad$ $x^2y^2 = 1$

2 A periodic sequence is defined by

$$U_n = (-1)^n$$

State the period of the sequence.

Circle your answer.

[1 mark]

-1 $\quad$ 0 $\quad$ 1 $\quad$ 2

3 The curve

$$y = \log_4 x$$

is transformed by a stretch, scale factor 2, parallel to the y-axis.

State the equation of the curve after it has been transformed.

Circle your answer.

[1 mark]

$y = \frac{1}{2}\log_4 x$ $\quad$ $y = 2\log_4 x$ $\quad$ $y = \log_4 2x$ $\quad$ $y = \log_8 x$

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ANSWER IN THE SPACES PROVIDED

4 The graph of

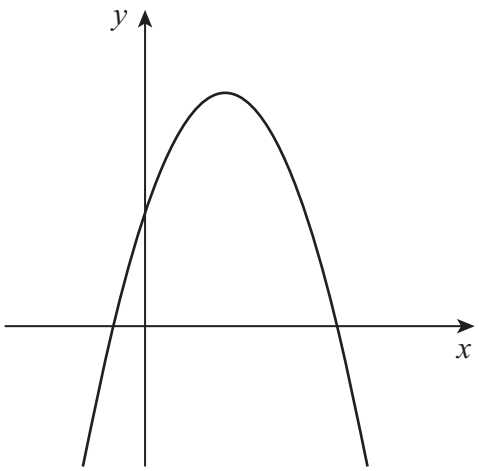
$$y = \mathrm{f}(x)$$

where

$$\mathrm{f}(x) = ax^2 + bx + c$$

is shown in **Figure 1**.

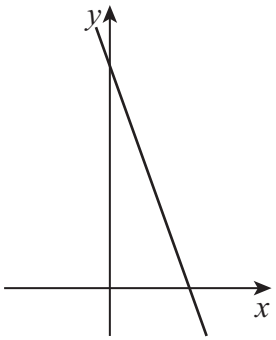
Figure 1



Which of the following shows the graph of $y = f'(x)$?

Tick (✓) **one** box.

[1 mark]



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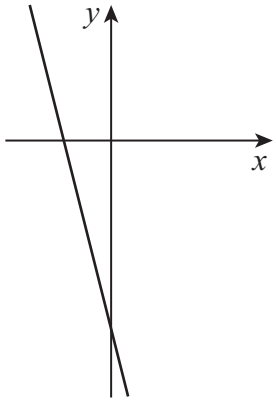

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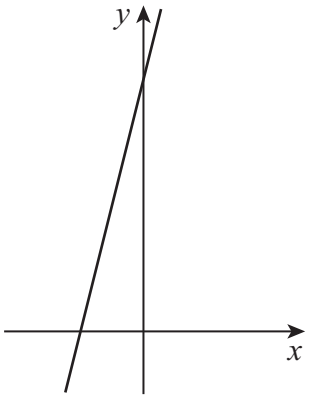

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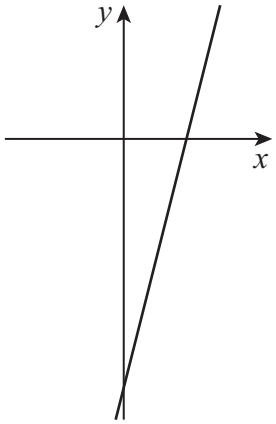

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5 Find an equation of the tangent to the curve

$$y = (x - 2)^4$$

at the point where $x = 0$

[3 marks]

6 (a) Find the first two terms, in ascending powers of x, of the binomial expansion of

$$\left(1-\frac{x}{2}\right)^{\frac{1}{2}}$$

[2 marks]

6 (b) Hence, for small values of x, show that

$$\sin 4x+\sqrt{\cos x}\approx A+Bx+Cx^2$$

where A, B and C are constants to be found.

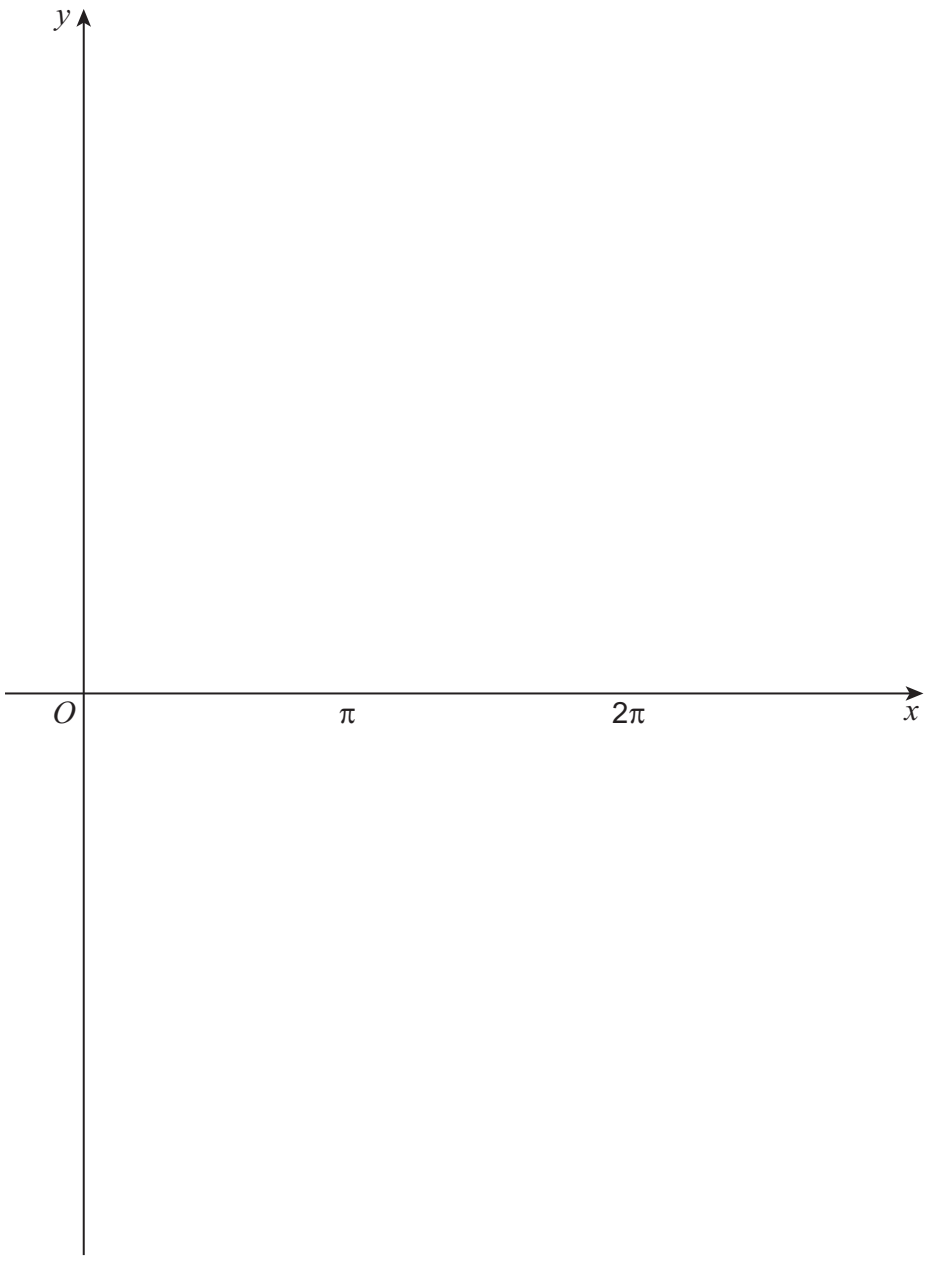
[4 marks]

7 Sketch the graph of

$$y = \cot\left(x - \frac{\pi}{2}\right)$$

for $0 \leq x \leq 2\pi$

[3 marks]



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8 The lines L_1 and L_2 are parallel.

L_1 has equation

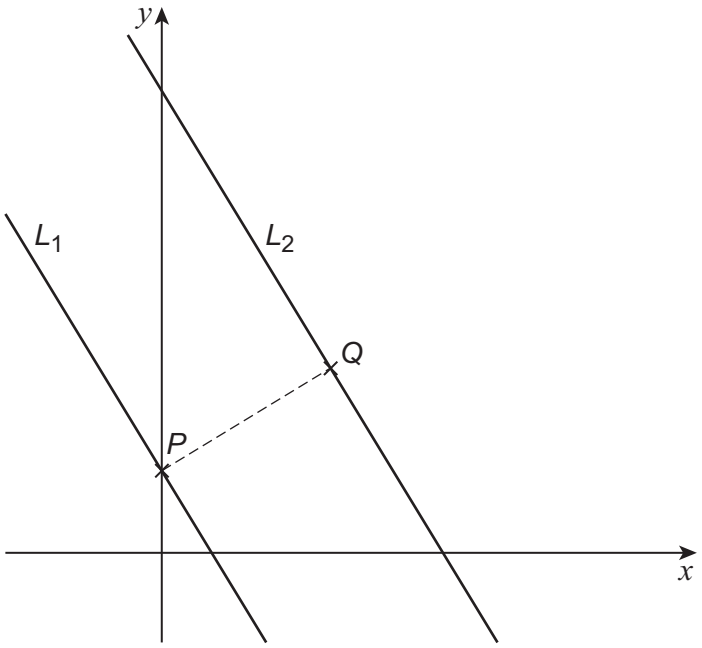
$$5x + 3y = 15$$

and L_2 has equation

$$5x + 3y = 83$$

L_1 intersects the y-axis at the point P.

The point Q is the point on L_2 closest to P, as shown in the diagram.



8 (a) (i) Find the coordinates of Q.

[5 marks]

8 (a) (ii) Hence show that $PQ = k\sqrt{34}$, where k is an integer to be found.

[2 marks]

8 (b) A circle, C, has centre $(a, -17)$.

L_1 and L_2 are both tangents to C.

8 (b) (i) Find a.

[2 marks]

8 (b) (ii) Find the equation of C.

[2 marks]

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9 The first three terms of an arithmetic sequence are given by

$$2x + 5 \qquad 5x + 1 \qquad 6x + 7$$

9 (a) Show that $x = 5$ is the only value which gives an arithmetic sequence.

[3 marks]

9 (b) (i) Write down the value of the first term of the sequence.

[1 mark]

9 (b) (ii) Find the value of the common difference of the sequence.

[1 mark]

9 (c) The sum of the first N terms of the arithmetic sequence is S_N where

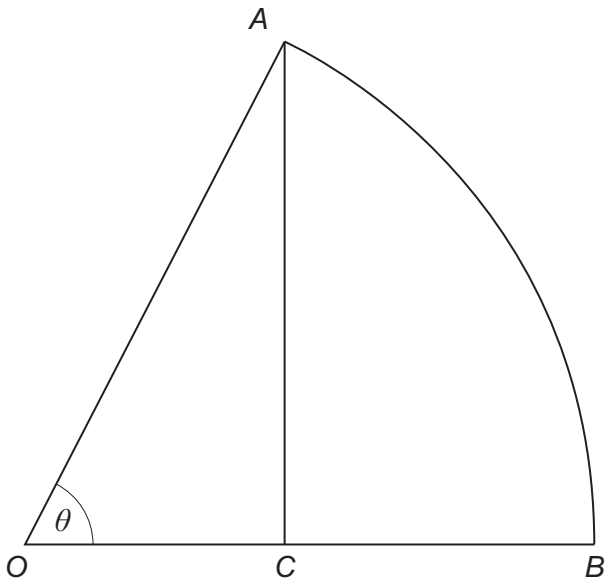
$$S_N < 100\,000$$

$$S_{N+1} > 100\,000$$

Find the value of N.

[4 marks]

10 The diagram shows a sector of a circle *OAB*.



The point *C* lies on *OB* such that *AC* is perpendicular to *OB*.

Angle *AOB* is θ radians.

10 (a) Given the area of the triangle *OAC* is half the area of the sector *OAB*, show that

$$\theta = \sin 2\theta$$

[4 marks]

10 (b) Use a suitable **change of sign** to show that a solution to the equation

$$\theta = \sin 2\theta$$

lies in the interval given by $\theta \in \left[\frac{\pi}{5}, \frac{2\pi}{5}\right]$

[2 marks]

Question 10 continues on the next page

10 (c) The Newton-Raphson method is used to find an approximate solution to the equation

$$\theta = \sin 2\theta$$

10 (c) (i) Using $\theta_1 = \frac{\pi}{5}$ as a first approximation for θ apply the Newton-Raphson method twice to find the value of θ_3

Give your answer to three decimal places.

[3 marks]

10 (c) (ii) Explain how a more accurate approximation for θ can be found using the Newton-Raphson method.

[1 mark]

10 (c) (iii) Explain why using $\theta_1 = \frac{\pi}{6}$ as a first approximation in the Newton-Raphson method does not lead to a solution for θ.

[2 marks]

Turn over for the next question

11 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 + (b+2)x^2 + 2(b+2)x + 8$$

where b is a constant.

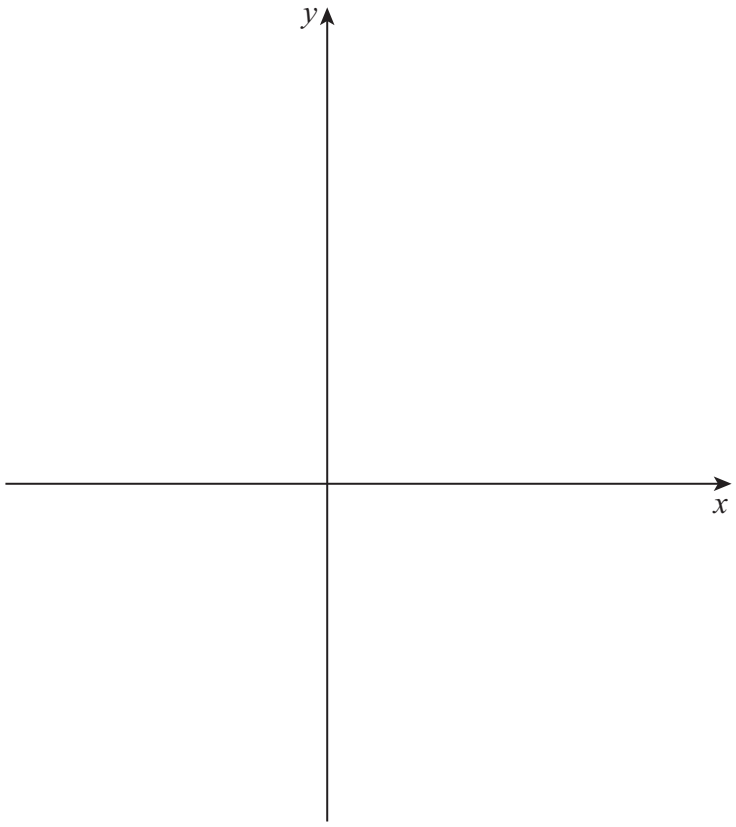
11 (a) Use the factor theorem to prove that $(x+2)$ is a factor of $\mathrm{p}(x)$ for **all** values of b.

[3 marks]

11 (b) The graph of $y = \mathrm{p}(x)$ meets the x-axis at exactly two points.

11 (b) (i) Sketch a possible graph of $y = \mathrm{p}(x)$

[3 marks]



11 (b) (ii) Given $\mathrm{p}(x)$ can be written as

$$\mathrm{p}(x) = (x + 2)(x^2 + bx + 4)$$

find the value of b.

Fully justify your answer.

[4 marks]

Turn over for the next question

12 (a) A geometric sequence has first term 1 and common ratio $\frac{1}{2}$

12 (a) (i) Find the sum to infinity of the sequence.

[2 marks]

12 (a) (ii) Hence, or otherwise, evaluate

$$\sum_{n=1}^{\infty} (\sin 30°)^n$$

[2 marks]

12 (b) Find the smallest positive exact value of θ, in **radians**, which satisfies the equation

$$\sum_{n=0}^{\infty} (\cos\theta)^n = 2 - \sqrt{2}$$

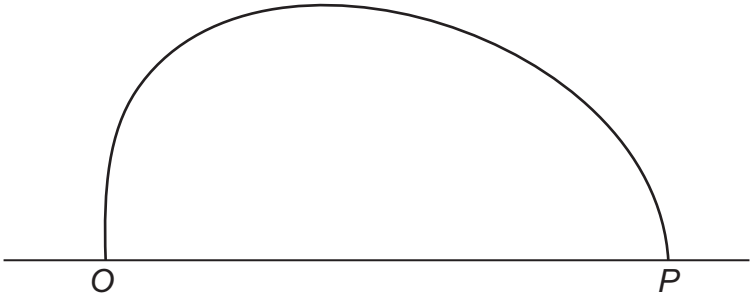
[4 marks]

Turn over for the next question

13 **Figure 2** shows the approximate shape of the vertical cross section of the entrance to a cave. The cave has a horizontal floor.

The entrance to the cave joins the floor at the points O and P.

Figure 2



Garry models the shape of the cross section of the entrance to the cave using the equation

$$x^2 + y^2 = a\sqrt{x} - y$$

where a is a constant, and x and y are the horizontal and vertical distances respectively, in metres, measured from O.

13 (a) The distance OP is 16 metres.

Find the value of a that Garry should use in the model.

[2 marks]

13 (b) Show that the maximum height of the cave above *OP* is approximately 10.5 metres.

[6 marks]

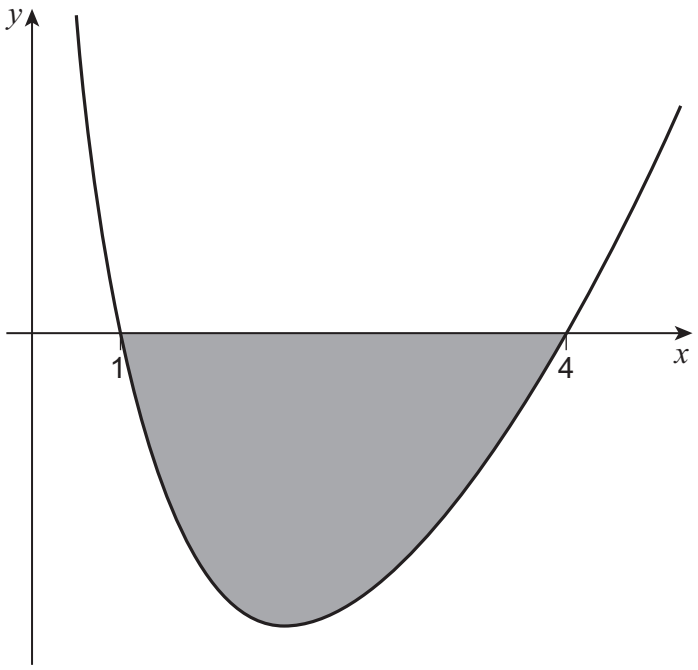
13 (c) Suggest one limitation of the model Garry has used.

[1 mark]

14 The region bounded by the curve

$$y = (2x - 8)\ln x$$

and the x-axis is shaded in the diagram below.



14 (a) Use the trapezium rule with **5 ordinates** to find an estimate for the area of the shaded region.

Give your answer correct to three significant figures.

[3 marks]

14 (b) Show that the exact area is given by

$$32\ln 2 - \frac{33}{2}$$

Fully justify your answer.

[6 marks]

15 (a) Given that

$$y = \text{cosec}\,\theta$$

15 (a) (i) Express y in terms of $\sin\theta$.

[1 mark]

15 (a) (ii) Hence, prove that

$$\frac{\mathrm{d}y}{\mathrm{d}\theta} = -\,\text{cosec}\,\theta\,\cot\theta$$

[3 marks]

15 (a) (iii) Show that

$$\frac{\sqrt{y^2-1}}{y} = \cos\theta \qquad \text{for } 0 < \theta < \frac{\pi}{2}$$

[3 marks]

15 (b) (i) Use the substitution

$$x = 2\,\text{cosec}\,u$$

to show that

$$\int \frac{1}{x^2\sqrt{x^2-4}}\,\mathrm{d}x \qquad \text{for } x > 2$$

can be written as

$$k\int \sin u\ \mathrm{d}u$$

where k is a constant to be found.

[6 marks]

15 (b) (ii) Hence, show

$$\int \frac{1}{x^2\sqrt{x^2-4}}\,\mathrm{d}x = \frac{\sqrt{x^2-4}}{4x} + c \qquad \text{for } x > 2$$

where c is a constant.

[3 marks]

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